MULTI-HOP, MULTI-ROUTE POWER MINIMISATION IN AD HOC NETWORK

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ABSTRACT

This paper studies the problem of finding the optimal route of a stream between the transmitter and the receiver in an ad hoc network while minimizing the total power consumption. The criterion is the minimum power, and the fact that the stream must reach its destination is used as a constraint. Compared to previous approaches, we introduce more flexibility by allowing the stream to be split between several intermediate destinations, and we do not use the specific approximations (mainly high SNR) required to make the problem convex. Therefore, the main difficulty is due to the non convexity of the problem. In this contribution, we address a simple situation (flat fading channels, constant during the optimization process, simple transmitters and receivers) in order to demonstrate some properties of the problem. Finally, we propose an algorithm which, at least in some cases, is able to overcome the non convexity. Illustrations of the path followed by the algorithm demonstrates that it is able to find the global optimum even is difficult cases.

Index Terms— Power allocation, wireless networks, ad hoc network, optimization, circuit theory

Introduction

Organizing a small scale ad hoc network so that the streams reach their destination while saving energy is an issue that becomes increasingly important. However, in general wireless networks power/rate optimisation is a complex and non convex problem. First solutions were assuming somewhat unrealistic assumptions, such as high SNR regime (which is hardly the case in such situations as expanding the coverage of a cell by user cooperation), or non interfering channels (which results in a waste of resources) in order to make the problem convex. Under these assumptions a centralised approach has been proposed by O'Neill, Julian and Boyd in [3]. This type of analysis is well suited to "small interference" cases such as the use of CDMA techniques that ensure low interference by orthogonality properties. In this paper we address a high interference situation, when interference between streams is allowed and appropriately addressed. We first describe the simple transmission scheme under study in section 1. The problem of power optimisation for a fixed transmission rate is stated and appears to be highly non-convex (section 2). Our analysis leads to rate asymptotical results presented in part 3. Finally, we propose an optimisation algorithm that has been observed to avoid non-convexity in many circumstances (section 4).

1. MODELING THE PROBLEM

Assume many wireless devices randomly distributed in a given area. Two of them want to communicate with a global rate R. All these devices can collaborate in order to achieve this objective, but they want to globally use the smallest possible power. The problem is to find the best solution in terms of multi-hop and multi-route to achieve this result. All devices are assumed to use the same spectrum, but for the fact that transmission and reception must occur on different time or frequency. Under these assumptions, every transmission between nodes interferes with the other ones and this has to be carefully taken into account. Also, if we assume that the nodes would like to use the smallest power, it is likely that we will not be in the high SNR regime.

1.1. Network graph model

There is an immediate issue if sharing spectrum is imposed for an ad hoc network: a node has to be able to relay, but reception and transmission are not really feasible on the same channel at the same time. So, even if the nodes want to share the bandwidth, ad hoc networks need at least two channels. This means that the set of nodes is divided into two subsets: nodes which transmit on channel one (subset 1), and nodes which transmit on channel two (subset 2). Two nodes cannot communicate directly if they are in the same set. The problem addressed in this paper (even if suboptimal) is power minimization with a priori given subsets.

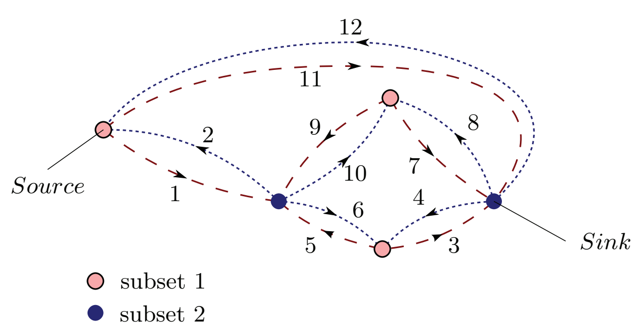

Fig. 1. Ad hoc network link graph example

Let $(\mathcal{G}, \zeta)$, be the graph representing the above described system. $\mathcal{G}$ is the set of wireless nodes (with $|\mathcal{G}| = N$) and ζ the set of edge represents the potential wireless links (with $|\zeta| = 2l$). We note P_k the power dedicated to the link represented by the edge indexed by k and use the same notation for the rate R_k^ζ. We write in this paper $k \in \zeta$ to design the indices corresponding to the edge in ζ. Also, considering a relaying node cannot create data, it cannot transmit more than it receives. On the opposite, if it emits fewer data than received, there is clearly a loss and the power used to bring these data would be wasted. Therefore, this cannot correspond to an optimal solution. These considerations lead to a data conservation equation (like Kirchhoff law for current).

