

TRANSMIT BEAMFORMING FOR THE MISO INTERFERENCE CHANNEL WITH ASYMMETRIC LINK GAINS

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ABSTRACT

This paper proposes a transmit beamforming technique to cope with the problem of the Multiple Input Single Output Interference Channel (MISO IC). In this scenario, the achievable sum-rate is highly conditioned by the inter-user interference. Furthermore, optimizing the sum-rate is known to be a cumbersome problem. In order to deal with this problem, we approximate the sum-rate when the desired received signal power level is higher than the multiuser interference power level (i.e. when the Signal-to-Interference Ratio (SIR) is high). For that case, the sum-rate expression can be optimized distributively and it leads to a transmit beamformer that outperforms the existing decentralized techniques when the channel gains are asymmetric. Numerical simulations show the performance of the proposed technique which improves the existing decentralized designs for the MISO IC.

Index Terms— Array signal processing, MISO Interference Channel.

1. INTRODUCTION

The interference channel models simultaneous communication among node pairs. Although it is the most general model and it encompasses other communication scenarios, it is also the most difficult to study. In addition, due to interference, network capacity does not scale with the size of the network. For open spectrum communication systems where users are meant to share both time and frequency resources, interference will become the truly bottleneck of the radio communication.

The capacity of the general MISO IC is not known and just partial results have been presented. For example, treating interference as noise has been stated to be optimal under the so-called noisy interference regime [1] (i.e. when the noise and interference power levels are equivalent). It is important to mention that, although treating interference as noise is not optimal in most cases, it is in fact what the mobile receivers will be able to do, due to their limited resources.

When the receivers implement single user detection, the Pareto rate region has been recently characterized in [2] for the K -user case. This characterization is of great importance since the optimal rate points can be achieved if all beamformers are designed according to a limited number of real parameters. Nevertheless, choosing those parameters for achieving a rate Pareto optimal point is a cumbersome task.

For the two user case, asymptotical results have been

obtained (they are summarized in Table I). When the SNR¹ tends to zero, [3] shows that the optimal transmit strategy is the matched filter to the intended user whereas when the SNR tends to infinity the zero forcer is the optimal option. On the other hand when the SINR is very high the optimal beamformer results to be different as [4] investigates. This transmit beamformer results in a MMSE design as the one previously presented in [5] for the MIMO broadcast channel. Similar design is presented in [6] by means of the heuristic measure of the virtual SINR and in [7] as the reciprocal transmit beamformer from the Matched Desired Impulse Response (MDIR) receiver (see [8]).

This paper presents a low complex solution for the MISO interference channel that performs better than the known designs in different scenarios. We derive this optimal beamformer when the SIR is high. Notice that this assumption is a more restrictive than the SINR: if the SINR is high the SIR is also high however, its counterpart might not be true. The resulting beamformer operates well in a wide range of transmitted power and shows the higher sum-rate when the link gains are asymmetric with respect to the existing transmit decentralized beamformers.

With this, the contribution of the paper is twofold:

- We derive the optimal transmit beamforming strategy when the SIR is high for the two user MISO interference channel.
- We evaluate the performance of our proposal for different scenarios, we claim its superior behaviour for a wide range of transmitted power and we extend the simulations for $K > 2$ case.

Numerical simulation show that our proposal outperforms the existing closed-form and decentralized designs. Therefore, the proposed beamformer is highly suitable for the future high data rate multicell protocols since it increases the sum-rate yet preserving a closed form solution and decentralized fashion.

The remainder of this paper is organized as follows. Section II presents the system model and states the problem. In section III is derived the proposed transmit beamformer by assuming high SIR. Section IV extends the design for the K user case. In section V numerical simulations are presented and section VI concludes.

¹The SNR is defined as the ratio between the desired signal and the noise power levels at the receiver.

Table I: Optimal Transmit Beamforming for the two user MISO-IC

Matched Filter, E. G. Larsson <i>et al.</i> [3]	$\text{SNR}_{kk} \rightarrow 0 \quad k = 1, 2$	$\mathbf{b}_k = \frac{\mathbf{h}_{kk}}{\ \mathbf{h}_{kk}\ }$
Zero Forcing, E. G. Larsson <i>et al.</i> [3]	$\text{SNR}_{kk} \rightarrow \infty \quad k = 1, 2$	$\mathbf{b}_k = \alpha \left(\mathbf{I} - \frac{\mathbf{h}_{kj} \mathbf{h}_{kj}^H}{\mathbf{h}_{kj}^H \mathbf{h}_{kj}} \right) \mathbf{h}_{kk} \quad \alpha$ is set so that $\ \mathbf{b}_k\ = 1$
Virtual SINR, R. Bhagavatula <i>et al.</i> [4]	$\text{SINR}_k \rightarrow \infty \quad k = 1, 2$	$(\mathbf{h}_{kk} \mathbf{h}_{kk}^H) \mathbf{b}_k = \lambda_{\max} \left(\text{SNR}_{kj} \mathbf{h}_{kj} \mathbf{h}_{kj}^H + \mathbf{I} \right) \mathbf{b}_k$

