DETECTION OF SPARSE RANDOM SIGNALS USING COMPRESSIVE MEASUREMENTS

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ABSTRACT

We consider the problem of detecting a sparse random signal from the compressive measurements without reconstructing the signal. Using a subspace model for the sparse signal where the signal parameters are drawn according to Gaussian law, we obtain the detector based on Neyman-Pearson criterion and analytically determine its operating characteristics when the signal covariance is known. These results are extended to situations where the covariance cannot be estimated. The results can be used to determine the number of measurements needed for a particular detector performance and also illustrate the presence of an optimal support for a given number of measurements.

***Index Terms*—** Compressive sensing, signal detection, sparse Gaussian vector, binary hypothesis, receiver operating characteristic

1. INTRODUCTION

The topic of *under-sampling*, where the objective is to achieve a lower sampling rate than the Shannon/Nyquist sampling rate, continues to attract significant attention. In recent times, compressive sampling (CS) has turned out to be the under-sampling mechanism that provides a *systematic* approach to acquire signal samples when the signal under acquisition is sparse in an appropriate domain [1], [2], [3]. The emerging theory of CS uses a system model comprising of an under-determined set of linear equations. In this standard CS setup, a higher dimensional sparse signal vector is sampled by a linear sampling matrix to a lower dimensional measurement vector (under-sampled measurement vector). The task, in general, is to reconstruct the higher dimensional sparse signal vector from the lower dimensional measurement vector. Following the rich literature, [4], [5], [6], [7], we note a significant effort towards development of reconstruction algorithms and their performance analysis.

In addition to signal reconstruction, it is important to draw inference from the measured data. Inference tasks include detection and classification of signals and also estimation of a signal parameters. Therefore, in the CS framework, it is interesting to investigate inference problems using lower dimensional CS measurement vector. A rational question is how to find a trade-off between level of under-sampling and inference performances. In this regard, an approach was proposed in [8] where the inferences (including detection) are performed directly using the CS measurement vector and without any reconstruction. An application of such a compressive detector for target detection in wireless sensor networks is considered in [9]. Employing a linear subspace model for sparse signals, a measurement matrix exploiting the model to reduce the number of measurements for a given performance is devised in [10]. Different detection algorithms, depending on the information available about the subspace and the sparse signal, have also been provided in [10]. On the other hand, an approach of using reconstruction algorithms along-with detection methods has been developed in [11], [12]. In all these works, the sparse signal is considered to be deterministic, and possibly, unknown. However, in many applications the underlying signal is inherently random. A typical example is spectrum sensing in cognitive OFDM (Orthogonal Frequency Division Multiplexing) systems where the task is to detect primary user transmissions that are inherently random in content. The literature on detection of random signals under the CS framework is limited, for e.g., detection of stochastic process using optimized projections and classification techniques is considered in [13]. Further, the approach of [13] is different to the standard rule based one. These motivate a study of detecting random signals using the CS framework based on the traditional rule based approach.

In our work, similar to [10], the sparse signal is described using a linear subspace model and its detection is performed based on the compressive measurements. In addition, we model the sparse signal to be random with values drawn according to a Gaussian distribution. For such a model, we obtain optimal detection rules, sufficient statistics and the receiver operating characteristics (ROC) when the signal covariance is known at the receiver [1]. This analysis allows us to study the effect of compressive measurements on the detection performance and determine the number of measurements guaranteeing a certain performance. To cater to the situation when estimation of signal covariance is not feasible, we obtain a sub-optimal detector and numerically study its performance. Further, we illustrate the presence of an optimal support when using the CS detector for a given number of measurements. This needs to be contrasted with traditional CS reconstruction algorithms whose performance degrades with an increase in the support of the sparse signal for a given number of measurements.

Notations: Uppercase boldface letters ($\mathbf{A}$) denote matrices, lowercase underlined letters ($\underline{x}$) represent vectors with $\underline{x}(k)$ is the kth element of a $\underline{x}$. Transpose operation is denoted by $(\cdot)^T$, $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\|\cdot\|$ denotes the l_2 norm [14] and $\lambda_i(\mathbf{A})$ is the ith eigenvalue of $\mathbf{A}$. For positive semi-definite matrices, these eigenvalues are arranged in the descending order. $\underline{x} \sim \mathcal{N}(\underline{\mu}, \mathbf{C})$ denotes a real Gaussian vector $\underline{x}$ with mean $\underline{\mu}$ and covariance $\mathbf{C}$.