1.2. Transmission model

This description is implicitly based on multiuser transmission techniques, because there is a power assigned to each edge (in other words, the power assigned to some node is shared between all the corresponding destinations). For the sake of simplicity, in a first step, we use the Bit Interleaved Coded Modulation (BICM) coding strategy as depicted on figure 2. Each user receives:

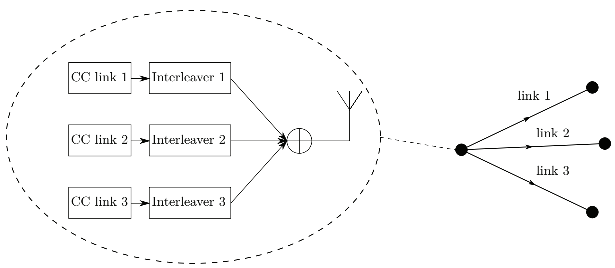

Fig. 2. Coding strategy

$$\forall k \in \zeta, Y_k = \sqrt{G_{kk}} X_k + \sum_{\substack{i \in \zeta \\ i \neq k}} \sqrt{G_{ik}} X_i + N \quad (1)$$

where $\sqrt{G_{ij}}$ is the gain from the interfering link i to the interfered link j. Each receiver is assumed to know only its own interleaving function and the interleavers make the statistical properties of interfering signal gaussian. This is clearly not really efficient, and can even be seen as the worst case if we do not assume any knowledge about the interferers. Finally, we suppose the channel coding is perfect and can transmit at capacity rate without any error for an infinite stream. With these assumptions, we obtain an achievable region based on the Shannon capacity when decoding $Y_k = \sqrt{G_{kk}} X_k + \left(\sum_{\substack{i \in \zeta \\ i \neq k}} \sqrt{G_{ik}} X_i + N \right)$ and considering the interference as gaussian. A more efficient scheme can be obtained by assuming that the receiver knows all interleavers and is able to do Successive Interference Cancellation (SIC), so that we can use some kind of interfering MAC and Broadcast channel in this general network. The best coding/decoding (or interference cancellation) strategy to be used in this context is an open problem. In this paper we want to keep a general formulation, and the capacity is evaluated as follows:

$$\forall k \in \zeta, \; R_k^\zeta \leq \log\left(1 + \gamma_k\right) \quad (2)$$

with $\gamma_k = \frac{P_k . G_k}{\sum_{\substack{i \in \zeta \\ i \neq k}} \Theta_{i,k} . P_i . G_{i,k}}$ and where matrix $\mathbf{\Theta}$ is able to model several possible coding/decoding strategies. Obviously both strategies mentioned above can be cast in this model. We also use matrix $\mathbf{\Theta}$ to model channel independence. On figure 1 the graph contains links that are independent (subsets 1 and 2) during the transmission process, since they correspond to two different channels. However, the transmission obviously makes use of both channels, and the routing problem must take this separation into account. Matrix $\mathbf{\Theta}$ is also used to model this independence. In practice it means if $i \in \zeta$ and $j \in \zeta$ are not in the same subset, then $\mathbf{\Theta}_{ij} = \mathbf{\Theta}_{ji} = 0$.

1.3. Formulation of the optimisation problem

Here we present a first formulation of the optimization problem deriving directly from the considerations above. This will be modified later based on properties of the problem. For $i \in \zeta$, we note $pred(i) \in \mathcal{G}$ the index of the node which is at the origin of link i. Similarly, $succ(i) \in \mathcal{G}$ denotes the index of the node which is the output of link i.

