ROBUST DOWNLINK BEAMFORMING IN MULTI-GROUP MULTICASTING USING TRACE BOUNDS ON THE COVARIANCE MISMATCHES

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ABSTRACT

We consider the problem of worst-case robust beamforming for multi-group multicasting network with erroneous channel state information (CSI). In previous beamforming techniques robustness is ensured for all mismatch matrices of bounded Frobenius norm. In contrast, we present an alternative method of bounding the channel uncertainties, where we only limit the trace of the mismatch matrices. This approach leads to a problem formulation of reduced complexity as compared to the previous methods. Our goal is to minimize the total transmitted power subject to the worst-case user quality-of service (QoS) constraints. Lagrange duality is used to obtain a simple reformulation of the worst-case beamforming problem. The resulting non-convex problem can then be converted into a convex form using semidefinite relaxation (SDR) that can be solved efficiently using interior point methods. The resulting problem is a linear second-order cone programming (SOCP) problem as opposed to the quadratic SOCP problems in the previous robust approaches. Simulation results also show that the proposed method offers a significantly improved performance in terms of transmitted power.

***Index Terms*—** Convex optimization, multicasting, broadcasting, downlink beamforming, robust adaptive beamforming.

1. INTRODUCTION

In multi-group multicasting systems mobile users are subscribed to different services such as video/audio broadcasting. Multiple subscriber group may be served on the same radio resource by one or multiple basestations using advanced multiantenna access technologies to improve quality-of-service (QoS) and mitigate multi-group interference [1]. Recently multi-group multicasting techniques have been implemented in modern wireless communications standards such as the Multimedia Broadcast Multicast Service (MBMS) in LTE and LTE-A.

In [1]-[2], multi-group multicasting beamforming techniques using perfect channel state information (CSI) have been studied. Since the second order statistic of the wireless channel usually evolves at a significantly lower rate as compared to the exact instantaneous channel, the use of covariance-based CSI can substantially reduce the feedback overhead, especially in fast fading scenarios. Therefore, in downlink beamforming, a number of approaches based on perfect covariance-based CSI have been proposed; see [3] and references therein. In practice, however, the CSI available at the transmitter is prone to errors resulting, e.g., from feedback quantization, channel estimation, and feedback delay. In the presence of CSI errors, the performance of the non-robust beamforming methods can significantly degrade. Therefore, the development of robust adaptive beamforming techniques has recently attracted considerable attention for its practical significance in wireless systems [3]-[5].

These techniques provide robustness against CSI errors by guaranteeing that the QoS constraints are satisfied for all possible mismatch matrices bounded by ellipsoids of given shapes and sizes. These ellipsoids are generally described by the Frobenius norm bounds, explicitly restricting all the entries of the mismatch matrices to fall within the bound of the ellipsoid. In this paper we present an alternative approach of modeling the uncertainty sets. In our approach we bound only the sum of the diagonal elements of the mismatch matrices to obtain a meaningful description of the uncertainty set. Interestingly we can show, that in worst case robust beamforming for multi-group multicasting systems, no restrictions for the off-diagonal elements of the mismatch matrix need to be imposed. This is due to the observation in our final problem reformulation, that for the worst-case mismatch matrices a positive-semidefinite constraint that bounds the off-diagonal elements in the mismatch matrices, is implicitly satisfied. Similarly, as in Frobenius norm based robust approaches of [3]-[5], our approach leads to semidefinite program (with non-convex rank-one constraints) in which a penalty term that quantifies the price for robustness can be identified. Our goal is to minimize the total transmitted power subject to the worst-case QoS constraints, which are expressed in minimum signal-to-interference-plus-noise ration (SINR) requirements. Using Lagrange duality theory and semidefinite relaxation (SDR), we approximate the original non-convex worst-case beamforming problem by a semidefinite programming (SPD) problem, which can be solved using convex optimization tools.

