# JOINT NETWORK OPTIMIZATION AND BEAMFORMING FOR COORDINATED MULTI-POINT TRANSMISSION USING MIXED INTEGER PROGRAMMING

*Yong Cheng*† *Sarah Drewes*§ *Anne Philipp*§ *Marius Pesavento*†

†Communication Systems Group, Technische Universität Darmstadt, Germany
§ Dept. of Mathematics, Technische Universität Darmstadt, Germany

## ABSTRACT

Coordinated Multi-Point Transmission (CoMP) has been proposed for 4G standards, like WiMAX and LTE-Advanced, as an effective mean to control intercell interference and to increase spectral efficiency in single frequency reuse networks. In practical systems, the remarkable benefits of CoMP operation over conventional single basestation transmission need to be traded against a significant overhead in network complexity and associated operational costs. In order to retain the benefits of CoMP at reasonable costs, we consider the problem of joint basestation selection and multicell beamforming (JBSB). We address this problem via a mixed integer second order cone programming (MI-SOCP) approach. We propose a novel MI-SOCP formulation of the JBSB problem and a reformulation with tighter continuous relaxations. Based on this formulation, we propose fast algorithms to find almost optimal feasible solutions. We show via simulations that the proposed algorithms outperform existing solutions in terms of both complexity and total transmitted power at a guaranteed signal-to-interference-plus-noise-ratio (SINR) level at each mobile station (MS).

***Index Terms*—** Coordinated Multi-Point Transmission, Basestation Selection, Beamforming, Mixed Integer Conic Programming

## 1. INTRODUCTION

Coordinated Multi-Point Transmission (CoMP) is a promising path towards energy efficient transmission and for intercell interference (ICI) reduction in current and future cellular networks with universal frequency reuse, such as WiMAX and LTE-Advanced [1–5]. CoMP has been demonstrated, in theory [4] and practice [5] to significantly increase spectral efficiency and coverage of cell-edge mobile stations (MSs) [3–5]. However, besides its unquestioned benefits in suppressing ICI, CoMP operation also induces significant overhead in network complexity and associated operational costs. According to [1], the additional expenses associated with CoMP are mainly due to the increased demand to acquire and exchange channel state information (CSI) among basestations (BSs) and MSs, signaling beamforming weights to each BS, and routing data between multiple BSs. In this paper, our objective is to retain the energy efficiency offered by CoMP operation while reducing the overall network complexity.

Apparently, selecting a limited number of cooperating BSs for each MS can simultaneously reduce the communication overhead for signaling beamforming weights and routing data to cooperating BSs. Several works have recently addressed the problem of basestation selection in cellular networks with CoMP (see, e.g., [3], for a comprehensive survey). More recently, the authors of [1] proposed a framework to jointly optimize basestation selection and multicell beamforming (JBSB), which aims for sparsity w.r.t. the number of connections between BSs and MSs. In this approach, the intractable $l_0$-norm sparseness constraint is approximated using the $l_1$-norm relaxation to heuristically generate approximate solutions [1]. Although the method of [1] has very low computational complexity and outperforms other existing heuristic methods, there are still several open issues. For example, the algorithm of [1] yields sparse beamformers, rather than sparse network topologies.

In contrast to [1], in this paper, we study the JBSB problem within a mixed integer second order cone programming (MI-SOCP) framework [6]. In this framework the binary variables represent the connections between BSs and MSs, i.e., a BS is connected to a MS if the BS is selected to serve the MS. When solving a MI-SOCP problem, the quality of its continuous relaxation, which arises from the MI-SOCP by relaxing the binary constraints, plays an important role [6, 7]. We present a reformulation of the MI-SOCP problem that has a tighter continuous relaxation which often leads to binary solutions (even if these have been relaxed) and also provides better lower bounds on the original objective function. This does not only give rise to good approximate solutions, but also plays an important role in the branch-and-bound (BB) procedure and its variations [7], which are the state of the art techniques in finding optimal solutions for MI-SOCP problems [7]. The continuous relaxation of the proposed MI-SOCP formulation significantly reduces the running time of BB based commercial software like IBM ILOG CPLEX [7].

