BEAMFORMING FOR MULTI-GROUP MULTICASTING WITH STATISTICAL CHANNEL STATE INFORMATION USING SECOND-ORDER CONE PROGRAMMING

Nils Bornhorst　　*Marius Pesavento*

Technische Universität Darmstadt, Merckstr. 25, D-64283 Darmstadt, Germany

ABSTRACT

We consider the problem of transmit beamforming in multi-group multicasting systems with covariance-based channel state information (CSI) available at the transmitter where the total transmitted power is minimized subject to quality-of-service (QoS) constraints at the receivers. Previous approaches for this problem are based on semidefinite relaxation (SDR) and require randomization and costly power scaling which is avoided in our approach. The proposed technique can be viewed as a non-trivial extension of the iterative second-order cone programming (SOCP) approach of [5], which is restricted to the case of instantaneous (rank-one) CSI, to the beamforming problem with higher-rank channel covariance matrices. Computer simulations reveal that the proposed technique exhibits superior performance in terms of total transmitted power at a reduced computational complexity as compared to the SDR method.

Index Terms— Multi-group multicasting, downlink beamforming, convex optimization, second-order cone programming, statistical channel state information

1. INTRODUCTION

In traditional cellular systems, multicasting has been considered as a task performed by efficient routing protocols at the network layer to enable, e.g., subscriber-based video streaming services. In emerging wireless networks, however, the broadcasting property of the wireless medium can be exploited to shift this task to the physical layer. The use of antenna arrays and CSI at the transmitter, which enables multicast beamforming, is provisioned by many wireless communication standards, such as, e.g., the Multimedia Broadcast Multicast Service (MBMS) in LTE-A. With multi-antenna techniques, more sophisticated transmission than in traditional broadcast radio becomes possible: Rather than radiating energy isotropically or with a fixed beam pattern, energy can be steered towards the subscribers with adaptive beams. Superimposing multiple beam patterns allows to multiplex different cochannel multicasting groups in space rather than in time or frequency, increasing the spectral efficiency of the system.

QoS based transmit beamformer designs which minimize the total transmitted power while guaranteeing a minimum received signal-to-interference-plus-noise ratio (SINR) for each user were first proposed in [1], [2] and [3]. The multiuser downlink scenario, where G independent data streams are transmitted to G mobile users, was treated in [1]. In [2], the broadcasting scenario, where the same stream is transmitted to M mobile users, also referred to as single group multicasting, was considered. Both beamforming scenarios mentioned above can be combined under the general multi-group multicasting framework of [3]. In this framework, the proposed QoS-based design yields a non-convex NP-hard optimization problem. Applying SDR, this problem can be approximated by a convex semidefinite programming (SDP) problem that is solvable in polynomial time [3]. However, the drawback of the relaxation approach is that it is not guaranteed that the solution of the SDP problem is feasible for the original problem. Therefore, randomization techniques (see, e.g., [2] and references therein) and proper power scaling are required to generate a feasible approximate solution from the solution of the SDP problem [3]. In the multi-group multicasting scenario, power scaling involves additional linear programming (LP) problems that need to be solved for each randomization instance.

In [4] and [5], we have proposed an alternative convex approximation approach for the multi-group multicasting scenario. We have approximated the original problem by a SOCP problem whose solution, provided it exists, is always feasible for the original problem. Hence, we can avoid the use of randomization and costly power control. As the SOCP problem is only an approximation of the original QoS beamformer design problem, we have proposed an iterative procedure in which the approximation is successively improved [5].

The two competing techniques, i.e., the SDR technique and the iterative SOCP approximation technique, have different requirements on the availability of CSI at the transmitter. The SDR technique of [3] can be applied in both cases, with instantaneous CSI available at the transmitter, as well as in the case where only second-order statistics of the CSI in form of channel covariance matrices are available. A major shortcoming of the iterative SOCP technique of [4] and [5] is that, despite its computational benefits, it is only applicable in the former case. However, instantaneous CSI is usually difficult to acquire, except for specific cases in which, e.g., channel reciprocity can be exploited in time division duplexing (TDD) systems to estimate the instantaneous downlink channels from the uplink [1]. In other scenarios, estimates of the instantaneous downlink channels need to be fed back from the users to the base station causing a prohibitive signaling overhead especially in fast fading scenarios. The second-order statistics of the downlink channel, however, usually evolve at a significantly lower rate than the corresponding instantaneous channel realizations. Therefore, beamformer designs based on channel covariance matrices are associated with a significantly reduced signaling overhead [1].