Notation: $E\{\cdot\}$, $\|\cdot\|_F$, $|\cdot|$, $\mathrm{tr}(\cdot)$, $(\cdot)^H$, and $\mathrm{rank}(\cdot)$ denote the statistical expectation, Frobenius norm of a matrix, absolute value of a complex number, trace of a matrix, Hermitian transpose, and rank of a matrix, respectively. $\mathbf{Y} \succeq 0$ means that $\mathbf{Y}$ is a positive semidefinite matrix. diag$\{\cdot\}$ denotes a diagonal matrix and $\lambda_{\max}\{\cdot\}$ denotes the principal eigenvalue of a matrix.

2. SYSTEM MODEL

Let us consider the multi-group multicasting scenario with a single N-antenna transmitter [1]. We assume that there are M users divided into G subscriber groups $\{\mathcal{G}_1, ..., \mathcal{G}_G\}$ such that $1 \leq G \leq M$, where $\mathcal{G}_i$ contains the indices of the users, which receive the information of the multicasting stream i, where $i \in \{1, ..., G\}$. Every user belongs to a single group; $\mathcal{G}_i \cap \mathcal{G}_l = \emptyset$ if $i \neq l$ and $\cup_{i=1}^{G} \mathcal{G}_i = \{1, ..., M\}$. We define $\mathbf{w} \triangleq [\mathbf{w}_1^T, ..., \mathbf{w}_G^T]^T$ where $\mathbf{w}_i$ is the $N \times 1$ beamforming weight vector of the ith group. Then the SINR of the kth user from the ith group $\mathcal{G}_i$ is given by

$$\mathrm{SINR}_k = \frac{\mathbf{w}_i^H \mathbf{R}_{\mathbf{h}_k} \mathbf{w}_i}{\sum_{j \neq i} \mathbf{w}_j^H \mathbf{R}_{\mathbf{h}_k} \mathbf{w}_j + \sigma_k^2},\ j \in \{1, ..., G\} \qquad (1)$$

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where $\mathbf{R}_{\mathbf{h}_k} \triangleq E\{\mathbf{h}_k\mathbf{h}_k^H\}$ and $\mathbf{h}_k$ are the $N \times N$ channel covariance matrix and $N \times 1$ channel vector of the kth user, respectively, and σ_k^2 is the noise variance at the receiver of kth user. The goal of the classic multi-group multicasting beamforming problem is to minimize the total transmitted power subject to the user QoS constraints. This problem can be formulated as [1]

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H\mathbf{w} \\ \text{s.t.} \quad & \text{SINR}_k \geq \gamma_k, \ \forall k \in \mathcal{G}_i, \ i \in \{1, ..., G\} \end{aligned} \tag{2}$$

where γ_k is the minimal required SINR for kth user in the ith group.

3. CHANNEL ERROR MODEL

Let $\mathbf{h}_k$ denote the exact random channel vector containing the flat-fading complex channel coefficients of the kth user and assume that $\hat{\mathbf{h}}_k$ is the corresponding estimated channel vector. If the random vector $\delta\mathbf{h}_k$ models the errors in the channel estimation, then $\hat{\mathbf{h}}_k = \mathbf{h}_k + \delta\mathbf{h}_k$. We can write $\mathbf{R}_{\hat{\mathbf{h}}_k} \triangleq E\{\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H\} = \mathbf{R}_{\mathbf{h}_k} + \tilde{\mathbf{\Delta}}_k$ where the error matrix $\tilde{\mathbf{\Delta}}_k \triangleq E\{\mathbf{h}_k\delta\mathbf{h}_k^H\} + E\{\delta\mathbf{h}_k\mathbf{h}_k^H\} + \mathbf{R}_{\delta\mathbf{h}_k}$ may be indefinite in general with $\mathbf{R}_{\delta\mathbf{h}_k} \triangleq E\{\delta\mathbf{h}_k\delta\mathbf{h}_k^H\}$.