2. DETECTOR WITH COMPRESSIVE MEASUREMENTS

2.1. Linear Subspace Model

We consider a detection problem with the aim of distinguishing the two hypotheses, $\mathcal{H}_0$ and $\mathcal{H}_1$ based on compressive measurements. In particular, these hypotheses are characterized as,

$$\begin{aligned} \mathcal{H}_0 &: \quad \underline{y} = \mathbf{\Phi}\underline{\eta}, \\ \mathcal{H}_1 &: \quad \underline{y} = \mathbf{\Phi}\left(\mathbf{T}\underline{x} + \underline{\eta}\right), \end{aligned} \tag{1}$$

[1]A Basis Pursuit De-noising algorithm, for example, can be used to estimate this information using training [10] for deterministic signals

where $\underline{x}$ is a $K \times 1$ real random vector, $\mathbf{T}$ is a $N \times K$ column orthonormal basis matrix (i. e, $\mathbf{T}^T\mathbf{T} = \mathbf{I}_K$), $\mathbf{\Phi}$ is a $M \times N$ compressive measurement matrix, $\underline{y}$ is the $M \times 1$ measurement vector and $\underline{\eta}$ is the $N \times 1$ receiver noise. We consider $K << N, M < N$ and assume a Gaussian model for signal and noise : $\underline{x} \sim \mathcal{N}(\underline{0}, \sigma_x^2\mathbf{I}_K), \underline{\eta} \sim \mathcal{N}(\underline{0}, \mathbf{I}_N)$. Further, $\underline{\eta}, \underline{x}$ are considered independent. In this model, the sparse signal $\underline{s} = \mathbf{T}\underline{x}$ lies in a K-dimensional subspace, $\mathcal{X}$, of $\mathbb{R}^N$ characterized by $\mathbf{T}$. While this model is similar to [10], it characterizes $\underline{x}$ as random unlike the deterministic formulation of [10].

Let $\mathbf{P}_\Phi = \mathbf{\Phi}^T\left(\mathbf{\Phi}\mathbf{\Phi}^T\right)^{-1}\mathbf{\Phi}$, be the projection matrix associated with $\mathbf{\Phi}$. It is assumed that $\sqrt{\frac{N}{M}}\mathbf{P}_\Phi$, provides a $\delta-$ stable embedding of $\{\mathcal{X}, 0\}$ [8]. Alternatively, with a probability of at least $1 - 2e^{-cM\delta^2}$ (for some positive c), $\mathbf{P}_\Phi$ satisfies,

$$(1-\delta)\frac{M}{N}\|\underline{s}\|^2 \leq \|\mathbf{P}_\Phi\underline{s}\|^2 \leq (1+\delta)\frac{M}{N}\|\underline{s}\|^2, \quad (2)$$

for all $\underline{s} \in \mathcal{X}$. Motivated by the discussion following equation (19), [8], and ease of analysis, we choose $\mathbf{\Phi}$ to contain orthonormal rows spanning a random subspace (i. e, $\mathbf{\Phi}\mathbf{\Phi}^T = \mathbf{I}_M$). For such a matrix, $\mathbf{P}_\Phi = \mathbf{\Phi}^T\mathbf{\Phi}$, and (2) holds with $\|\mathbf{P}_\Phi\underline{s}\|^2$ replaced by $\|\mathbf{\Phi}\underline{s}\|^2$ [8].

2.2. Compressive Detector

Using the Neyman-Pearson criterion, the signal is detected if,

$$\frac{p(\underline{y};\mathcal{H}_1)}{p(\underline{y};\mathcal{H}_0)} \geq \gamma_{NP}, \quad (3)$$

where $p(\underline{y};\mathcal{H}_k)$ is the probability density function (pdf) under $\mathcal{H}_k$ and γ_{NP} is chosen based on the receiver operating point. For the linear subspace model of Section 2.1, we have,