In this paper, we propose a non-trivial extension of our technique proposed in [5] to the case where statical CSI is available at the transmitter. In our extension, the aforementioned advantages over the SDR technique are preserved, i.e., the use of randomization techniques and costly power control is avoided. As in [5], we approximate the original problem by a convex SOCP problem and successively improve the approximate solution. A trivial way to keep the technique of [5] applicable would be to simply use only the principal component of the channel covariance matrix to formulate

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the SOCP approximation. However, if the covariance matrix is of higher rank, this approximation is poor and leads to unsatisfactory beamforming results. Therefore, we propose an l_1-norm approximation of the Euclidian norm which involves all eigenvectors in the optimization procedure. Similar to [5], we prove that our algorithm yields an improved solution in each iteration. Our simulation results reveal that the proposed technique is an attractive alternative to the state-of-the-art methods enjoying a significantly reduced computational complexity.

2. SIGNAL MODEL AND PROBLEM STATEMENT

Let us consider the wireless multicasting scenario of [3] where a single base station equipped with an array of N antenna elements transmits information symbols to M single-antenna mobile users. Let the $N \times 1$ vector of complex channel gains from each transmit antenna element to the receive antenna of user $m \in \{1, \dots, M\}$ be denoted as $\boldsymbol{h}_m$. Each user belongs to one out of $1 \leq G \leq M$ multicast groups, $\{\mathcal{G}_1, \dots, \mathcal{G}_G\}$, where $\mathcal{G}_k$ contains the indices of the users interested in the message of the kth multicasting stream, and $k \in \mathcal{K}$ where $\mathcal{K} = \{1, \dots, G\}$. We assume that each user can only receive a single stream and thus, the multicasting groups are disjoint, i.e., $\mathcal{G}_k \cap \mathcal{G}_l = \emptyset$, for $l \neq k$ and $\cup_k \mathcal{G}_k = \{1, \dots, M\}$. The general multi-group multicasting scenario contains the multiuser downlink case of [1] with $\mathcal{G}_k = k, \ \forall k \in \mathcal{K}$, and the broadcasting case of [2] with $M = 1$ as special cases.

Let $\boldsymbol{w}_k^H \in \mathbb{C}^N$, where $(\cdot)^H$ stands for the Hermitian transpose, be the vector of weighting coefficients applied to the N transmitting antenna elements to form a beam towards group k. The total power radiated by the transmitting antenna array can then be expressed as $\sum_{k=1}^{G} ||\boldsymbol{w}_k||_2^2$. Assuming instantaneous CSI at the receiver might not be valid in practice since feeding instantaneous CSI back to the base station causes a large signaling overhead. Therefore, we assume that at the transmitter, the CSI is only available in terms of the channel covariance matrices, i.e., $\boldsymbol{R}_m = \mathrm{E}\{\boldsymbol{h}_m \boldsymbol{h}_m^H\}, \ \forall m = 1, \dots, M$. The receiver noise powers $\{\sigma_m^2\}_{m=1}^M$ are also assumed to be known at the transmitter. With this information, we can design the multicast beamformer such that the total transmitted power is minimized while the SINR levels at the receivers are kept above prescribed QoS thresholds:

$$\begin{aligned} \min_{\{\boldsymbol{w}_k \in \mathbb{C}^N\}_{k=1}^G} \quad & \sum_{k=1}^{G} ||\boldsymbol{w}_k||_2^2 \\ \text{subject to} \quad & \frac{\boldsymbol{w}_k^H \boldsymbol{R}_m \boldsymbol{w}_k}{\sum_{l \neq k} \boldsymbol{w}_l^H \boldsymbol{R}_m \boldsymbol{w}_l + \sigma_m^2} \geq \gamma_m \\ & \forall m \in \mathcal{G}_k, \ \forall k, l \in \mathcal{K} \end{aligned} \tag{1}$$

As shown in [3], the problem (1) is non-convex and NP-hard. However, convex approximation techniques can be applied to find approximate solutions in polynomial time.