Let us consider the specific example when the Least Squares (LS) channel estimation is used and the only error in the channel is due to the errors in the estimation process. The error in the channel estimate can then be expressed as $\delta\mathbf{h}_k = \mathbf{M}_k^\dagger\mathbf{n}_k$ where $\mathbf{n}_k$ is the training noise vector, $\mathbf{M}_k^\dagger \triangleq (\mathbf{M}_k^H\mathbf{M}_k)^{-1}\mathbf{M}_k^H$ with $\mathbf{M}_k$ being the training matrix; see [6] for details. Assuming $\delta\mathbf{h}_k$ to be zero mean and independent to the channel $\mathbf{h}_k$, the error matrix $\tilde{\mathbf{\Delta}}_k$ can be written as

$$\tilde{\mathbf{\Delta}}_k = \mathbf{M}_k^\dagger E\{\mathbf{n}_k\mathbf{n}_k^H\}\mathbf{M}_k^{\dagger H} \tag{3}$$

which is a positive semidefinite matrix.

4. ROBUST BEAMFORMING

In practical systems the true channel covariance matrices $\mathbf{R}_{\mathbf{h}_k}$ are unknown and only erroneous estimates $\hat{\mathbf{R}}_{\mathbf{h}_k}$ are available. In this case, in the QoS based worst-case robust beamforming proposed in [3]-[5], the beamformig weight vectors are designed such that the minimum SINR requirements are satisfied for all possible mismatched covariances $\hat{\mathbf{R}}_{\mathbf{h}_k} - \mathbf{\Delta}_k$ for which the mismatch matrices $\mathbf{\Delta}_k$ fall within a unit-sphere defined by

$$\|\mathbf{\Delta}_k\|_F \leq \epsilon_k, \ k = 1, ..., M \tag{4}$$

for predefined bounds ϵ_k. In this paper we propose an alternative worst-case robust beamforming approach in which the uncertainty sets are defined as:

$$\text{tr}(\mathbf{\Delta}_k) \leq \mu_k, \ \mathbf{\Delta}_k \succeq 0, \ k = 1, ..., M. \tag{5}$$

Comparing the uncertainty set in (5) with the one in (4) we observe that in the latter set the Frobenius norm explicitly restricts all entries of the $\mathbf{\Delta}_k$, whereas in our set only the diagonal elements of mismatch matrices are explicitly limited. However, with the positive semidefinite constraint in (5) the off-diagonal entries of $\mathbf{\Delta}_k$ are implicitly bounded, see [7, p. 398]. Interestingly, as shown later in Section 4, this positive semidefinite constraint can also be omitted, and hence the uncertainty set in (5) can be extended, without changing the solution of the robust beamforming problem. For the example discussed at the end of Section 3 the bounds defined in (4) and (5) can be found using (3). Finally, we remark that for $\mathbf{\Delta}_k \succeq 0$, the inequality $\|\mathbf{\Delta}_k\|_F \leq \text{tr}(\mathbf{\Delta}_k)$ holds, which shows that for the special choice $\epsilon_k = \mu_k$, the uncertainty set (5) is contained in the set (4). The worst-case robust multicasting beamforming problem can be formulated as

$$\begin{aligned} \min_{\mathbf{w}} \ & \mathbf{w}^H\mathbf{w} \\ \text{s.t.} \ & \min_{\text{tr}(\mathbf{\Delta}_k)\leq\mu_k} \frac{\mathbf{w}_i^H(\mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k)\mathbf{w}_i}{\sum_{j\neq i}\mathbf{w}_j^H(\mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k)\mathbf{w}_j + \sigma_k^2} \geq \gamma_k \\ & \mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k \succeq 0, \ \mathbf{\Delta}_k \succeq 0, \ \forall k \in \mathcal{G}_i, \ \forall i, j \in \{1, ..., G\} \end{aligned} \tag{6}$$