2. SYSTEM MODEL AND PROBLEM STATEMENT

Consider an IC channel with K transmission pairs between a single antenna receiver and a transmitter with N antennas. Focusing on one particular time instant, denote the received signal at receiver k by y_k

$$y_k = \sum_{j=1}^K \mathbf{h}_{jk}^H \mathbf{b}_j x_j + w_k \quad k = 1, \dots, K \quad (1)$$

where x_j are the transmitted symbols, and $\mathbf{h}_{jk} \in \mathbb{C}^{N \times 1}$ is the complex channel vector from transmitter j to receiver k , which is assumed to be at the transmitter. Vector $\mathbf{b}_j \in \mathbb{C}^{N \times 1}$ is the norm-one beamformer used by transmitter j . The additive noise w_k at user k is zero mean complex Gaussian with variance σ_k^2 . The transmitter power for user j is $P_j = \mathbf{E}[|x_j|^2]$.

It is clear to see that the received signal y_k is composed by the desired signal plus the multiuser interference plus noise. Let us define the power of those signals as

$$S_k = P_k |\mathbf{h}_{kk}^H \mathbf{b}_k|^2 \quad I_{jk} = P_j |\mathbf{h}_{jk}^H \mathbf{b}_j|^2 \quad (2)$$

where S_k denotes the desired power level and I_{jk} the interference power level created by transmitter j to receiver k ($j \neq k$).

The achievable rate by user k when receivers do not try to decode the multiuser interfering signals is

$$r_k = \log_2 \left(1 + \frac{S_k}{I_k + \sigma_k^2} \right) \quad (3)$$

where it has been defined $I_k = \sum_{j \neq k} I_{jk}$ and it is assumed to be gaussian distributed. The scope of this paper is to maximize the sum-rate

$$R = \sum_{k=1}^K r_k \quad (4)$$

which is a measurement of the overall network performance. With this, we can define another measures of performance

$$\text{SNR}_{kl} = \frac{S_k}{\sigma_l^2} \quad \text{SIR}_k = \frac{S_k}{I_{jk}} \quad \text{SINR}_k = \frac{S_k}{I_{jk} + \sigma_k^2} \quad (5)$$

3. SUM-RATE OPTIMIZATION AT HIGH SIR

For the two user case the sum-rate (4) becomes

$$R = \log_2 \left[\left(1 + \frac{S_1}{I_1 + \sigma_1^2} \right) \left(1 + \frac{S_2}{I_2 + \sigma_2^2} \right) \right] \quad (6)$$

which can be rewritten as

$$R = \log_2 \left[\left(\frac{S_1 + I_1 + \sigma_1^2}{I_1 + \sigma_1^2} \right) \left(\frac{S_2 + I_2 + \sigma_2^2}{I_2 + \sigma_2^2} \right) \right] \quad (7)$$

The maximization of (7) w.r.t $\mathbf{b}_1, \mathbf{b}_2$ is a difficult nonconvex problem. Our proposal relies on the high SIR assumption supported by beamforming, this is when

$$\text{SIR}_k \rightarrow \infty \quad k = 1, 2 \quad (8)$$

When (8) holds we can approximate (7) by

$$R \approx \log_2 \left[\left(\frac{S_1 + \sigma_1^2}{I_1 + \sigma_1^2} \right) \left(\frac{S_2 + \sigma_2^2}{I_2 + \sigma_2^2} \right) \right] \quad (9)$$

Notice the difference between our approximation with the high SINR assumption. For that case, the sum-rate becomes

$$R \approx \log_2 \left[\left(\frac{S_1}{I_1 + \sigma_1^2} \right) \left(\frac{S_2}{I_2 + \sigma_2^2} \right) \right] \quad (10)$$

Note that, both (10) and (9) lead to a separate maximization of both terms. However, (10) the noise level at the receiver plays a key role since it is present in the numerator and the denominator. This is due to the fact that at high SIR regime, noise level might be high compared to the desired one. In fact, the basic idea behind is closer to antenna cancellation or RF interference cancellation. First, the interference signal is diminished by means of an analog spatial filter in order to provide a specific SINR before detection that is required for a target row BER. Therefore, the approximation (9) focuses on the interference as the harmful signal, which can be get ride of by means of a spatial preprocessing.