In [3], problem (1) is approximated by a SDP problem using the SDR technique. The relaxation consists in approximating the non-convex feasible set of problem (1) by a convex set which contains the original set as a subset. As a consequence, the solution of this relaxed problem may not be feasible for the original problem. Generating a feasible (but generally only sub-optimal) solution from the solution of the SDP problem can be achieved by means of the Gaussian randomization technique [2]. However, this necessitates to solve additional power control problems, as proposed in [3]. In specific, a few hundred candidate weight vectors are generated in the randomization technique and for each candidate, a LP problem is solved to obtain a feasible scale. Despite this rather costly procedure, it is not guaranteed that a feasible solution can always be obtained in this way.

In this paper, we tackle the problem from a different perspective. Rather than relaxing the original feasible set, we restrict it in the sense that we use a subset as a convex approximation of the original set. Then, we use an iterative algorithm where in each iteration, this approximation is adapted to the current solution. In this approach, the approximate solution is successively improved.

3. ITERATIVE POWER MINIMIZATION

For the special case of multiuser downlink beamforming, where $G = M$, with instantaneous CSI at the transmitter, an equivalent convex SOCP reformulation exists [1]. However, if only second-order statistics CSI can be assumed and/or we consider the general multi-group multicasting case, such an equivalent reformulation of the original problem is no longer possible. However, the originally non-convex problem can be approximated by a SOCP problem in this case.

Towards this end, let us rewrite the mth QoS constraint in problem (1) as

$$\sqrt{\boldsymbol{w}_k^H \boldsymbol{R}_m \boldsymbol{w}_k} \geq \sqrt{\gamma_m \left(\sum_{l \neq k} \boldsymbol{w}_l^H \boldsymbol{R}_m \boldsymbol{w}_l + \sigma_m^2 \right)}. \tag{2}$$

Further, let us introduce the matrices and the vector

$$\begin{aligned} \boldsymbol{H}_{k,m} &\triangleq \begin{bmatrix} \sigma_m^2 & \boldsymbol{0}^T \\ \boldsymbol{0} & (\boldsymbol{I} - \mathrm{diag}\{\boldsymbol{e}_k\}) \otimes \boldsymbol{R}_m \end{bmatrix}^{1/2} \\ \forall m \in \mathcal{G}_k, \ \forall k \in \mathcal{K}, & \qquad \boldsymbol{w} \triangleq [1, \boldsymbol{w}_1^T, \dots, \boldsymbol{w}_G^T]^T \end{aligned} \tag{3}$$

where $\otimes$ stands for the Kronecker product, $\boldsymbol{I}$ denotes the $G \times G$ identity matrix, $\boldsymbol{e}_k$ denotes the kth column of $\boldsymbol{I}$ and $\mathrm{diag}\{\boldsymbol{a}\}$ stands for a diagonal matrix whose diagonal entries are the elements of vector $\boldsymbol{a}$. Then, we can rewrite (2) as

$$\sqrt{\boldsymbol{w}_k^H \boldsymbol{R}_m \boldsymbol{w}_k} \geq \sqrt{\gamma_m} ||\boldsymbol{H}_{k,m} \boldsymbol{w}||_2. \tag{4}$$

In order to find a second-order cone (SOC) approximation of the constraint in (4), its left hand side (LHS) needs to be linearized. In the linearization proposed below, we exploit the fact that the l_1-norm of a vector $\boldsymbol{a} = [a_1, \dots, a_N]^T$ is a lower approximation of the corresponding Euclidian norm as follows.

$$\sqrt{\sum_{n=1}^{N} |a_n|^2} \geq \frac{1}{\sqrt{N}} \sum_{n=1}^{N} |a_n| \tag{5}$$

Note that in (5), equality holds if and only if all elements of $\boldsymbol{a}$ have the same magnitude. Further, note that this inequality still holds if we use only a subset of the elements in $\boldsymbol{a}$ for the approximation, i.e.,

$$\sqrt{\sum_{n=1}^{N} |a_n|^2} \geq \sqrt{\sum_{n \in \bar{\mathcal{N}}} |a_n|^2} \geq \frac{1}{\sqrt{\bar{N}}} \sum_{n \in \bar{\mathcal{N}}} |a_n| \tag{6}$$

where we define the index set $\bar{\mathcal{N}}$ as a subset of the original set $\mathcal{N}$, i.e., $\bar{\mathcal{N}} \subseteq \mathcal{N} = \{1, \dots, N\}$ and $\bar{N} = |\bar{\mathcal{N}}|$. To see that an appropriate choice of the index set can improve the approximation, let us consider the specific example of $|a_1| = 0$ and $|a_2| = \cdots = |a_N| > 0$ in which the approximation in (5) is not tight. However, if we define the subset $\bar{\mathcal{N}} = \{2, \dots, N\}$, the inequalities in (6) are satisfied with equality. With these observations, we next derive the proposed linearization of the LHS of (4).