where $\mathbf{\Delta}_k$ is a Hermitian matrix. The constraint $\mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k \succeq 0$ in (6) guarantees that the only those mismatched covariance matrices are considered in the robust approach which are positive semidefinite. Defining $\mathbf{A}_{ik} \triangleq \mathbf{w}_i\mathbf{w}_i^H - \gamma_k\sum_{j\neq i}\mathbf{w}_j\mathbf{w}_j^H$, the problem (6) can be reformulated as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H\mathbf{w} \\ \text{s.t.} \quad & \min_{\text{tr}(\mathbf{\Delta}_k)\leq\mu_k} \text{tr}((\mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k)\mathbf{A}_{ik}) \geq \sigma_k^2\gamma_k \\ & \mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k \succeq 0, \ \mathbf{\Delta}_k \succeq 0, \ \forall k \in \mathcal{G}_i, \ \forall i \in \{1, ..., G\}. \end{aligned} \tag{7}$$

Taking a similar approach as in [5] we consider the worst-case QoS constraint in (7) as a separate optimization problem for which the following Lemma applies:

Lemma 1 (*equivalence of worst-case QoS constraints*): If we fix the non-zero weight vector $\bar{\mathbf{w}}$ (and hence the matrices $\mathbf{A}_{ik}$), then the following problems have the same optimal value:

$$\begin{aligned} \min_{\mathbf{\Delta}_k} \quad & \text{tr}((\mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k)\mathbf{A}_{ik}) \\ \text{s.t.} \quad & \mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k \succeq 0, \ \mathbf{\Delta}_k \succeq 0, \ \text{tr}(\mathbf{\Delta}_k) \leq \mu_k, \end{aligned} \tag{8}$$

and

$$\begin{aligned} \max_{\beta_k, \mathbf{Z}_k} \quad & \text{tr}(\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_{ik} - \mathbf{Z}_k)) - \beta_k\mu_k \\ \text{s.t.} \quad & \beta_k\mathbf{I} - \mathbf{A}_{ik} + \mathbf{Z}_k \succeq 0, \ \mathbf{Z}_k \succeq 0, \ \beta_k \geq 0. \end{aligned} \tag{9}$$

Proof: The Lagrangian associated with problem (8) is given by

$$\begin{aligned} f_k(\mathbf{\Delta}_k, \beta_k, \mathbf{Z}_k, \mathbf{Q}_k) = & \text{tr}((\mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k)\mathbf{A}_{ik}) + \beta_k(\text{tr}(\mathbf{\Delta}_k) - \mu_k) \\ & - \text{tr}((\mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k)\mathbf{Z}_k) - \text{tr}(\mathbf{\Delta}_k\mathbf{Q}_k). \end{aligned} \tag{10}$$

where β_k is the Lagrange multipliers corresponding to the trace constraint in (8) and the Hermitian matrix $\mathbf{Z}_k$ and $\mathbf{Q}_k$ are the Lagrange multipliers corresponding the first and the second positive semidefinite constraint in (8), respectively. Maximizing with respect to $\mathbf{\Delta}_k$ and substituting $\mathbf{Y}_k \triangleq \beta_k\mathbf{I} - \mathbf{A}_{ik} + \mathbf{Z}_k - \mathbf{Q}_k$ yields the Lagrange dual problem corresponding to (8)

$$\inf_{\mathbf{\Delta}_k} f_k(\mathbf{\Delta}_k, \beta_k, \mathbf{Z}_k, \mathbf{Q}_k) = \begin{cases} \text{tr}(\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_{ik} - \mathbf{Z}_k)) - \beta_k\mu_k, & \mathbf{Y}_k \succeq 0 \\ -\infty, & \text{otherwise} \end{cases} \tag{11}$$

which can also be written as

$$\begin{aligned} \max_{\beta_k, \mathbf{Z}_k} \quad & \text{tr}(\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_{ik} - \mathbf{Z}_k)) - \beta_k\mu_k \\ \text{s.t.} \quad & \beta_k\mathbf{I} - \mathbf{A}_{ik} + \mathbf{Z}_k - \mathbf{Q}_k \succeq 0 \\ & \mathbf{Z}_k \succeq 0, \ \mathbf{Q}_k \succeq 0, \ \beta_k \geq 0. \end{aligned} \tag{12}$$