Problem 1.

$$\min_{(\mathbf{P}, \mathbf{R}^\zeta) \in \mathbb{R}_+^{2l} * \mathbb{R}_+^{2l}} \|\mathbf{P}\|_1 \; \textit{subject to} \quad (3)$$

$$\forall k \in \zeta, \; R_k^\zeta \leq \log\left(1 + \frac{P_k . G_k}{\sigma^2 + I_k}\right) \quad (4)$$

$$\textit{with } I_k = \sum_{m \in \zeta, m \neq k} \Theta_{m,k} . P_m . G_{m,k}$$

$$\forall N \in \mathcal{G}, \sum_{\substack{k \in \zeta \\ pred(k) = N}} R_k^\zeta - \sum_{\substack{m \in \zeta \\ succ(m) = N}} R_m^\zeta = \begin{cases} R & \textit{if } N = Source \\ -R & \textit{if } N = Sink \\ 0 & \textit{elsewhere} \end{cases} \quad (5)$$

This optimisation problem admits $(\mathbf{P}, \mathbf{R}^\zeta) \in \mathbb{R}_+^{2l} * \mathbb{R}_+^{2l}$ as variables, which are dependant. We have chosen to express the problem in terms of variables $\mathbf{R}^\zeta$, from which $\mathbf{P}$ is determined. However, as illustrated below, such a $\mathbf{P}$ does not always exist. A $\mathbf{R}^\zeta$ is said to be **feasible** if there exists $\mathbf{P} \geq \mathbf{0}$ such that $(\mathbf{P}, \mathbf{R}^\zeta)$ satisfies the constraints of the problem (i.e. eq. (4) and (5)). The characterization of $\mathbf{R}^\zeta$ meeting (4) is studied in section 2.1 while section 2.2 is concerned with the "Kirchhoff laws" and the global rate constraint. Such a $\mathbf{R}^\zeta$ is said in this paper to be **admissible**. To summarize, if a $\mathbf{R}^\zeta$ is **admissible** and **feasible**, then there exists $\mathbf{P} \geq 0$ such that $(\mathbf{P}, \mathbf{R}^\zeta)$ is a solution of the problem 1.

2. REFORMULATING THE PROBLEM

In this section, we decrease the number of unknowns and demonstrate some properties of the constraints.

2.1. The capacity constraint

We assume a fixed $\mathbf{R}^{\zeta} \in \mathbb{R}_{+}^{2l}$. We are interested in finding the corresponding power allocation meeting the constraints (4),

$$\forall k \in \zeta,\ R_k^{\zeta} \leq \log\left(1 + \frac{P_k.G_k}{\sigma^2 + I_k}\right)$$
$$I_k = \sum_{m \in \zeta, m \neq k} \Theta_{m,k}.P_m.G_{m,k}$$

Claim 1. *Let* $\mathbf{R}^{\zeta} \in \mathbb{R}_{+}^{2l}$, arg $\left(\min_{\mathbf{P} \in \mathbb{R}_{+}^{2l}} \|\mathbf{P}\|_1\right)$ *saturates the set of constraints* (4)

The precise proof is omitted due to lack of space. Since the rate constraints are saturated, one can see that the problem can be written either in terms of the powers or equivalently in terms of the rates. We choose to work with rate as optimisation variable. Hence for a given rate vector, the power vector is determined (provided that it corresponds to positive values, this is addressed below). Eq. (4) thus needs to be reformulated in terms of $\mathbf{R}^{\zeta}$.