Based on the proposed MI-SOCP formulations of the JBSB problem, we further develop fast heuristic algorithms which are particularly useful for applications in large networks. We show by Monte Carlo simulations that the proposed fast algorithms outperform in terms of both computational complexity and total transmitted power the method of [1], and the proposed heuristic algorithms consume much less time than the BB implementation in CPLEX [7].

## 2. SYSTEM MODEL

Consider a cellular network with $L$ multiple-antenna BSs and $K$ single-antenna MSs, as shown in Fig. 1, where the $l$th BS is equipped with $M_l > 1$ antennas, $\forall l \in \mathcal{L} \triangleq \{1, 2, \cdots, L\}$.

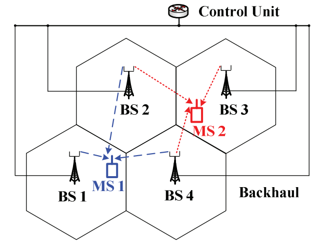



**Fig. 1**: A cellular network with CoMP ($K = 2, L = 4, M_l = 2$)

This work was supported by the European Commission under FP7 Project CROWN, the European Research Council (ERC) Advanced Investigator Grants Program under Grant 227477-ROSE, and the LOEWE Priority Program Cocoon (www.cocoon.tu-darmstadt.de).

It is assumed that CoMP is employed by multiple BSs in the downlink transmission. We consider that all BSs are synchronized and mutually fully connected over a BS interface (e.g., by a X2 type interface in LTE), and therefore, the data of all users can be made available at each BS with associated operational costs. Let $\mathbf{h}_{k,l} \in \mathbb{C}^{M_l \times 1}$ denote the frequency-flat channel vector from the $l$th BS to the $k$th MS, $\forall l \in \mathcal{L}, k \in \mathcal{K} \triangleq \{1, 2, \cdots, K\}$, and define $\mathbf{h}_k \triangleq \left[\mathbf{h}_{k,1}^T,\ \mathbf{h}_{k,2}^T,\ \cdots,\ \mathbf{h}_{k,L}^T\right]^T \in \mathbb{C}^{M \times 1}$, with $M \triangleq \sum_{l=1}^{L} M_l, \forall k \in \mathcal{K}$. Accordingly, we denote $\mathbf{w}_{k,l} \in \mathbb{C}^{M_l \times 1}$ as the beamforming vector used at the $l$th BS for transmitting to the $k$th MS, $\forall l \in \mathcal{L}, k \in \mathcal{K}$, and we define $\mathbf{w}_k \triangleq \left[\mathbf{w}_{k,1}^T,\ \mathbf{w}_{k,2}^T,\ \cdots,\ \mathbf{w}_{k,L}^T\right]^T \in \mathbb{C}^{M \times 1}$, $\forall k \in \mathcal{K}$. Assuming that all BSs share the same subchannels, the received signal $y_k \in \mathbb{C}$ at the $k$th MS can be written as

$$y_k = \mathbf{h}_k^H \mathbf{w}_k x_k + \sum_{j=1, j\neq k}^{K} \mathbf{h}_k^H \mathbf{w}_j x_j + z_k, \forall k \in \mathcal{K}, \tag{1}$$

where $x_k \in \mathbb{C}$ denotes the unit-power data symbol designated for the $k$th MS, and $z_k \in \mathbb{C}$ denotes the additive white Gaussian noise at the $k$th MS, with mean zero and variance $\sigma_k^2, \forall k \in \mathcal{K}$.

Note that there is no need to share the data of the $k$th MS with the $l$th BS if the $l$th BS is not selected for transmitting to the $k$th MS. Further, in this case the beamformer $\mathbf{w}_{k,l} = \mathbf{0}$ and the communication overhead required for signaling the beamforming weights $\mathbf{w}_{k,l}$ to the $l$th BS can also be saved.