$$\begin{aligned} p(\underline{y};\mathcal{H}_1) &= \mathcal{N}\left(\underline{0}, \mathbf{C}_n + \mathbf{C}_s\right), \\ p(\underline{y};\mathcal{H}_0) &= \mathcal{N}\left(\underline{0}, \mathbf{C}_n\right), \end{aligned} \quad (4)$$

where $\mathbf{C}_s$ and $\mathbf{C}_n$ are the covariance matrices of $\mathbf{\Phi}\mathbf{T}\underline{x}$ and $\underline{\eta}$ respectively. Exploiting the statistics of $\underline{x}, \underline{\eta}$ and row orthonormality of $\mathbf{\Phi}$, it follows that, $\mathbf{C}_s = \sigma_x^2\mathbf{\Phi}\mathbf{T}\mathbf{T}^T\mathbf{\Phi}^T$ and $\mathbf{C}_n = \mathbf{I}_M$.

Let $\mathbf{C}_s = \mathbf{V}_s\mathbf{\Lambda}_s\mathbf{V}_s^T$ denote the eigen-value decomposition of $\mathbf{C}_s$ and $\underline{z} = \mathbf{V}_s^T\underline{y}$. Since $\mathrm{rank}(\mathbf{C}_s) \leq K$, $\mathbf{\Lambda}_s$ contains utmost K non-zero diagonal elements. Using these, it can be shown after simple algebra that (3) reduces to the following detection rule,

$$T(\underline{z}) \geq \gamma'. \quad (5)$$

Here $T(\underline{z}) = \underline{z}^T\mathbf{\Lambda}_s\left(\mathbf{I}_M + \mathbf{\Lambda}_s\right)^{-1}\underline{z}$ and $\gamma' = (2\ln\gamma_{NP} + \ln\det\left(\mathbf{I}_M + \mathbf{C}_s\right))$. Thus, $T(\underline{z})$ is the sufficient statistic for the detection problem and $P_{FA} = Prob\{T(\underline{z}) \geq \gamma'; \mathcal{H}_0\}$ denotes the probability of false alarm, while $P_D = Prob\{T(\underline{z}) \geq \gamma'; \mathcal{H}_1\}$ denotes detection probability. Evaluating P_D and P_{FA} requires the pdf of $T(\underline{z})$. Towards this, exploiting unitary $\mathbf{V}_s$, we can show,

$$p(\underline{z};\mathcal{H}_k) = \mathcal{N}\left(\underline{0}, \mathbf{I}_M + k\mathbf{\Lambda}_s\right), k = 0, 1. \quad (6)$$

Recalling that $\lambda_i(\mathbf{C}_s)$ is nothing but the ith diagonal entry of $\mathbf{\Lambda}_s$, we have $T(\underline{z}) = \sum_{i=1}^K\left(\frac{[\underline{z}(i)]^2\lambda_i(\mathbf{C}_s)}{1+\lambda_i(\mathbf{C}_s)}\right)$. Thus, $T(\underline{z})$ involves weighted sum of independent $\chi^2(1)$ variables and its pdf can be obtained using the standard results in literature [15]. This leads to, P_{FA} and P_D being evaluated as,

$$P_{FA} = \frac{1}{2\pi}\int_{\gamma'}^{\infty}\int_{-\infty}^{-\infty}\left(\prod_{i=1}^K\frac{1}{\sqrt{1-\frac{2j\lambda_i(\mathbf{C}_s)\omega}{1+\lambda_i(\mathbf{C}_s)}}}\right)e^{-j\omega r}\,d\omega\,dr, \quad (7)$$

$$P_D = \frac{1}{2\pi}\int_{\gamma'}^{\infty}\int_{-\infty}^{-\infty}\left(\prod_{i=1}^K\frac{1}{\sqrt{1-2j\lambda_i(\mathbf{C}_s)\omega}}\right)e^{-j\omega r}\,d\omega\,dr. \quad (8)$$

In the following, we further analyze the detector ROC by simplifying the expressions for P_{FA} and P_D.