$$\forall k \in \zeta,\ R_k^{\mathcal{E}} = \log\left(1 + \frac{P_k.G_k}{\sigma^2 + I_k}\right) \tag{6}$$
$$,\left(2^{R_k^{\zeta}} - 1\right)\left(\sigma^2 + \sum_{m \in \zeta, m \neq k} \Theta_{m,k}.P_m.G_{m,k}\right) = P_k.G_k$$

and in matrix notation,

$$\mathbf{A}(\mathbf{R}^{\zeta}).\mathbf{P} = \mathbf{B}(\mathbf{R}^{\zeta}) \tag{7}$$

with $\mathbf{A} =$

$$\begin{bmatrix} G_{1,1} & -\left(2^{R_1^{\zeta}} - 1\right)\Theta_{1,2}.G_{1,2} & \cdots \\ -\left(2^{R_2^{\zeta}} - 1\right)\Theta_{2,1}.G_{2,1} & G_{2,2} & \cdots \\ -\left(2^{R_3^{\zeta}} - 1\right)\Theta_{3,1}.G_{3,1} & -\left(2^{R_3^{\zeta}} - 1\right)\Theta_{3,2}.G_{3,2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{8}$$

$$\mathbf{B} = \begin{bmatrix} \left(2^{R_1^{\zeta}} - 1\right).\sigma^2 \\ \left(2^{R_2^{\zeta}} - 1\right).\sigma^2 \\ \vdots \end{bmatrix} \tag{9}$$

Corollary 1. *Let* $\mathbf{R}^{\zeta}$ *an admissible rate vector.* $\mathbf{R}^{\zeta}$ *is also feasible if and only if* $\exists \mathbf{P} \ \geq \mathbf{0},\ \mathbf{A}(\mathbf{R}^{\zeta}).\mathbf{P} = \mathbf{B}(\mathbf{R}^{\zeta})$

Proof. $\Leftarrow$ It is immediate, if $\exists \mathbf{P} \ \geq \mathbf{0},\ \mathbf{A}(\mathbf{R}^{\zeta}).\mathbf{P} = \mathbf{B}(\mathbf{R}^{\zeta})$, there exist $\mathbf{P}$ satisfying the constraints (4), so $(\mathbf{R}^{\zeta}, \mathbf{P})$ satisfies all the problem constraints so $\mathbf{R}^{\zeta}$ is feasible.

$\Rightarrow$ If $\mathbf{R}^{\zeta}$ is admissible and feasible by definition there exist $\mathbf{P}$ such as $(\mathbf{R}^{\zeta}, \mathbf{P})$ satisfies all the constraints and the claim 1 assure that at least one $\mathbf{P}$ associated to $\mathbf{R}^{\zeta}$ satisfy $\mathbf{A}(\mathbf{R}^{\zeta}).\mathbf{P} = \mathbf{B}(\mathbf{R}^{\zeta})$ □

This corollary is important because such a $\mathbf{P}$ does not always exist and in this case there is no admissible solution, which makes the minimisation problem difficult to solve: the feasible set can be non connex. In such cases there is no power (even very large) which will allow to realize the corresponding rate allocations in the graph. From now on, matrix $\mathbf{A}$ is assumed non-singular for all rate we consider. This is not true in general but it can be proved that this assumption is met within the explored area. The problem can now be written as

Problem 2.

$$\min_{\mathbf{R}^{\zeta} \in \mathbb{R}_{+}^{2l}} \|\mathbf{A}^{-1}(\mathbf{R}^{\zeta})\mathbf{B}(\mathbf{R}^{\zeta})\|_1 \text{ verifying the set of constraints } (5)$$

2.2. The routing constraint

This section addresses a characterization of the set of constraint (5) expressing that, except at the source and sink nodes, no information can be created or lost. This set of equations is similar to that obtained in circuit theory, denoted as the Kirchhoff law of current, but for the fact that the quantities should be positive. Therefore, finding such $\mathbf{R}^{\zeta}$ is very close to an already solved problem of circuit theory [1]. This connection is important in that it provides practical ways of reducing the number of unknowns thanks to the set of linear equations (5) while ensuring that the corresponding rates remain positive. Due to lack of space, this cannot be detailed here, but we provide the main tool.

Theorem 1. *In an oriented graph, all current vectors satisfying the Kirchhoff law of current can be generated by currents freely chosen on a co-tree of the graph. For a determined co-tree, denoting* $\mathbf{R}_{cT}^{\mathcal{E}}$ *the current vector associated to the co-tree, there exists a matrix* $\mathbf{M} \in \mathcal{M}^{l, l-N+2}$ *whose all elements are in* $\{-1, 0, 1\}$ *such that:* $\mathbf{R}^{\mathcal{E}} = \mathbf{M}.\mathbf{R}_{cT}^{\mathcal{E}}$.