The maximization of (9) can be done now separately by each of the users, then we have

$$\max_{\mathbf{b}_k} \frac{P_k |\mathbf{h}_{kk}^H \mathbf{b}_k|^2 + \sigma_k^2}{P_k |\mathbf{h}_{kl}^H \mathbf{b}_k|^2 + \sigma_l^2} \quad (11)$$

for $k, l = 1, 2 \quad k \neq l$. We can be benefited from the fact that the beamformer has unit norm

$$\max_{\mathbf{b}_k} \frac{P_k |\mathbf{h}_{kk}^H \mathbf{b}_k|^2 + \sigma_k^2 \|\mathbf{b}_k\|^2}{P_k |\mathbf{h}_{kl}^H \mathbf{b}_k|^2 + \sigma_l^2 \|\mathbf{b}_k\|^2} \quad (12)$$

which can be rewritten as

$$\max_{\mathbf{b}_k} \frac{\mathbf{b}_k^H (P_k \mathbf{h}_{kk} \mathbf{h}_{kk}^H + \sigma_k^2 \mathbf{I}) \mathbf{b}_k}{\mathbf{b}_k^H (P_k \mathbf{h}_{kl} \mathbf{h}_{kl}^H + \sigma_l^2 \mathbf{I}) \mathbf{b}_k} \quad (13)$$

The optimization problem (13) is well known and it is ref-

ered to the generalized Rayleigh quotient. The solution to this problem is the generalized eigenvector associated to the maximum generalized eigenvalue

$$(P_k \mathbf{h}_{kk} \mathbf{h}_{kk}^H + \sigma_k^2 \mathbf{I}) \mathbf{b}_k = \lambda_{\max} (P_k \mathbf{h}_{kl} \mathbf{h}_{kl}^H + \sigma_l^2 \mathbf{I}) \mathbf{b}_k \quad (14)$$

Notice that the design of \mathbf{b}_k does not require knowledge of \mathbf{h}_{ll} or P_l in other words, transmitters do not need to exchange information. Thus, it becomes a decentralized design. We coined this beamformer as EIG beamformer.

It is important to mention the relation with the high SINR approximation. Clearly, when SINR is high, the SIR is also high. However, its counterpart is not generally true: a high SIR does not imply a high SINR. This is specially true when the noise power level is relatively high. Thus, our proposal is a more specific approximation.

The proposed transmit beamformer (14) was first presented in [9] as an optimal solution to the MISO wiretap channel (i.e. achieve secret communication with a transmitted equipped with several antennas in the presence of one potential eavesdropper with just one antenna).

Bearing in mind the definitions in (5) we can rewrite (14) as

$$\begin{aligned} (\text{SNR}_{kk} \mathbf{h}_{kk} \mathbf{h}_{kk}^H + \mathbf{I}) \mathbf{b}_k = \\ \lambda_{\max} (\text{SNR}_{kl} \mathbf{h}_{kl} \mathbf{h}_{kl}^H + \mathbf{I}) \mathbf{b}_k \end{aligned} \quad (15)$$

where for the first time the ratio SNR_{kk} appears. Numerical simulations will show the sum-rate gain when considering both SNR_{kk} and SNR_{kl} .

4. NUMERICAL RESULTS

In this section, we compare EIG beamforming with other decentralized designs (Table I) for the MISO IC. All numerical results are obtained via Monte Carlo simulations with fading rayleigh channel model over 10000 realizations. Moreover, we assume that $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 1$. In order to evaluate the impact of assymetric channel gains we parametrize the ratio between the direct channel gains so that

$$\beta = \frac{|\mathbf{h}_{11}|}{|\mathbf{h}_{22}|} \quad (16)$$

and the interfered channel gains are set to one.

Fig. 1 is depicted the sum-rate versus the SNR for the 2 antenna case when $\beta = 0dB$. In this scenario it can be seen that the zero forcer is the best solution at high SNR. Both virtual SINR and EIG perform well in the middle-low SNR range. On the other hand, when we set $\beta = 10dB$ (Fig. 2) EIG presents an outstanding performance. Indeed, EIG can increase the sum-rate in 0.3 bits per channel use at 2 dB of SNR over the rest of existing techniques. Note that under this range both SINR and SNR are high but EIG performs better than Virtual SINR tough.