3. PERFORMANCE ANALYSIS

3.1. Complete Measurements

When all the N measurements are available, i.e., $M = N$, we have, $\mathbf{\Phi}^T\mathbf{\Phi} = \mathbf{\Phi}\mathbf{\Phi}^T = \mathbf{I}_N$. This results in a column orthogonal $\mathbf{\Phi}\mathbf{T}$. As a result, using standard matrix identities, it can be shown that $\lambda_i(\mathbf{C}_s) = \sigma_x^2, i \in [1, K]$ and $\lambda_i(\mathbf{C}_s) = 0, i > K$ [14]. Using this in (7) and (8), and from the well known results on cdf of chi-squared variables [16], we have,

$$P_{FA} = \frac{\Gamma(\frac{K}{2}, \frac{(1+\sigma_x^2)\gamma'}{2\sigma_x^2})}{\Gamma(\frac{K}{2})}, \quad P_D = \frac{\Gamma(\frac{K}{2}, \frac{\gamma'}{2\sigma_x^2})}{\Gamma(\frac{K}{2})}, \quad (9)$$

where, $\Gamma(s, x)$ is the upper incomplete Gamma function and $\Gamma(s)$ is the standard Gamma function [16]. When K to be even, we further have, $\Gamma(\frac{K}{2}, \theta) = \Gamma(K/2)e^{-\theta}\sum_{n=0}^{\frac{K}{2}-1}\frac{\theta^n}{n!}$ (Chapter 6 of [16]). Assuming K to be even, P_{FA} and P_D of (9) reduce to

$$P_{FA} = e^{-\theta}\sum_{n=0}^{\frac{K}{2}-1}\frac{\theta^n}{n!}, \quad \theta = \frac{(1+\sigma_x^2)\gamma'}{2\sigma_x^2} \quad (10)$$

$$P_D = e^{-\theta}\sum_{n=0}^{\frac{K}{2}-1}\frac{\theta^n}{n!}, \quad \theta = \frac{\gamma'}{2\sigma_x^2}, \quad (11)$$

where γ' is used to choose an operating point on the ROC. Signal covariance information is needed to obtain the K useful components of $\underline{z}(i)$ needed in the evaluation of $T(\underline{z})$.

Since P_D, P_{FA} decrease with increasing γ', it is follows that $e^{-\theta}\sum_{n=0}^{\frac{K}{2}-1}\frac{\theta^n}{n!}$ reduces with increase in $\theta > 0$. This would be used to derive bounds in the next section.

3.2. Compressive Measurements

We now analyze the detector ROC when $M < N$ by deriving bounds and approximations on P_{FA} and P_D.

3.2.1. Bounds on P_D

Recalling $T(\underline{z}) = \sum_{i=1}^K\left(\frac{[\underline{z}(i)]^2\lambda_i(\mathbf{C}_s)}{1+\lambda_i(\mathbf{C}_s)}\right)$, it immediately follows that, $T_{D,l}(\underline{z}) \leq T(\underline{z}) \leq T_{D,u}(\underline{z})$, where $T_{D,u}(\underline{z}) = \lambda_{max}\sum_{i=1}^K\frac{[\underline{z}(i)]^2}{1+\lambda_i(\mathbf{C}_s)}$ and $T_{D,l}(\underline{z}) = \lambda_{min}\sum_{i=1}^K\frac{[\underline{z}(i)]^2}{1+\lambda_i(\mathbf{C}_s)}$ (here, $\lambda_{max} = \max_i\lambda_i(\mathbf{C}_s), \lambda_{min} = \min_{i\in[1,K]}\lambda_i(\mathbf{C}_s)$). This leads to $Prob\{T_{D,l}(\underline{z}) > \gamma'; \mathcal{H}_1\} \leq P_D \leq Prob\{T_{D,u}(\underline{z}) > \gamma'; \mathcal{H}_1\}$. Using ideas similar to the Section 3.1 and letting $\theta_l = \frac{\gamma'}{2\lambda_{min}}, \theta_u = \frac{\gamma'}{2\lambda_{max}}$, these probabilities can be evaluated to yield,

$$e^{-\theta_l}\sum_{n=0}^{\frac{K}{2}-1}\frac{\theta_l^n}{n!} \leq P_D \leq e^{-\theta_u}\sum_{n=0}^{\frac{K}{2}-1}\frac{\theta_u^n}{n!}. \quad (12)$$