Note that problem (8) is convex and bounded below. Moreover, let $\mathbf{U}_k\mathbf{\Lambda}_k\mathbf{U}_k^H$ be the matrix decomposition of $\mathbf{R}_{\hat{\mathbf{h}}_k}$, then there exists

$\mathbf{\Delta}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{U}_k^H$ such that $[\mathbf{D}_k]_{ij} = \min\{[\mathbf{\Lambda}_k]_{ij}, \tilde{\mu}_k\}$ with $0 < \tilde{\mu}_k < \mu_k/N$ which is strictly feasible. Using the Slater's condition [8], we have strong duality between (8) and (9).

It is clear from (12) that when $(\beta_k^\star, \mathbf{Z}_k^\star, \mathbf{Q}_k^\star)$ is an optimal solution of (12), then $(\beta_k^\star, \mathbf{Z}_k^\star, \mathbf{0})$ is also a feasible point corresponding to the same objective function value. Therefore, we can set w.l.o.g. $\mathbf{Q}_k^\star = \mathbf{0}$, and (12) reduces to (9). □

Since $\min_x x$ s.t. $x\mathbf{I} - \mathbf{M} \succeq 0$ is equivalent to $\lambda_{\max}(\mathbf{M})$, the problem (9) can be compactly written as

$$\max_{\mathbf{Z}_k \succeq 0} \quad \mathrm{tr}(\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_{ik} - \mathbf{Z}_k)) - \lambda_{\max}(\mathbf{A}_{ik} - \mathbf{Z}_k)\mu_k. \tag{13}$$

Using Lemma 1 and (13), the problem (7) can be formulated as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{w} \\ \text{s.t.} \quad & \max_{\mathbf{Z}_k \succeq 0} \mathrm{tr}(\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_{ik} - \mathbf{Z}_k)) - \lambda_{\max}(\mathbf{A}_{ik} - \mathbf{Z}_k)\mu_k \geq \sigma_k^2 \gamma_k \\ & \forall k \in \mathcal{G}_i, \ \forall i \in \{1, ..., G\}. \end{aligned} \tag{14}$$

The first constraint in (14) is satisfied if there exists some $\mathbf{Z}_k \succeq 0$ for which

$$\mathrm{tr}(\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_{ik} - \mathbf{Z}_k)) - \lambda_{\max}(\mathbf{A}_{ik} - \mathbf{Z}_k)\mu_k \geq \sigma_k^2 \gamma_k. \tag{15}$$

The problem (14) then reduces to

$$\begin{aligned} \min_{\mathbf{w}, \{\mathbf{Z}_k\}} \quad & \mathbf{w}^H \mathbf{w} \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_{ik} - \mathbf{Z}_k)) - \lambda_{\max}(\mathbf{A}_{ik} - \mathbf{Z}_k)\mu_k \geq \sigma_k^2 \gamma_k \\ & \mathbf{Z}_k \succeq 0, \ \forall k \in \mathcal{G}_i, \ \forall i \in \{1, ..., G\}. \end{aligned} \tag{16}$$