We assume that the data symbols for different MSs are mutually statistically independent and independent from the noise. Further assuming that single user detection is performed at the MS receivers, i.e., the cochannel interference is treated as noise at the MSs, the receive SINR of the $k$th MS, denoted by $\mathrm{SINR}_k$, is given by [1, 2]

$$\mathrm{SINR}_k = \frac{\mathbf{w}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_k}{\sum_{j=1, j\neq k}^{K} \mathbf{w}_j^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_j + \sigma_k^2}, \forall k \in \mathcal{K}. \tag{2}$$

Throughout this paper, it is assumed that there exists a central unit with full CSI for all possible MS-BS connections that dynamically selects the BSs serving each MS and computes the optimal beamforming vectors corresponding to that network topology.

## 3. PROBLEM FORMULATION

### 3.1. Related work

The JSBS problem has previously been addressed in [1], where the following problem formulation has been proposed:

$$\min_{\{\mathbf{w}_{k,l}\}} \sum_{k=1}^{K}\sum_{l=1}^{L} \|\mathbf{w}_{k,l}\|_2^2 + \sum_{k=1}^{K}\sum_{l=1}^{L} \lambda_{k,l} U\left(\|\mathbf{w}_{k,l}\|_2\right) \tag{3a}$$

$$\text{s. t.} \quad \sum_{l=1}^{L} U\left(\|\mathbf{w}_{k,l}\|_2\right) \leq c_k, \forall k \in \mathcal{K}, \tag{3b}$$

$$\mathrm{SINR}_k \geq \gamma_k^{(\min)}, \forall k \in \mathcal{K}, \tag{3c}$$

$$\sum_{k=1}^{K} \|\mathbf{w}_{k,l}\|_2^2 \leq P_l^{(\max)}, \forall l \in \mathcal{L}, \tag{3d}$$

$$U(\tau) = \begin{cases} 0, & \text{if } \tau = 0, \\ 1, & \text{if } \tau > 0, \end{cases} .$$

The constant $\lambda_{k,l} \geq 0$ quantifies the penalty for selecting the $l$th BS to serve the $k$th MS, $\forall l \in \mathcal{L}, \forall k \in \mathcal{K}$, and the integer[1] $c_k \in [1,\ L]$ denotes the maximum number of BSs that can be selected for the $k$th MS, $\forall k \in \mathcal{K}$. Further, the constant $\gamma_k^{(\min)} > 0$ denotes the SINR level at the $k$th MS required to provide sufficient quality-of-service (QoS), $\forall k \in \mathcal{K}$, and the constant $P_l^{(\max)} > 0$ denotes the maximum available transmit power at the $l$th BS, $\forall l \in \mathcal{L}$.

[1] As argued in [1], due to limited backhaul capacity, there may be an upper bound on the number of cooperating BSs for each MS.

It is well-known that the optimal beamformers $\{\mathbf{w}_k, \forall k \in \mathcal{K}\}$ for the JSBS problem (3) are rotation-invariant in the sense that the following second order cone reformulation of the SINR constraints (3c) can be used [1, 8], with $\mathbf{W} \triangleq [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_K] \in \mathbb{C}^{M \times K}$,

$$\left\|\left[\mathbf{h}_k^H \mathbf{W},\ \sigma_k\right]\right\|_2 \leq \sqrt{1 + 1/\gamma_k^{(\min)}} \mathrm{Re}\{\mathbf{h}_k^H \mathbf{w}_k\}, \forall k \in \mathcal{K}, \tag{4a}$$

$$\mathrm{Im}\{\mathbf{h}_k^H \mathbf{w}_k\} = 0, \forall k \in \mathcal{K}. \tag{4b}$$

Even with the convex reformulation of the SINR constraints in (4), the JSBS problem (3) is intractable due to the non-convexity of the *Step* functions in (3a) and (3b). In order to find approximate solutions to the JSBS problem (3), the authors of [1] have proposed the following convex approximation of problem (3):

$$\min_{\{\mathbf{w}_{k,l}\}} \sum_{k=1}^{K}\sum_{l=1}^{L} \|\mathbf{w}_{k,l}\|_2^2 + \sum_{k=1}^{K}\sum_{l=1}^{L} \lambda_{k,l} \|\mathbf{w}_{k,l}\|_1 \tag{5a}$$

$$\text{s. t.} \quad \text{(3d) and (4)}, \tag{5b}$$

After solving problem (5), the constraints in (3b) are re-enforced in the basestation selection procedure [1]. It has been demonstrated in [1] that the method described above outperforms in terms of both computational complexity and total transmitted power the other heuristic approaches [1]. However, an essential drawback of this method is that the $l_1$-norm approximation in (5) favors sparse beamforming vectors (i.e., antenna selections), rather than sparse network topologies (i.e., basestation selections) as requested by the original problem formulation (3).