It now remains to estimate $\lambda_{max}, \lambda_{min}$. Towards this, we exploit (2) and the fact that $\mathbf{P}_\Phi = \mathbf{\Phi}\mathbf{\Phi}^T$. Since (2) holds for all $\underline{s} = \mathbf{T}\underline{x}$, and that $\|\mathbf{T}\underline{x}\| = \|\underline{x}\|$ as $\mathbf{T}$ is column orthonormal, we have,

$$(1-\delta)\tfrac{M}{N}\|\underline{x}\|^2 \leq \underline{x}^T\mathbf{T}^T\mathbf{\Phi}^T\mathbf{\Phi}\mathbf{T}\underline{x} \leq (1+\delta)\tfrac{M}{N}\|\underline{x}\|^2, \quad (13)$$

$$(1-\delta)\tfrac{M}{N} \leq \lambda_i(\mathbf{T}^T\mathbf{\Phi}^T\mathbf{\Phi}\mathbf{T}) \leq (1+\delta)\tfrac{M}{N}, 1 \leq i \leq K. \quad (14)$$

Equation (14) follows from the Courant-Fischer theorem [14]. Noting that the non-zero eigenvalues $\mathbf{AB}$ and $\mathbf{BA}$ are identical, we have, $\lambda_i(\mathbf{C}_s) = \sigma_x^2 \lambda_i(\mathbf{T}^T\mathbf{\Phi}^T\mathbf{\Phi}\mathbf{T}), i \in [1, K]$ and hence $\lambda_{max} < \frac{M}{N}\sigma_x^2(1+\delta)$ and $\lambda_{min} > \sigma_x^2(1-\delta)\frac{M}{N}$. Noting that $e^{-\theta}\sum_{n=0}^{p}\frac{\theta^n}{n!}$ is inversely proportional to θ, bounds on P_D are obtained by using the aforementioned inequalities on $\lambda_{max}, \lambda_{min}$. In particular, using $\theta_l = \frac{N\gamma'}{2M\sigma_x^2(1-\delta)}, \theta_u = \frac{N\gamma'}{2M\sigma_x^2(1+\delta)}$ in (12) yields the necessary bounds on P_D. When $\delta \approx 0$, P_D has the following approximation,

$$P_D \approx e^{-\theta}\sum_{n=0}^{\frac{K}{2}-1}\frac{\theta^n}{n!}, \quad \theta = \frac{N\gamma'}{2M\sigma_x^2}. \tag{15}$$

3.2.2. Bounds on P_{FA}

Letting $T_{FA,u}(\underline{z}) = \frac{\lambda_{max}}{1+\lambda_{max}}\sum_{i=1}^{K}[\underline{z}(i)]^2$ and $T_{FA,l}(\underline{z}) = \frac{\lambda_{min}}{1+\lambda_{min}}\sum_{i=1}^{K}[\underline{z}(i)]^2$, it can be shown that $T_{FA,l}(\underline{z}) \le T(\underline{z}) \le T_{FA,u}(\underline{z})$. Following the methodology pursued in Section 3.2.1, we can obtain bounds similar to (12) with $\theta_l = \frac{\gamma'(1+\sigma_x^2(1-\delta)\frac{M}{N})}{2\sigma_x^2(1-\delta)\frac{M}{N}}$ and $\theta_u = \frac{\gamma'(1+\sigma_x^2(1+\delta)\frac{M}{N})}{2\sigma_x^2(1+\delta)\frac{M}{N}}$. Further, when $\delta \approx 0$,

$$P_{FA} \approx e^{-\theta}\sum_{n=0}^{\frac{K}{2}-1}\frac{\theta^n}{n!}, \quad \theta = \frac{N(1+\sigma_x^2)\gamma'}{2M\sigma_x^2}. \tag{16}$$

3.2.3. Observations

Remark 1 Tightness of bounds: *When $M = N$, it can be shown that the upper and lower bounds on P_D in (12) are identical (a similar result holds for P_{FA}) and the approximations in (15), (16) become equalities. For other M, tightness of the bounds depend on the spread of $\lambda_i(\mathbf{T}^T\mathbf{\Phi}^T\mathbf{\Phi}\mathbf{T})$, which, in turn, is governed by the choice of $\mathbf{\Phi}$. Better results are obtained by designing $\mathbf{\Phi}$ for a smaller δ.*