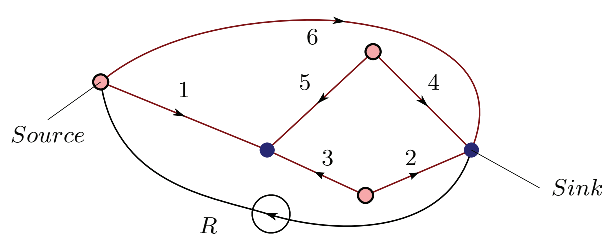

Fig. 3. Graph used for routing constraint. The initial graph $(\mathcal{G}, \zeta)$ is associated to a graph $(\mathcal{G}, \mathcal{E})$ obtained by merging the double edges and adding a virtual edge representing the global rate constraint. In $(\mathcal{G}, \mathcal{E})$ the rate can be positive or negative so the orientation of the element of $\mathcal{E}$ is arbitrary.

Corollary 2. *The set of all admissible* $\mathbf{R}^{\zeta}$ *can be generated using only* $l - N + 1$ *rate variables and the Kirchhoff laws equations. These variables are denoted below as* $\mathbf{R}_{free}^{\mathcal{E}}$.

As a result of this section, we can reformulate the initial problem as a non constrained one, with $\mathbf{R}_{free}^{\mathcal{E}}$ as variables, that explicitly generate all admissible $\mathbf{R}^{\zeta}$

$$\min_{\mathbf{R}_{free}^{\mathcal{E}} \in \mathbb{R}^{l-N-1}} \|\mathbf{P}((\mathbf{A}^{-1}\mathbf{B})(\mathbf{R}_{free}^{\mathcal{E}})\|_1 \tag{10}$$

3. ASYMPTOTIC RESULTS

This section analyses the influence of the global rate on the existence of a solution. Let $\mathbf{R}^{\zeta} = R.(r_1^{\zeta}, r_2^{\zeta}, \cdots, r_{2l}^{\zeta}) = R.\mathbf{r}^{\zeta}$. As with $\mathbf{R}^{\zeta}$, we consider admissible and/or feasible $\mathbf{r}^{\zeta}$. A first result is that, when $R \to 0$, this implies high SINR, a situation proved to be convex in [3]. Therefore, this always correspond to a feasible situation. The next result is concerned with the other extreme: when $R \to_+ \infty$. Lack of space prevents us to provide the proof, but it can be shown that for such large R, the only feasible solution is the direct link between the transmitting node and the receiving node. This shows the limit of sharing the same channel.

4. OPTIMISATION ALGORITHM

This section proposes an iterative algorithm for solving the non convex optimization problem 2.2 which has been shown to be equivalent to problem 1. We introduce here an intermediate step in which the powers involved in the interference term are considered as constant quantities. Let $\mathbf{U} = diag(\mathbf{A}(\mathbf{R}^{\zeta}))$ and $\mathbf{V}(\mathbf{R}^{\zeta}) = \mathbf{U} - \mathbf{A}(\mathbf{R}^{\zeta})$, then eq. (7) reads

$$\mathbf{U}. \underbrace{\mathbf{P}}_{\text{Variable}} - \mathbf{V}(\mathbf{R}^{\zeta}). \underbrace{\mathbf{P}_{cst}}_{\text{Constant}} = \mathbf{B}(\mathbf{R}^{\zeta})$$

$$\hat{\mathbf{P}} = \mathbf{U}^{-1}\left(\mathbf{B}(\mathbf{R}^{\zeta}) + \mathbf{V}(\mathbf{R}^{\zeta}).\mathbf{P}_{cst}\right) \quad (11)$$

where $\mathbf{U}$ is associated with the useful power and $\mathbf{V}$ with the interference term. The minimisation problem

$$\min_{\mathbf{R}^{\mathcal{E}} \in \mathbb{R}^{l-N+1}} \hat{\mathbf{P}} = \mathbf{U}^{-1}\left(\mathbf{B}(\mathbf{R}^{\zeta}) + \mathbf{V}(\mathbf{R}^{\zeta}).\mathbf{P}_{cst}\right)$$

is a strictly convex optimisation problem. Now, a practical scheme for solving (10) can be proposed.