### 3.2. Proposed MI-SOCP formulation

As an alternative to the problem formulation (3), we propose a novel formulation within the MI-SOCP framework. Later in Section 4.2 we propose a variation of this MI-SOCP formulation that exhibits desirable properties of the associated continuous relaxation.

Let the *binary* variable $a_{k,l} \in \{0, 1\}$ take the value $a_{k,l} = 1$ if the $l$th BS serves the $k$th MS, and $a_{k,l} = 0$ otherwise. Further, we postulate that $\mathbf{w}_{k,l} = \mathbf{0}$ if $a_{k,l} = 0$, $\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. The JBSB problem (3) can then be reformulated as the following MI-SOCP:

$$\min_{\{\mathbf{w}_{k,l}, a_{k,l}\}} \sum_{k=1}^{K}\sum_{l=1}^{L} \|\mathbf{w}_{k,l}\|_2^2 + \sum_{k=1}^{K}\sum_{l=1}^{L} \lambda_{k,l} a_{k,l} \tag{6a}$$

$$\text{s. t.} \quad a_{k,l} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \tag{6b}$$

$$\sum_{l=1}^{L} a_{k,l} \leq c_k, \forall k \in \mathcal{K}, \tag{6c}$$

$$\|\mathbf{w}_{k,l}\|_2 \leq a_{k,l} \sqrt{P_l^{(\max)}}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \tag{6d}$$

$$\text{(3d) and (4)}. \tag{6e}$$

Here, we use the well-known *big-M* formulation in (6d) to ensure that $\mathbf{w}_{k,l} = \mathbf{0}$, if $a_{k,l} = 0$, and that no additional constraint is enforced on $\mathbf{w}_{k,l}$, if $a_{k,l} = 1$ [9]. This follows since $\sqrt{P_l^{(\max)}}$ is the associated upper bound according to (3d), such that the constraint in (6d) is implicit if the $l$th BS is selected to serve the $k$th MS (i.e., if $a_{k,l} = 1$). Otherwise, when $a_{k,l} = 0$, the constraint (6d) ensures that the corresponding beamformer is zero, i.e., $\mathbf{w}_{k,l} = \mathbf{0}$, $\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. However, the bound $P_l^{(\max)}$ on $\|\mathbf{w}_{k,l}\|_2^2$ in (6d) is generally loose as according to (3d) this bound represents the total available transmit power of the $l$th BS, $\forall l \in \mathcal{L}$.

## 4. OPTIMAL SOLUTIONS

### 4.1. MI-SOCP solvers and continuous relaxation

The popular BB procedure and its variations [7] are commonly applied to obtain exact (or also approximate) solutions to the MI-SOCPs at reasonable computational complexity. These procedures are based on the continuous relaxation of the original MI-SOCP problem and subproblems arising from fixing binary (or integer) variables. Considering the JBSB problem (6), its continuous relaxation of all binary variables is given by

$$\min_{\{\mathbf{w}_{k,l},a_{k,l}\}} \sum_{k=1}^{K}\sum_{l=1}^{L}\|\mathbf{w}_{k,l}\|_2^2+\sum_{k=1}^{K}\sum_{l=1}^{L}\lambda_{k,l}a_{k,l} \tag{7a}$$

$$\text{s. t.}\quad 0\le a_{k,l}\le 1,\forall k\in\mathcal{K},\forall l\in\mathcal{L}, \tag{7b}$$

$$\text{(3d), (4), (6c), and (6d),} \tag{7c}$$

where the *binary* variable $a_{k,l}$ has been relaxed to be a *continuous* variable taking value in $[0,\ 1]$, $\forall k\in\mathcal{K},\forall l\in\mathcal{L}$. After solving the continuous relaxation, the BB procedure continues searching for feasible points via recursively creating subproblems by setting binary variables to 0 or 1, respectively, and comparing objective function values of the different subproblems [7].