Fig. 3 shows the performance within the previous described scenario but in this case the number of antennas is set to three. It is clear that EIG still performs better in a wide range of SNR. Moreover, at high SNR in both Fig. 2 and 3 EIG shows a behaviour closer to zero forcing technique. This is because EIG beamforming is more colinear to zero forcing

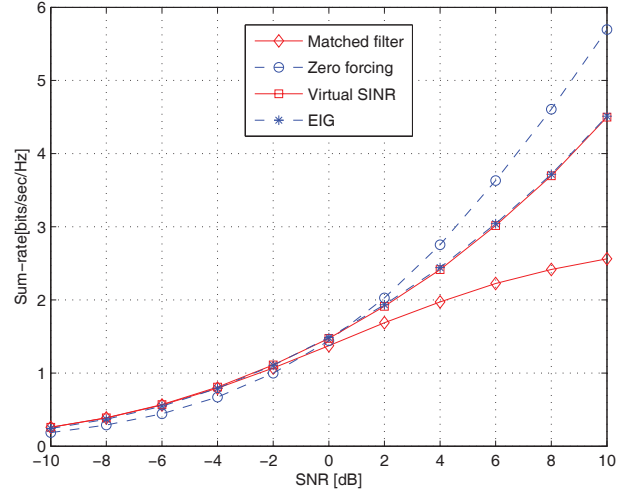


Fig. 1. Sum-rate evaluation for 2-user MISO IC with $N = 2$ antennas

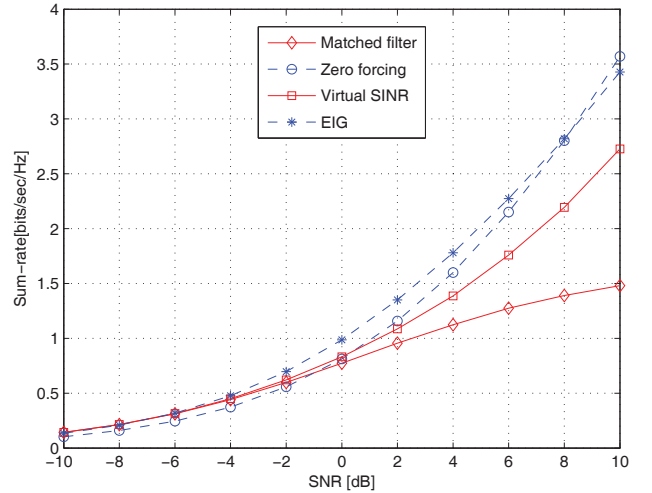


Fig. 2. Sum-rate evaluation for 2-user MISO IC with $N = 2$ antennas, $\beta = 10dB$

solution than virtual SINR design. As a matter of fact, this is of great importance in the asymmetric case where one channel link might have a very low SNR and thus communication will not be reliable. In that case, for the sake of maximum sum-rate, the considered transmitter should do not harm the communication of the other link (i.e. use a zero forcing design). EIG can be aware of this setting from the SNR feedback and nearly null the interference to the other communication pair.

Although not derived in this paper, the natural extension for the K user case will be the generalized eigenvector associ-

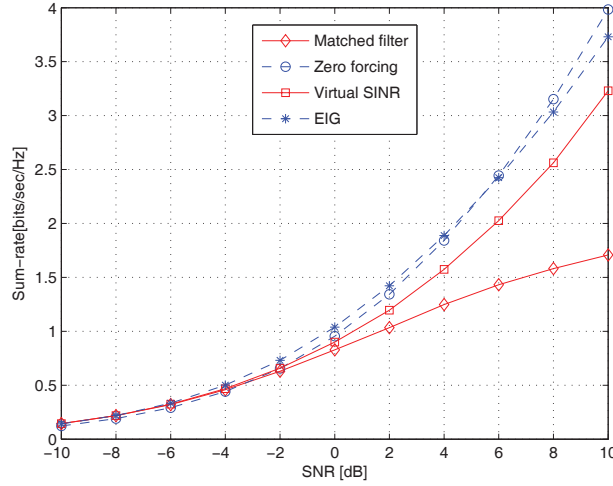


Fig. 3. Sum-rate evaluation for 2-user MISO IC with $N = 3$ antennas, $\beta = 10\text{dB}$

ated to the largest generalized eigenvalue of the matrix pencil

$$\left(\text{SNR}_{kk} \mathbf{h}_{kk} \mathbf{h}_{kk}^H + \mathbf{I}, \sum_{l \neq k}^K \text{SNR}_{kl} \mathbf{h}_{kl} \mathbf{h}_{kl}^H + \mathbf{I} \right) \quad (17)$$

This new proposal is now evaluated with the other decentralized designs for $K = 3$ and $N = 3$ antennas. The channel gains are set so that

$$\beta = \frac{|\mathbf{h}_{11}|}{|\mathbf{h}_{22}|} = \frac{|\mathbf{h}_{11}|}{|\mathbf{h}_{33}|} = 10\text{dB} \quad (18)$$

Fig. 4 shows clearly that the proposed technique results in a higher sum-rate from SNR 0 to 7.