Algorithm 1.
1. *Choose an admissible* $\mathbf{R}^{\zeta}[i]$ *(initialisation on direct transmission point, always a solution)*
2. *Solving problem on that point* $\mathbf{P}[i] = \mathbf{A}(\mathbf{R}^{\mathcal{E}}[i])^{-1}.\mathbf{B}(\mathbf{R}^{\mathcal{E}}[i])$.
3. *We minimise power with fixed interference.*

$$\hat{\mathbf{P}}[i+1] = \mathbf{U}^{-1}.(\mathbf{V}(\mathbf{R}^{\mathcal{E}}[i]).\mathbf{P}[i] + \mathbf{B}(\mathbf{R}^{\mathcal{E}}[i]))$$

$$\mathbf{R}^{\mathcal{E}}[i+1] = \arg\left(\min_{\mathbf{R}^{\mathcal{E}} \in \mathbb{R}^{l-N-1}}, \|\hat{\mathbf{P}}[i+1]\|_1\right) \quad (12)$$

4. *increment* i *by* 1 *and go to 2*

Each step of the minimisation problem is strictly convex in terms of $\mathbf{R}^{\mathcal{E}}_{free}$ variables, hence has a unique solution. We are not able to prove the convergence of Algorithm 1. However, we observed that, as long as the feasible domain is convex, the algorithm seems to converge to a global minimum, even when the path followed by the algorithm corresponds to a local increase the total power (see figure 4) . The global rate constraints in figure 4 is $R = 1.3 bits.s^{-1}.Hz^{-1}$.

The plots in this document all correspond to the graph presented in figure 1. We used $\mathbf{\Theta}. * \mathbf{G}$, where $.*$ is the term by term product, as provided below. This matrix corresponds to the fully interfering strategy (no interference is canceled). The line/column numbering corresponds to figure 1.

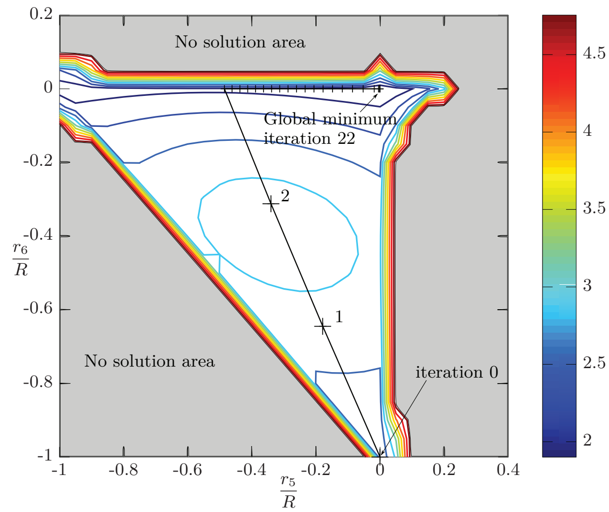

Fig. 4. Avoiding non convexity
Algorithm convergence in the free rate space. We represent the logarithm of the sum power as a function of two free link rate (link 5 and 6 see figure 3). The initial point is the direct transmission. We note that this algorithm strictly increase the criterion from iteration 0 to 2, and converge from 3 to 22 to the global minimum.

Conclusion

This paper addresses the problem of power and rate allocation in ad hoc networks, in the low SINR regime, where convexity of the criterion is a coarse approximation. A study of the problem allows to change the initial problem, to an unconstrained one. An analysis of the resulting problem demonstrates that if the rate to be transmitted is too high, cooperation will not improve over direct link. However, for reasonable rate requirements, the problem will have solutions, even under low SINR assumptions. We proposed an iterative algorithm which has been observed to be able to converge to the global optimum, even when initialized in the basin of attraction of a local minimum. However, we do not have any proof of convergence yet. Further work will be reported.

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