Remark 2 Performance Dependence on M: *Equations (15) and (11) indicate that the detection performance deteriorates with decrease in M when other parameters are fixed (recall that $e^{-\theta}\cdot\sum_{n=0}^{p}\frac{\theta^n}{n!}$ is inversely proportional to θ). In particular, the compressive detector needs an additional signal power of about $10\log\frac{N}{M}\,dB$ to achieve a performance similar to traditional detectors. A similar inference can be obtained from [8]. On the other hand, from a design perspective, the approximations provide a framework for numerically obtaining M required for a certain performance.*

Remark 3 Performance Dependence on K: *When K increases for a fixed M and N, the sparsity level decreases, and CS theory indicates poor reconstruction performance. However, when detecting a signal with K independent components, increasing K provides additional degrees of freedom (diversity), thereby hinting at improved performance. This predicts a set of values of K that optimize the performance for a given M, N. Simulations corroborate this behavior and obtaining optimal K analytically is left for future work.*

Remark 4 Knowledge of $\mathbf{T}$ at the detector: *When the signal subspace $\mathbf{T}$ is known, the use of $\mathbf{\Phi} = \mathbf{G}\mathbf{T}^T$ is suggested in [10], where $\mathbf{G}$ is a $M \times K$ compressive sensing matrix with $M < K$. This choice allows for a much lower M for a given performance compared to the original scheme. For appropriate orthonormal $\mathbf{G}$ satisfying the required isometry properties, expressions derived earlier continue to hold with K replacing N.*

3.2.4. Detection without information about Signal Covariance

In the earlier development, signal covariance was implicitly assumed at the detector when evaluating $T(\underline{z})$. Alternatively, having access to $\mathbf{T}$ and σ_x^2 also suffices. This information can be obtained from measurements in a straightforward manner. To address the scenario when such an estimation is not possible, either due to limited data record or complexity constraints, we develop a sub-optimal detector where the computation of the test statistic does not require information about $\mathbf{C}_s$.

Let $S(\underline{y}) = \sum_{i=1}^{M}[\underline{y}(i)]^2$ and the detector decides $\mathcal{H}_1$ if $S(\underline{y}) > \beta$. Since $\mathbf{V}_s$ is unitary, $\sum_{i=1}^{M}[\underline{y}(i)]^2 = \sum_{i=1}^{M}[\underline{z}(i)]^2$. Using this, it can be shown that $S(\underline{y}) \ge T(\underline{z})$, thereby indicating a poorer P_D for the sub-optimal detector. We now provide expressions for P_{FA}, P_D, and the steps are omitted for lack of space. In particular, when M is even, it can be shown that,

$$P_{FA} = e^{-\theta}\sum_{n=0}^{\frac{M}{2}-1}\frac{\theta^n}{n!}, \quad \theta = \frac{\gamma'}{2\sigma_x^2}, \tag{17}$$

$$P_D = \frac{1}{2\pi}\int_{\gamma'}^{\infty}\int_{-\infty}^{-\infty}\left(\prod_{i=1}^{M}\frac{1}{\sqrt{1-2j(1+\lambda_i(\mathbf{C}_s))\omega}}\right)e^{-j\omega r}\,d\omega\,dr. \tag{18}$$

While P_{FA} has been evaluated in close-form, P_D requires numerical evaluation. While formulating the test-statistic does not warrant the knowledge of $\mathbf{C}_s$, determining the thresholds require information about signal statistics. For simplicity, it is assumed that the threshold is provided to the receiver by a central entity periodically. Simplifying the error expressions of this detector further as well as exploring other sub-optimal detectors is left for future work.