Defining $\mathbf{W}_i \triangleq \mathbf{w}_i \mathbf{w}_i^H$ and $\mathbf{B}_{ik} \triangleq \mathbf{W}_i - \gamma_k \sum_{j \neq i} \mathbf{W}_j$, the problem (16) can be equivalently reformulated as

$$\begin{aligned} \min_{\{\mathbf{W}_i\}, \{\mathbf{Z}_k\}} \quad & \mathrm{tr}(\sum_{i=1}^{G} \mathbf{W}_i) \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{B}_{ik} - \mathbf{Z}_k)) - \lambda_{\max}(\mathbf{B}_{ik} - \mathbf{Z}_k)\mu_k \geq \sigma_k^2 \gamma_k \\ & \mathbf{W}_i \succeq 0, \ \mathrm{rank}(\mathbf{W}_i) = 1 \\ & \mathbf{Z}_k \succeq 0, \ \forall k \in \mathcal{G}_i, \ \forall i \in \{1, ..., G\}. \end{aligned} \tag{17}$$

Non-convexity in the problem (17) is only due to the rank-one constraint. Therefore, following the so-called SDR technique [3], [9], we remove the rank-one constraint from problem (17) to obtain a convex SDP problem. The resulting problem can be solved efficiently using convex optimization tools such as CVX [10].

There is an interesting implication following from observation made above, that the Lagrange multipliers corresponding to the positive semidefinite constraint in (8) at the optimum of (12) can be chosen as $\mathbf{Q}_k^\star = \mathbf{0}$. This implies, from the complementary slackness conditions of the corresponding Karush-Kuhn-Tucker (KKT) optimality system [8], that the positive semidefinite constraint $\mathbf{\Delta}_k \succeq 0$ is inactive at optimum and can therefore be removed in the original problem (6) without changing the resulting problems in (16) and (17). In other words, even if the positive semidefinite constraints $\mathbf{\Delta}_k \succeq 0$, that together with the trace bound $\mathrm{tr}(\mathbf{\Delta}_k) \leq \mu_k$ restricts the off-diagonal entries of the mismatch matrices, are omitted the off-diagonal entries in the worst-case mismatch matrix are in fact bounded [7, p. 398].

While $\mathbf{Q}_k^\star$ can be chosen to be a zero matrix, the optimal Lagrange multiplier matrix $\mathbf{Z}_k^\star$ corresponding to the first positive-semidefinite constraint in (6) is a non-zero matrix in general. To illustrate this consider the counter-example of a single group multicasting system with $M = 2$ users, $G = 1$, $\sigma_k^2 = 1$, $\gamma_k = 1$, and estimated channel covariances as

$$\mathbf{R}_{\hat{\mathbf{h}}_1} = \mathrm{diag}\{0, 1.5\} \ \mathbf{R}_{\hat{\mathbf{h}}_2} = \mathrm{diag}\{2, 0\}. \tag{18}$$

It can readily be verified that in this case a optimal solution to (16) is given by

$$\mathbf{W}_1^\star = \mathrm{diag}\{1, 2\} \ \mathbf{Z}_1^\star = \mathrm{diag}\{1, 0\} \ \mathbf{Z}_2^\star = \mathrm{diag}\{0, 2\} \tag{19}$$

and there exists no optimal solution with $\mathbf{Z}_k^\star = \mathbf{0}$. This means that the positive definite constraint on the mismatched channel covariance matrix in (6), i.e. $\mathbf{R}_{\hat{\mathbf{h}}_k} - \mathbf{\Delta}_k \succeq 0$, is active. Therefore, the robust beamforming problem with this constraint provides beamformers with a lower total transmitted power compared to the problem without the constraints. Further note that for $\mu_k = 0$, the problem (17) reduces to the non-robust problem and, therefore, the terms $\lambda_{\max}(\mathbf{B}_{ik} - \mathbf{Z}_k)\mu_k$ in problem (17) quantify the penalty paid for achieving the robustness. From (9) and (13), we can see that the penalty terms in problem (17) are linear. On the other hand, the penalty terms in the problem formulation of [5] are quadratic. Therefore, our problem formulation is comparatively less complex.