For simplicity of notation, we omit the index m in the following discussion. Let us first use the eigenvalue decomposition of the channel covariance matrix given by $\boldsymbol{R} = \sum_{n=1}^{N} \lambda_n \boldsymbol{v}_n \boldsymbol{v}_n^H$ to rewrite the LHS of inequality (4) as

$$\sqrt{\sum_{n=1}^{N} \lambda_n \boldsymbol{w}_k^H \boldsymbol{v}_n \boldsymbol{v}_n^H \boldsymbol{w}_k} = \sqrt{\sum_{n=1}^{N} |\sqrt{\lambda_n} \boldsymbol{w}_k^H \boldsymbol{v}_n|^2} \geq \frac{1}{\sqrt{\bar{N}}} \sum_{n \in \bar{\mathcal{N}}} \sqrt{\lambda_n} |\boldsymbol{w}_k^H \boldsymbol{v}_n| \tag{7}$$

where λ_n and $\boldsymbol{v}_n$ are the nth eigenvalue and eigenvector of $\boldsymbol{R}$, respectively, and $\bar{\mathcal{N}}$ can be any subset of $\mathcal{N}$. Note that (6) has been used to obtain the approximation in (7). The final step in our linearization is similar to the procedure proposed in [5] and consists in approximating the magnitude of a complex scalar by its real-part, hence,

$$\frac{1}{\sqrt{\bar{N}}} \sum_{n \in \bar{\mathcal{N}}} \sqrt{\lambda_n} |\boldsymbol{w}_k^H \boldsymbol{v}_n| \geq \frac{1}{\sqrt{\bar{N}}} \sum_{n \in \bar{\mathcal{N}}} \sqrt{\lambda_n} \mathrm{Re}\{\boldsymbol{w}_k^H \boldsymbol{v}_n\}. \tag{8}$$

Note that the LHS of (8) is independent of a multiplication of each $\boldsymbol{v}_n$ by a rotation factor $\exp(-j\alpha_n)$, where α_n is the corresponding rotation angle. However, this multiplication changes the RHS of (8) and can be used to improve the approximation [5] as we describe later. Thus, we generalize the approximation (8) by replacing $\boldsymbol{v}_n$ with $\bar{\boldsymbol{v}}_n = \boldsymbol{v}_n \exp(-j\alpha_n)$.

We can summarize the linearization steps in (7) and (8) as

$$\sqrt{\boldsymbol{w}_k^H \boldsymbol{R} \boldsymbol{w}_k} \geq \frac{1}{\sqrt{\bar{N}}} \sum_{n \in \bar{\mathcal{N}}} \sqrt{\lambda_n} \mathrm{Re}\{\boldsymbol{w}_k^H \bar{\boldsymbol{v}}_n\}, \tag{9}$$

where $\bar{\mathcal{N}} \subseteq \mathcal{N}$ and $\bar{\boldsymbol{v}}_n = \boldsymbol{v}_n \exp(-j\alpha_n)$, and approximate the original, non-convex problem (1) by the following convex SOCP problem

$$\begin{aligned} \min_{t \in \mathbb{R},\, \boldsymbol{w} \in \mathbb{C}^{GN}} \quad & t \\ \text{subject to} \quad & t \geq ||\boldsymbol{w}||_2, \qquad w_1 = 1, \\ & \frac{1}{\sqrt{\bar{N}_m}} \sum_{n \in \bar{\mathcal{N}}_m} \sqrt{\lambda_{m,n}} \mathrm{Re}\{\boldsymbol{w}_k^H \bar{\boldsymbol{v}}_{m,n}\} \\ & \geq \sqrt{\gamma_m} ||\boldsymbol{H}_{k,m} \boldsymbol{w}||_2, \ \forall m \in \mathcal{G}_k, \ \forall k \in \mathcal{K} \end{aligned} \tag{10}$$

where $\bar{\mathcal{N}}_m \subseteq \mathcal{N}_m$, $\bar{\boldsymbol{v}}_{m,n} = \boldsymbol{v}_{m,n} \exp(-j\alpha_{m,n})$ and the index m has been reinstalled. Problem (10) can be solved efficiently using interior-point methods [6]. Note that there are some degrees of freedom in parameterizing this approximation. We can choose the subsets $\bar{\mathcal{N}}_m$ as well as the rotation angles $\alpha_{m,n}$. Thus, the goal is to find those $\bar{\mathcal{N}}_m$ and $\alpha_{m,n}$ which yield the best approximate solution. Based on the idea in [5], we propose to pursue this goal using an iterative algorithm where the SOCP approximation is successively improved.