We remark that, in the continuous relaxation (7), the second term in (7a), i.e., $\sum_{k=1}^{K}\sum_{l=1}^{L}\lambda_{k,l}a_{k,l}$, can be interpreted as the $l_1$-norm of the vector $\breve{\mathbf{a}}\in\mathbb{R}_+^{KL\times 1}$ that is obtained by stacking the variables $\{\lambda_{k,l}a_{k,l},\forall k\in\mathcal{K},\forall l\in\mathcal{L}\}$. However, as can be observed from (6d) the components inside the $l_1$-norm term in (7a) relate to the $l_2$-norm of the beamforming vectors $\mathbf{w}_{k,l}$ instead of to its single component like in (5). For that reason, problem (7) favors sparse solutions of the vector $\breve{\mathbf{a}}$, and thus sparse network topologies.

### 4.2. An improved MI-SOCP formulation

A known drawback of the *big-M* formulations of problem (7) is the associated poor running-time performance of BB solvers [9]. This can be explained by the looseness of the bound $P_l^{(\max)}$ used in (6d), which correspond to weak lower bounds on the objective of the exact problem obtained during the BB procedure [9]. In this section we provide a variation of the MI-SOCP formulation in (7) which is associated with a much tighter continuous relaxation. Specifically, by introducing auxiliary variables $\{t_{k,l}\ge 0,\forall k\in\mathcal{K},\forall l\in\mathcal{L}\}$, we can rewrite the JBSB problem in (6) as follows:

$$\min_{\{\mathbf{w}_{k,l},a_{k,l},t_{k,l}\}} \sum_{k=1}^{K}\sum_{l=1}^{L}(t_{k,l}+\lambda_{k,l}a_{k,l}) \tag{8a}$$

$$\text{s. t.}\quad \|\mathbf{w}_{k,l}\|_2^2\le a_{k,l}t_{k,l},\forall k\in\mathcal{K},\forall l\in\mathcal{L}, \tag{8b}$$

$$\sum_{k=1}^{K}t_{k,l}\le P_l^{(\max)},\forall l\in\mathcal{L}, \tag{8c}$$

$$\text{(4), (6b), and (6c),} \tag{8d}$$

where the on-off hyperbolic constraints in (8b) can be transformed into the following second order cone constraints:

$$\|[2\mathbf{w}_{k,l}^T,\ \ a_{k,l}-t_{k,l}]\|_2\le a_{k,l}+t_{k,l},\forall k\in\mathcal{K},\forall l\in\mathcal{L}. \tag{9}$$

Note that, since $a_{k,l}\in\{0,1\},\forall k\in\mathcal{K},\forall l\in\mathcal{L}$, problem (8) is equivalent to problem (6) and thus also the original formulation (3). The *continuous relaxation* of all binary variables in (8) is given by

$$\min_{\{\mathbf{w}_{k,l},a_{k,l},t_{k,l}\}} \sum_{k=1}^{K}\sum_{l=1}^{L}t_{k,l}+\sum_{k=1}^{K}\sum_{l=1}^{L}\lambda_{k,l}a_{k,l} \tag{10a}$$

$$\text{s. t.}\quad 0\le a_{k,l}\le 1,\forall k\in\mathcal{K},\forall l\in\mathcal{L}, \tag{10b}$$

$$\text{(4), (6c), (8c), and (9).} \tag{10c}$$

Comparing the continuous relaxation (10) with the one in (7) we observe that the former formulation has the following advantages:

**i)** The variable $t_{k,l}$ in (8b) can be interpreted as an upper bound on $\|\mathbf{w}_{k,l}\|_2^2$. This upper bound is much tighter than the corresponding bound $P_l^{(\max)}$ in (6d) as $t_{k,l}$ models the power required for a single connection, i.e., between the $l$th BS and $k$th MS, whereas $P_l$ denotes the power available for all connections of the $l$th BS. In addition, in the MI-SOCP problem (8), the variable $t_{k,l}$ represents the actually transmitted power from the $l$th BS to the $k$th MS as used in the objective and not a power budget that may be deceeded.