Similar to the SDR approach in [4], it is generally not guaranteed that the solutions of SDR-based problems are rank-one. To obtain rank-one solutions, we follow the same approach as in [1]. We find candidate beamforming vectors using the Gaussian randomization technique [11] and then solve the multi-group multicast power control problem on (16) to obtain a candidate solution. This requires solving a linear programming problem. This process is repeated for a predetermined number of iterations and the best candidate with respect to the objective function is chosen as the solution.

5. SIMULATION RESULTS

In the first part of our simulations, we consider a multicasting network with G multicast groups of equal size, $N = 6$ antennas at the transmitter and $M = 10$ users. The channel model of [12] is considered. We assume that the true channel vector $\mathbf{h}_k(t)$ can be written as $\mathbf{h}_k(t) = \bar{\mathbf{h}}_k + \tilde{\mathbf{h}}_k(t)$ where $\bar{\mathbf{h}}_k$ is the mean of $\mathbf{h}_k(t)$, and $\tilde{\mathbf{h}}_k(t)$ is a zero-mean random variable vector. For any $\mathbf{h}_k(t)$, we choose $\bar{\mathbf{h}}_k = (1/\sqrt{1+\alpha_k})[e^{j\theta_{1k}}, ..., e^{j\theta_{Nk}}]^T$ where θ_{ik} is a uniform random variable chosen from $[0, 2\pi]$ and α_k is the parameter modeling the degree of uncertainty in the CSI. Then the covariance of $\mathbf{h}_k$ can be written as $\mathbf{R}_{\mathbf{h}_k} = \bar{\mathbf{h}}_k \bar{\mathbf{h}}_k^H + \alpha_k \mathbf{I}/(1+\alpha_k)$. Note that the diagonal elements of $\mathbf{R}_{\mathbf{h}_k}$ are all equal to one. This means that the power gain of each channel coefficient is one. The noise power at the receiver is assumed to be 10 times smaller than the power gain of the channel coefficient. We assume that the estimated channel $\hat{\mathbf{h}}_k = \mathbf{h}_k + \delta\mathbf{h}_k$ where $\delta\mathbf{h}_k$ is a zero-mean i.i.d. Gaussian noise vector with $E\{\mathbf{h}_k \delta\mathbf{h}_k^H\} = \mathbf{0}$, $E\{\delta\mathbf{h}_k^H \delta\mathbf{h}_k\} = \eta_k^2$ and $\eta_k^2 = 0.1$ dB. Then, the covariance of the estimated channel $\hat{\mathbf{h}}_k$ can be expressed as $\mathbf{R}_{\hat{\mathbf{h}}_k} = \mathbf{R}_{\mathbf{h}_k} + \mathbf{R}_{\delta\mathbf{h}_k}$ and, correspondingly, we choose the trace bound as $\mu_k = \mathrm{tr}(\mathbf{R}_{\delta\mathbf{h}_k}) = \eta_k^2$. We introduce the normalized constraint value ζ_k as

$$\zeta_k = \frac{\mathbf{w}_i^H \mathbf{R}_{\mathbf{h}_k} \mathbf{w}_i}{\gamma_k(\sum_{j \neq i} \mathbf{w}_j^H \mathbf{R}_{\mathbf{h}_k} \mathbf{w}_j + \sigma_k^2)}, \tag{20}$$

as an abstract measure of the constraint satisfaction, i.e., the corresponding QoS constraint is satisfied if and only if $\zeta_k \geq 1$, to evaluate the performance of our proposed method and the non-robust method of [1].