Let $\{\boldsymbol{w}_{k,\mathrm{opt}}^{(i)}\}_{k=1}^{G}$ denote the solution to the SOCP problem (10) for $\bar{\mathcal{N}}_m = \bar{\mathcal{N}}_m^{(i)}$ and $\bar{\boldsymbol{v}}_{m,n} = \bar{\boldsymbol{v}}_{m,n}^{(i)}$ in the ith iteration of our algorithm. We can then find $\bar{\mathcal{N}}_m^{(i+1)}$ and $\bar{\boldsymbol{v}}_{m,n}^{(i+1)}$ of the next iteration such that the approximation in the next iteration is optimized around $\boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$ of the current iteration. This makes it possible to find an improved solution $\boldsymbol{w}_{k,\mathrm{opt}}^{(i+1)}$ in the next iteration which will be shown below. Therefore, the idea is to maximize the norm bound on the RHS of (9) for the current $\boldsymbol{w}_k = \boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$ of the ith iteration through the choice of $\bar{\mathcal{N}}_m^{(i+1)}$ and $\bar{\boldsymbol{v}}_{m,n}^{(i+1)}$ in order to make the approximation as tight as possible. Instead of finding the optimal $\bar{\mathcal{N}}_m^{(i+1)}$ using an exhaustive search, we can search more efficiently for an optimal $\bar{N}_m^{(i+1)} = |\bar{\mathcal{N}}_m^{(i+1)}|$ using the following deflation approach. For $\boldsymbol{w}_k = \boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$, we sort the eigenvalues and eigenvectors such that the summands in the LHS of (7) are arranged in descending order, i.e., $\lambda_{m,\kappa_1} |\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \boldsymbol{v}_{m,\kappa_1}|^2 \geq \ldots \geq \lambda_{m,\kappa_N} |\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \boldsymbol{v}_{m,\kappa_N}|^2$ where $\kappa_n \in \{1, \ldots, N\}$ and $\cup_n \kappa_n = \{1, \ldots, N\}$. Then, we choose $\bar{N}_m^{(i+1)}$ according to

$$\bar{N}_m^{(i+1)} = \underset{\bar{N}_m}{\operatorname{argmax}} \frac{1}{\sqrt{\bar{N}_m}} \sum_{n=1}^{\bar{N}_m} \sqrt{\lambda_{m,\kappa_n}} |\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \bar{\boldsymbol{v}}_{m,\kappa_n}^{(i)}| \tag{11}$$

so that the approximation in (7) becomes closest to the Euclidian norm. The optimal value $\bar{N}_m^{(i+1)}$ can efficiently be computed by successively reducing $\bar{N}_m$ in (11) starting form $\bar{N}_m = N$. After computing the optimal index set $\bar{\mathcal{N}}_m^{(i+1)}$, the optimal rotation angles $\alpha_{m,n}^{(i)}$ associated with the beamforming vector $\boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$ obtained in the ith iteration are obtained when $\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \bar{\boldsymbol{v}}_{m,n}^{(i+1)}$ becomes real and positive, hence for:

$$\bar{\boldsymbol{v}}_{m,n}^{(i+1)} = \bar{\boldsymbol{v}}_{m,n}^{(i)} \exp\left(-j\alpha_{m,n}^{(i)}\right) \tag{12}$$

where $\alpha_{m,n}^{(i)} = \angle(\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \bar{\boldsymbol{v}}_{m,n}^{(i)})$. In this case, the inequality (8) is satisfied with exact equality for $\boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$ and $\bar{\boldsymbol{v}}_{m,n}^{(i+1)}$ [5]. For the propose algorithm summarized in Table 1, the following lemma applies which guarantees the convergence to a local optimum of the original problem (1).