4. SIMULATIONS

In this section, we simulate the performance of detectors based on compressive measurements. In the figure (1), performance of the following are numerically depicted : (i) detector based on (5) with a $M \times N$ row orthonormal $\mathbf{\Phi}$ (termed as Full CSD), (ii) detector based on (5) with $\mathbf{\Phi} = \mathbf{G}\mathbf{T}^T$ [10] (termed as Reduced CSD) and (3) sub-optimal detector developed in Section 3.2.4. $\mathbf{G}$ and $\mathbf{\Phi}$ (for cases (1) and (3)) are obtained as orthonormal rows spanning the subspace generated by a matrix consisting of i.i.d $\mathcal{N}(0,1)$ variables (of appropriate dimensions). Further, $SNR = \sigma_x^2 = 10$ [2], $K = 10, N = 500, M = 200$ for Full CSD ($\frac{M}{N} = 0.4$), $M = 4$ for Reduced CSD ($\frac{M}{K} = 0.4$) and $M = 300$ for the suboptimal detector. The hypotheses are equally likely and the results are averaged over 500 realizations of $\mathbf{\Phi}$ and 1000 realizations of $\underline{x}$ per realization of $\mathbf{\Phi}$. Analytical results obtained from (15) and (16) for Full CSD case are also presented. Clearly, for the chosen parameters, Full and Reduced CS exhibit high P_D for a low P_{FA}. Figure (1) also illustrates that the analytical expressions and numerical evaluations show a high degree of similarity, thereby allowing the use of derived expressions for benchmarking. Further, the advantages of exploiting the signal covariance (reduced CSD) in terms of number of measurements and performance *vis-a-vis* the blind detector are also shown.

Figure (2) evaluates the P_D and P_{FA} expressions in (15), (16) for $N = 500, M = 200$ and various K. The total signal power is held constant, with $SNR = \sigma_x^2 = \frac{100}{K}$. Performance initially improves as K is increased from 4 to 12; it then remains mostly unchanged for $K \in [12, 20]$ and subsequently reduces, as indicated in Remark (3). In general, the optimal value(s) of K depends on factors including SNR, M, N and its derivation would be considered in future. We do not report the performance variations with M, partly

[2] measured per component

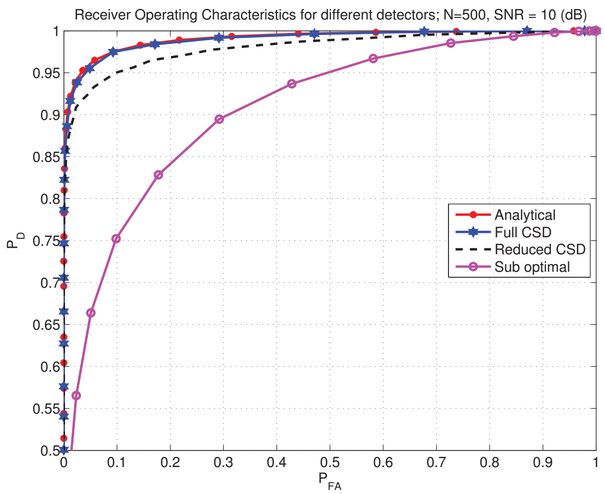

Fig. 1. Performance of various detectors

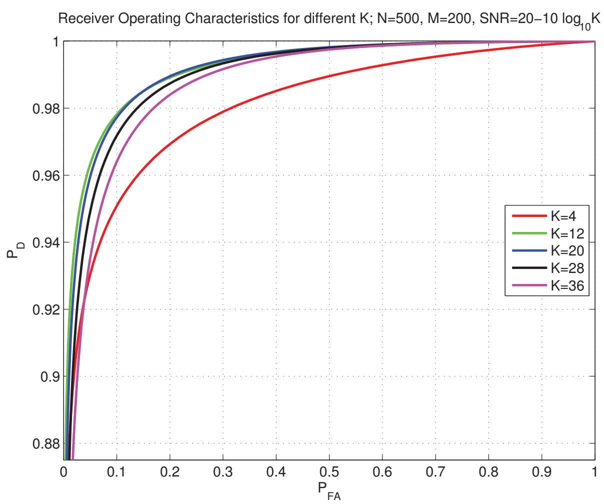

Fig. 2. Effect of K on the ROC based on approximations for P_D, P_{FA}.

due to the space constraints and also due to the availability of derived expressions that provide a close match to the actual performance.

5. CONCLUSIONS

A binary hypothesis problem of detecting a random signal in noise with compressive measurements is formulated. Focussing on the known signal covariance case, approximations for Receiver Operating Characteristics are derived. These approximations are shown, numerically, to emulate the actual performance, thereby providing insights into the effects of various parameters – number of measurements, sparse signal support, SNR – on performance. A sub-optimal detector blind to signal covariance is also devised followed by an evaluation of its performance. Apart from an understanding of the detector characteristics, the work indicates an optimal sparse signal support for a given number of measurements.

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