**ii)** In the continuous relaxation (10), it can be proved by contradiction that at optimum the constraints (9) are all active. This suggests that at optimum, we have, $\forall k\in\mathcal{K},\forall l\in\mathcal{L}$,

$$t_{k,l}=\frac{\|\mathbf{w}_{k,l}\|_2^2}{a_{k,l}}\ge\|\mathbf{w}_{k,l}\|_2^2,\ \text{if}\ a_{k,l}\in(0,1], \tag{11}$$

which implies that in the continuous relaxation (10), the per-BS sum-power constraints are strengthened if the relaxed variable $a_{k,l}\in(0,1)$. In this sense the constraints in (9) promote binary solutions.

**iii)** Equation (11) also suggests that the first term of the objective function in (10a) favors binary solutions.

Each of the above three points suggests that the continuous relaxation (10) provides better lower bounds of the objective function (8a) as compared to the continuous relaxation in (7).

## 5. PROPOSED HEURISTIC SOLUTIONS

### 5.1. Proposed inflation procedure

Although available BB procedure based MI-SOCP solvers, like IBM ILOG CPLEX [7], have made much progress recently to solve even larger problems in reasonable time, many difficult and large-scale problems are still intractable. This motivates the development of fast heuristic algorithms to find suboptimal solutions for large-scale JBSB problems (8) at low computational complexity. We propose in this section the following inflation procedure.

In the first step, the continuous relaxation (10) is solved, e.g., using interior-point methods. We denote the obtained solutions as $\{\mathbf{w}_{k,l}^\star,a_{k,l}^\star,t_{k,l}^\star,\forall k\in\mathcal{K},\forall l\in\mathcal{L}\}$. If problem (10) is infeasible, the JBSB problem in (8) is also infeasible and the algorithm stops.

In the second step, exactly $c_k$ BSs out of $L$ BSs are selected to serve the $k$th MS, $\forall k\in\mathcal{K}$. This is done by choosing the largest $c_k$ entries of $\{a_{k,l}^\star,\forall l\in\mathcal{L}\}$. If two entries of $\{a_{k,l}^\star,\forall l\in\mathcal{L}\}$, e.g., $a_{k,m}^\star$ and $a_{k,n}^\star$, have the same value and only one of the BSs $\{m,n\}$ can be selected then we compare the $l_2$-norm of the vectors, $\mathbf{w}_{k,m}^\star$ and $\mathbf{w}_{k,n}^\star$, and we select the BS corresponding to a *smaller* $l_2$-norm.

Denote the set of serving BSs selected by the proposed method by $\mathcal{L}_k^\star\subseteq\mathcal{L}$, $\forall k\in\mathcal{K}$. After the BSs are selected for each MS, the following convex optimization problem:

$$\min_{\{\mathbf{w}_{k,l}\}} \sum_{k=1}^{K}\sum_{l=1}^{L}\|\mathbf{w}_{k,l}\|_2^2 \tag{12a}$$

$$\text{s. t.}\quad \mathbf{w}_{k,l}=\mathbf{0},\forall k\in\mathcal{K},\forall l\notin\mathcal{L}_k^\star, \tag{12b}$$

$$\text{(3d) and (4).} \tag{12c}$$

is solved to refine the beamformers and to further optimize the total transmitted power. If problem (12) is infeasible, the algorithm stops. The proposed inflation procedure is summarized in Tab. 1.

We remark that the proposed Alg. 1 has the same computational complexity as the algorithm of [1], which both mainly consist in solving two SOCPs. The two algorithms mainly differ in that the continuous relaxation (10) is used in Alg. 1, whereas the $l_1$-norm approximation (5) is used in [1]. Further, an interesting observation is that, for $c_k = L$, $\forall k \in \mathcal{K}$, problem (8) (also problem (3)) is feasible if and only if the continuous relaxation (10) is feasible.