Lemma 1: The iterative procedure summarized in Table 1 yields an improved approximate solution in each iteration until the active constraints remain unchanged in consecutive iterations.

Proof: Let us first proof that for $\boldsymbol{w}_k = \boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$, the LHS of the QoS constraints of problem (10) in iteration $(i+1)$ is greater or equal to the corresponding LHS in iteration i.

$$\begin{aligned} & \frac{1}{\sqrt{\bar{N}_m^{(i+1)}}} \sum_{n \in \bar{\mathcal{N}}_m^{(i+1)}} \sqrt{\lambda_{m,n}} \mathrm{Re}\{\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \bar{\boldsymbol{v}}_{m,n}^{(i+1)}\} \\ &= \frac{1}{\sqrt{\bar{N}_m^{(i+1)}}} \sum_{n \in \bar{\mathcal{N}}_m^{(i+1)}} \sqrt{\lambda_{m,n}} |\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \boldsymbol{v}_{m,n}| \\ &\geq \frac{1}{\sqrt{\bar{N}_m^{(i)}}} \sum_{n \in \bar{\mathcal{N}}_m^{(i)}} \sqrt{\lambda_{m,n}} |\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \boldsymbol{v}_{m,n}| \\ &\geq \frac{1}{\sqrt{\bar{N}_m^{(i)}}} \sum_{n \in \bar{\mathcal{N}}_m^{(i)}} \sqrt{\lambda_{m,n}} \mathrm{Re}\{\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \bar{\boldsymbol{v}}_{m,n}^{(i)}\} \end{aligned} \tag{13}$$

We proceed proving Lemma 1 by contradiction as follows. Assume that the solution in iteration $(i+1)$ is the same as that in the previous iteration i, i.e., $\boldsymbol{w}_{k,\mathrm{opt}}^{(i+1)} = \boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$. Assume further that the approximation of all active constraints in iteration i changes from iteration i to iteration $(i+1)$. Then, we can see from inequality (13) and the QoS constraints in (10) that the solution in iteration $(i+1)$ can be scaled down, still satisfying all QoS constraints. This contradicts optimality of $\boldsymbol{w}_{k,\mathrm{opt}}^{(i+1)} = \boldsymbol{w}_{k,\mathrm{opt}}^{(i)}$ and we conclude that $||\boldsymbol{w}_{k,\mathrm{opt}}^{(i+1)}||_2 < ||\boldsymbol{w}_{k,\mathrm{opt}}^{(i)}||_2$ in this case. □

We initialize our algorithm with the unrotated principal component of each channel covariance matrix, i.e., we choose $\bar{\mathcal{N}}_m^{(1)} = p_m$, where p_m denotes the index corresponding to the principal component of $\boldsymbol{R}_m$, and $\bar{\boldsymbol{v}}_{m,n}^{(1)} = \boldsymbol{v}_{m,n}$. The iterative scheme is summarized in Table 1 where I stands for the total number of iterations.

According to [7], the SOCP problem (10) can be solved with a worst-case complexity of $\mathcal{O}(G^3 N^3 M^{1.5})$ using efficient interior-point methods. This complexity scales linearly with the number of iterations I. Our simulations have shown that a very small number of iterations I (e.g. $I = 3$) is sufficient to achieve performance comparable to or even better than that achieved by the SDR-based

Initialization: $\bar{\mathcal{N}}_m^{(1)} = p_m,\ \bar{\boldsymbol{v}}_{m,n}^{(1)} = \boldsymbol{v}_{m,n}$
for $i = 1, \ldots, I$
Solve problem (10) with $\bar{\mathcal{N}}_m = \bar{\mathcal{N}}_m^{(i)}$ and $\bar{\boldsymbol{v}}_{m,n} = \bar{\boldsymbol{v}}_{m,n}^{(i)}$.
Perform the rotation of (12) with $\alpha_{m,n}^{(i)} = \angle(\boldsymbol{w}_{k,\mathrm{opt}}^{(i)H} \bar{\boldsymbol{v}}_{m,n}^{(i)})$.
Find $\bar{\mathcal{N}}_m^{(i+1)}$ according to (11).
end

Table 1: Proposed iterative procedure

approach of [3]. The overall complexity of the SDR-based approach is that of a single SDP problem along with N_{rand} times the complexity of a LP problem. Here, N_{rand} stands for the number of generated candidate vectors of which a few hundred are usually required, according to [3]. The mentioned SDP problem can be solved with a worst-case complexity of $\mathcal{O}(M^2(GN+M)^{2.5})$ [8] whereas the LP requires a worst-case complexity of $\mathcal{O}(G^{3.5}+MG^{3.5})$ [3]. Thus, as M increases, the computational complexity of our method decreases relative to that of the SDR-based method.