5. CONCLUSION

This paper proposes a new decentralized beamformer as an alternative to solve the beamforming problem for the MISO IC. Motivated by the practical constraint that ADC and other reception elements impose on the dynamics of the total amount of received signal and by the impact that the interference has on this dynamic, the presented solution is derived by means of refining the non-convex optimization of the sum-rate with a high SIR assumption. The proposed scheme shows to perform better than the nowadays non-iterative decentralized solutions when the channel gains are assymmetric irrespectively of the SNR. Monte Carlo simulations demonstrate the better behaviour of EIG as compared to other designs.

6. REFERENCES

[1] B. Bandemer, A. Sezgin, and A. Paulraj, "On the noisy interference regime of the MISO Gaussian interference channel," in *Signals, Systems and Computers, 2008 42nd Asilomar Conference on*. IEEE, 2008, pp. 1098–1102.

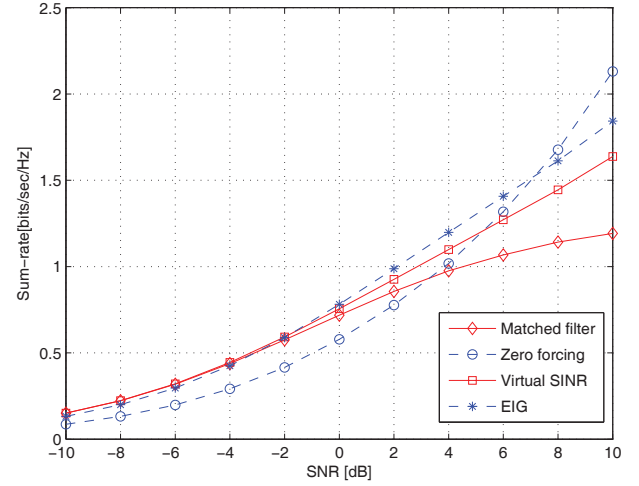


Fig. 4. Sum-rate evaluation for 3-user MISO IC with $N = 3$ antennas, $\gamma = 10\text{dB}$

- [2] R. Mochaourab and E. Jorswieck, "Optimal Beamforming in Interference Networks with Perfect Local Channel Information," *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 1128–1141, Mar. 2011.
- [3] E.G. Larsson, D. Danev, and E.A. Jorswieck, "Asymptotically optimal transmit strategies for the multiple antenna interference channel," in *Communication, Control, and Computing, 2008 46th Annual Allerton Conference on*. IEEE, 2008, pp. 708–714.
- [4] R. Bhagavatula and R.W. Heath, "Adaptive limited feedback for sum-rate maximizing beamforming in cooperative multicell systems," *Signal Processing, IEEE Transactions on*, vol. 59, no. 2, pp. 800–811, 2011.
- [5] M. Sadek, A. Tarighat, and A.H. Sayed, "A leakage-based precoding scheme for downlink multi-user mimo channels," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 5, pp. 1711–1721, 2007.
- [6] R. Zakhour and D. Gesbert, "Distributed multicell-MISO precoding using the layered virtual SINR framework," *Wireless Communications, IEEE Transactions on*, vol. 9, no. 8, pp. 2444–2448, 2010.
- [7] M.A. Vazquez, A. Perez-Neira, and M.A. Lagunas, "Space-time ml receiver and its reciprocal transmitter design for interference networks," in *Smart Antennas (WSA), 2011 International ITG Workshop on*. IEEE, pp. 1–8.
- [8] M.A. Lagunas, J. Vidal, and A.I. Pérez-Neira, "Joint array combining and mlse for single-user receivers in multipath gaussian multiuser channels," *Selected Areas in Communications, IEEE Journal on*, vol. 18, no. 11, pp. 2252–2259, 2000.
- [9] S. Shafiee and S. Ulukus, "Achievable rates in Gaussian MISO channels with secrecy constraints," in *Information Theory, 2007. ISIT 2007. IEEE International Symposium on*. IEEE, 2007, pp. 2466–2470.