**Initialization:** Setting $\mathcal{L}_k^\star$ to be empty set, $\forall k \in \mathcal{K}$.
Solving problem (10). If it is infeasible, stop.
**for** $k = 1$ **to** $K$ **do**
  **while** $|\mathcal{L}_k^\star| < c_k$ **do**
    Finding BS: $\bar{l} = \underset{\forall l \in \mathcal{L} \setminus \mathcal{L}_k^\star}{\operatorname{argmax}}\, a_{k,l}^\star$. If more than one is found, the one with the smallest $\left\|\mathbf{w}_{k,l}^\star\right\|_2$ is chosen.
    Updating the selected sets of BSs: $\mathcal{L}_k^\star = \mathcal{L}_k^\star + \bar{l}$
  **end**
**end**
Solving problem (12) with $\{\mathcal{L}_k^\star, \forall k \in \mathcal{K}\}$.

**Tab. 1**: The proposed inflation procedure (Alg. 1)

### 5.2. Proposed deflation procedure

A shortcoming of Alg. 1 (and the algorithm of [1]) is that it selects exactly $c_k$ BSs to serve the $k$th MS, $\forall k \in \mathcal{K}$, resulting in a total $\sum_{k=1}^{K} c_k$ connections. To find solutions with less connections than $\sum_{k=1}^{K} c_k$, we propose a deflation procedure in the following.

We start from feasible sets $\{\overline{\mathcal{L}}_k^\star \subseteq \mathcal{L}, \forall k \in \mathcal{K}\}$ of selected connections in the network, e.g., obtained from Alg. 1. Then, in the proposed deflation procedure, we successively delete BSs from the current sets of serving BSs $\{\overline{\mathcal{L}}_k^\star, \forall k \in \mathcal{K}\}$ and solve the corresponding problem (12) with the updated sets $\{\overline{\mathcal{L}}_k^\star, \forall k \in \mathcal{K}\}$ unless the problem becomes infeasible. Specifically, in the first step of each iteration, a connection, e.g., between the $\bar{k}$th MS and the $\bar{l}$th BS, is found and deleted, i.e., the set $\overline{\mathcal{L}}_{\bar{k}}^\star$ is updated as: $\overline{\mathcal{L}}_{\bar{k}}^\star = \overline{\mathcal{L}}_{\bar{k}}^\star - \bar{l}$. In the second step of each iteration, the feasibility of the corresponding problem (12) is checked with the updated sets of serving BSs $\{\overline{\mathcal{L}}_k^\star, \forall k \in \mathcal{K}\}$. If it is feasible, the deflation procedure goes back to the first step and repeats. Otherwise, the deflation procedure stops.

The metrics used in the deflation procedure is received powers, i.e., $\{|\mathbf{h}_{k,l}^H \overline{\mathbf{w}}_{k,l}^\star|^2, \forall k \in \mathcal{K}, l \in \mathcal{L}\}$, with $\{\overline{\mathbf{w}}_{k,l}^\star, \forall k \in \mathcal{K}, l \in \mathcal{L}\}$ being the solutions to problem (12) with the sets of serving BSs $\{\overline{\mathcal{L}}_k^\star, \forall k \in \mathcal{K}\}$. In each iteration, we de-select the BS that gives the smallest received power. If two entries have the same value, we de-select the BS corresponding to a *larger* $l_2$-norm of the associated beamformers. The deflation procedure is summarized in Tab. 2.

1 Finding BS to delete: $(\bar{k}, \bar{l}) = \underset{\forall k \in \mathcal{K}, l \in \overline{\mathcal{L}}_k^\star}{\operatorname{argmin}} |\mathbf{h}_{k,l}^H \overline{\mathbf{w}}_{k,l}^\star|$. If more than one is found, the one with the largest $\|\overline{\mathbf{w}}_{k,l}^\star\|_2$ is chosen.
2 Updating the selected sets of BSs: $\overline{\mathcal{L}}_{\bar{k}}^\star = \overline{\mathcal{L}}_{\bar{k}}^\star - \bar{l}$;
3 Solving problem (12) with $\{\overline{\mathcal{L}}_k^\star, \forall k \in \mathcal{K}\}$. If feasible, go to step 1 and repeat; otherwise, set $\overline{\mathcal{L}}_{\bar{k}}^\star = \overline{\mathcal{L}}_{\bar{k}}^\star + \bar{l}$, and stop.