4. ITERATIVE FEASIBILITY SEARCH

Similar to [5], the SOCP problem in (10) might not be feasible when initialized as in Table 1. This is the case when the feasible set shrinks to the empty set. However, the iterative feasibility search proposed in [5] can straightforwardly be applied to find an initial approximation of the QoS constraints for which problem (10) becomes feasible. Furthermore, the admission control scheme of [5] can be used as well. We refer to [5] for details concerning the iterative feasibility search and the admission control procedure.

5. SIMULATION RESULTS

In our numerical simulations, we consider the following scenario similar to that of [9]. $M = 16$ users are located in angular directions distributed uniformly between 0° and 360°. Users are partitioned in $G = 3$ multicasting groups of similar size which are separated in space. We consider a uniform linear transmit antenna array with $N = 6$ elements spaced half a wavelength. According to [9], the channel covariance matrix can be approximated by

$$[\boldsymbol{R}(\theta, \sigma_\theta)]_{k,l} = \exp(j\pi(k-l)\sin\theta)\exp\left(-\frac{(\pi(k-l)\sigma_\theta\cos\theta)^2}{2}\right) \tag{14}$$

where θ and σ_θ denote the angular direction of the user and the spread angle, respectively. The spread angle models local scattering. The larger this angle, the larger the number of local scatterers by which the users are surrounded and the higher the rank of the corresponding channel covariance matrix $\boldsymbol{R}$. If $\sigma_\theta = 0$, $\boldsymbol{R}$ is rank-one. We compare the performance, in terms of transmitted power, of the following three competing methods: the proposed method, the method of [3] and the method of [5]. As mentioned earlier, the latter one is actually only applicable if the matrices $\boldsymbol{R}_m$ are rank-one. However, independent of its rank, we can always approximate $\boldsymbol{R}_m$ with its principal component to make the method of [5] applicable in the higher-rank case. This corresponds to a less sophisticated version of the proposed method where, instead of choosing the optimal subset $\bar{\mathcal{N}}_m^{(i+1)}$ according to (11), only the principal eigenvector is chosen in every iteration, i.e., $\bar{\mathcal{N}}_m^{(i)} = p_m$, $i = 1, \ldots, I$. For the method of [3], 300 candidate vectors have been generated in the randomization procedure. In the other two methods, we have used only $I = 3$ iterations.

Figure 1 depicts the total transmitted power versus the spread angle, which is varied from 0° to 15°, for the three methods. An idealistic lower bound is also shown. It corresponds to the solution to the

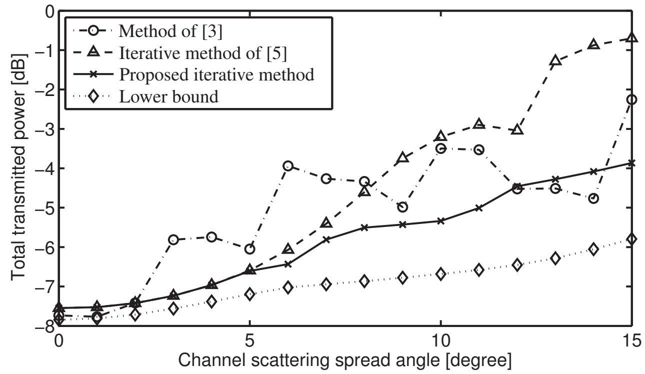

Fig. 1: Total transmitted power versus spread angle.

SDP problem which is obtained after applying the SDR to problem (1) (see [3], [5]). It is clear from the figure, that for small spread angles, the method of [5] and the proposed method have the same performance. The reason is that in this region, the optimal subset contains only the principal eigenvector in every iteration of the proposed method. As the spread angle is further increased, the curves deviate, revealing superior performance of the proposed method. Further, we can observe that, for the wide range of spread angles between 3° and 11°, our method clearly outperforms the method of [3]. Angles beyond 11° correspond to unrealistically strong scattering in the vicinity of the users.

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