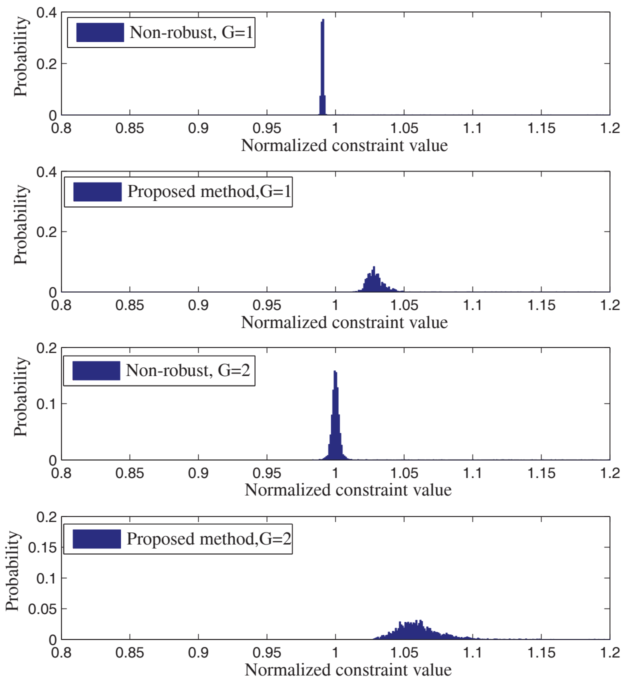

Fig. 1. Histogram of normalized constraint value.

Fig. 1 displays the histograms of ζ_k for $G = 1, 2$, $\gamma_k = 0$ dB, and $\alpha_k = -20$ dB. As can be observed from Fig. 1, the non-robust technique of [1] violates almost all of the constraints when $G = 1$ and only satisfies about 50% of the constraints when $G = 2$. However, our proposed approach satisfies all of the constraints for both the cases.

In order to compare our technique to the non-robust and robust methods of [3] and [5], we consider the downlink beamforming scenario, which is a special case of the multigroup multicasting scenario with $G = M$. We consider the case $M = 3$ without changing the remaining parameters from the first simulation. The true covariance matrices $\mathbf{R}_{\mathbf{h}_k}$ are generated using the same channel model as used in [3] and [5] with the users located at $\theta_1 = 3^\circ$, $\theta_2 = 10^\circ$ and $\theta_3 = 17^\circ$ relative to the array broadside. The angular spread of $\sigma_\theta = 2^\circ$ is considered. The channel error $\delta\mathbf{h}_k$ is generated similar to the previous simulation. For a fair comparison, we compute the bound for the robust method of [5] as $\epsilon_k = \|\mathbf{R}_{\delta\mathbf{h}_k}\|_F = \eta_k^2/\sqrt{N}$. In Fig. 2, we plot the minimum total transmitted power versus the minimal required SINR for all the three approaches. It can be seen that both robust techniques show similar performance for lower SINRs. However at higher SINRs the penalty for the robustness reflected in the total transmitted power is lower for the proposed method. Further, it is interesting to note that the penalty for robustness for the proposed approach is only moderate when comparing the total transmitted power of our approach to the respective power in the non-robust approach.

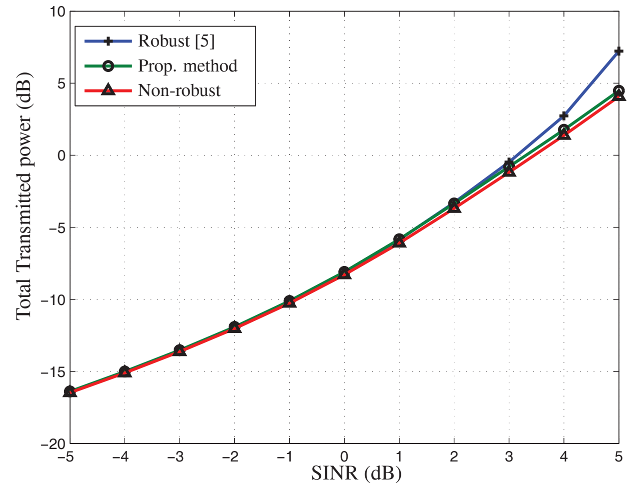

Fig. 2. Total transmitted power versus SINR for $G = M = 3$.

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