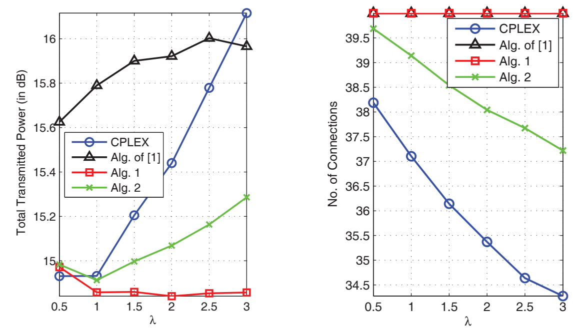
**Tab. 2**: The proposed deflation procedure (Alg. 2)

We remark that the *worst-case* computational complexity of the proposed Alg. 2 mainly consists in solving $K(L-1)$ SOCPs.

## 6. NUMERICAL RESULTS AND DISCUSSIONS

We simulate a cellular network with $L = 7$ BSs and $K = 10$ MSs, and we choose $c_k = 4$, $\gamma_k^{(\min)} = 10$ dB, and $M_l = 2$, and $\lambda_{k,l} = \lambda$, $\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. The channel model and parameters are set the same as in [1]. The simulation results are averaged over 400 Monte Carlo runs. The results labeled with "CPLEX" are the best feasible solutions of the JBSB problem (8) found by the BB implementation in CPLEX [7] under a run-time limit of 45 seconds.

In Fig. 2 the overall transmitted power and the total number of connections vs. the penalty factor $\lambda$ are shown. We can observe that, the proposed Alg. 1 provides solutions with a lower total transmitted power while using the same number of connections as compared to the method of [1]. In addition, the proposed Alg. 2, initialized with Alg. 1, yields solutions with notably less total transmitted power while using slightly more connections as compared with the solutions found by the BB implementation in CPLEX [7].



(a) Total Transmitted power (b) Total No. of Connections

**Fig. 2**: Total Transmitted power and No. of Connections vs. $\lambda$.

In Tab. 3 the run-time (in seconds) is listed. It is clear that the proposed Alg. 1 consumes almost the same time as the algorithm of [1], and both the proposed Alg. 1 and Alg. 2 consume much less time than the BB procedure implementation in CPLEX [7].

| Methods | Alg. of [1] | Alg. 1 | Alg. 2 | CPLEX |
|---|---|---|---|---|
| CPU Time | 3.39 | 3.48 | 7.73 | 48.03 |

**Tab. 3**: Average Running Time (in seconds)

## 7. REFERENCES

[1] Y. Zeng, E. Gunawan, Y. L. Guan, and J. Liu, "Joint base station selection and linear precoding for cellular networks with multi-cell processing," in *Proc. of TENCON*, Nov. 2010.

[2] E. Björnson, R. Zakhour, and et al, "Cooperative multicell precoding: Rate region characterization and distributed strategies with instantaneous and statistical csi," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4298–4310, Aug. 2010.

[3] D. Gesbert, S. Hanly, and et al, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1380–1408, Dec. 2010.

[4] G. J. Foschini, K. Karakayali, and R. A. Valenzuela, "Coordinating multiple antenna cellular networks to achieve enormous spectral efficiency," *IEE Proceedings of Communications*, vol. 153, no. 4, pp. 548–555, Aug. 2006.

[5] V. Jungnickel, A. Forck, and et al, *Realtime Implementation and Field Trials for Downlink CoMP*. In: Coordinated Multi-Point in Mobile Communications: From Theory to Practice, Cambridge University Press, 2011.

[6] S. Drewes, *Mixed Integer Second Order Cone Programming*. PhD thesis, Technische Universität Darmstadt, 2009.

[7] P. Bonami, M. Kilinc, and J. T. Linderoth, *Algorithms and Software for Solving Convex Mixed Integer Nonlinear Programs*. IMA Volumes, to appear, 2011.

[8] M. Bengtsson and B. Ottersten, *Optimal and Suboptimal Transmit Beamforming*. In: Handbook of Antennas in Wireless Communications, CRC Press, Aug. 2001.

[9] A. Khurana, A. Sundaramoorthy, and I. A. Karimi, "Improving mixed integer linear programming formulations," in *Proc. of AIChE*, Oct. 2005.