LOW COMPLEXITY EQUALIZATION FOR SINGLE CARRIER TRANSMISSIONS OVER DOUBLY-SELECTIVE CHANNELS USING LINEAR APPROXIMATION

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ABSTRACT

In this paper, a low complexity equalizer for single carrier transmissions over doubly selective channels is developed. The main attribute of the proposed equalizer is its low complexity compared to the existing equalizers, which is attained at the expense of a slight performance loss. Simulation results show that the performance of our proposed low complexity equalizer is as good as that of the conventional equalizers with extensive computation complexity.

Index Terms— Equalizer, single carrier, doubly selective channel.

1. INTRODUCTION

Recently, the demands for higher data rates have grown to an unprecedented level. Frequency selective propagation in channel is one of the major challenges in high data rate transmission systems. Conventional single carrier (SC) transmissions system suffers from inter symbol interference (ISI), which limits (or degrades) the system performance. On the other hand, insertion of the cyclic prefix at the transmitter and its removal at the receiver can mitigate the effects of ISI as in OFDM systems.

OFDM systems have been adopted in many wireless standards such as IEEE 802.11a/g, digital video broadcasting (DVB) and WiMAX due to its robustness against frequency selective channels. For time invariant channels, a computationally efficient single tap equalizer in frequency domain is sufficient for OFDM. However, OFDM transmits signals after IFFT and CP insertion and thereby exhibit high peak to average power ratio (PAPR). To process high PAPR signals properly, transmitters need to be equipped with good power amplifiers, which are relatively expensive. This drawback of OFDM systems motivates the adoption of the SC transmissions with CP in high data rate transmissions.

To achieve the transmission with low PAPR, the SC transmissions with CP, transmits the time domain signals after (CP) insertion [1]. In general, the computational complexities of the transmitters used for SC transmissions with CP are relatively lower than the complexities of OFDM transmitters. Thus SC transmissions with CP are suitable for uplink transmissions. Typical examples of the SC transmissions with CP are SC Frequency Division Multiple Access (FDMA), utilized for uplink in Long Term Evolution (LTE), and SC-OFDM.

For the fast moving mobile terminals, where the channel is time varying, to obtain sufficient performance of transmissions over time and frequency (doubly) selective channels, sophisticated equalizers at the receiver are necessary.

The channel between the transmitted block and the received block after CP removal can be described by a channel matrix. In general, $O(N^3)$ computations are required to invert a matrix of size N. Thus, without any modification, zero forcing (ZF) and minimum mean squared error (MMSE) equalizers of transmissions with CP have a complexity of $O(N^3)$. Fortunately, the channel matrix of a doubly selective channel is "pseudo-circulant", which enables us to develop equalizers with low complexities.

Due to IFFT at the transmitter and FFT at the receiver in OFDM systems, the composite channel over a doubly selective channel can be approximated as a circularly banded matrix [2, 3]. A linear equalization with $O(B^2N)$ computations has been proposed in [2], where B is the upper and the lower bandwidth of the banded matrix. An iterative soft equalization with $O(B^2N)$ computations per iteration has been presented in [3] by utilizing the banded approximation.

Since SC transmissions with CP and OFDM are dual, the equalizers developed for OFDM can be modified for SC transmissions with CP as in [4]. But, in general, additional computations are necessary to construct the equalizers for SC transmissions. In [5], low complexity iterative equalizations have been developed based on basis expansion models (BEM) for channels [6]. However, the disadvantages induced by the iterative algorithms still persist.

Here, we study SC transmission with CP and develop a non-iterative equalization requiring only $O(5N \log_2 N)$ computations. Our equalizer is an approximate version of the parallel FFT equalizer proposed in [7] that has the complexity of $O(2N^2 \log_2 N)$. The major reduction of complexity is achieved by using linearly interpolated channel coefficients and some approximations at the expense of a slight performance loss. Numerical simulations are provided to show that for a practical range of Doppler frequency, our equalizer exhibits a comparable performance to ZF equalizer with much less complexity.

2. TRANSMISSIONS OVER DOUBLY-SELECTIVE CHANNELS AND EQUALIZATION

Let us consider single carrier transmissions over time and frequency (doubly) selective channels. We assume that the continuous received signal is sampled at the information symbol transmission rate and that the maximum length of the discrete-time baseband equivalent channel is given by L.

Without loss of generality, the equivalent discrete-time baseband description of the ith received sample $y(i)$ can be expressed as [6]

$$y(i) = \sum_{l=0}^{L} h(i;l)u(i-l) + w(i), \quad l \in [0, L], \tag{1}$$

where $h(i;l)$ denotes the lth channel tap at time i, $u(i)$ the transmitted sample at time i and $w(i)$ the additive white Gaussian noise (AWGN) with zero mean and variance σ_w^2.

To mitigate the inter symbol interference (ISI), we utilize cyclic prefix (CP), which is adopted in many systems e.g., Orthogonal Frequency Division Multiplexing (OFDM), single carrier (SC-) OFDM, and SC Frequency Division Multiple Access (FDMA).

In OFDM, prior to CP insertion, inverse fast Fourier transform (IFFT) is performed on the serial data sequence to convert it to slow parallel data. On the other hand, in SC-OFDM and SC-FDMA, CP is inserted into the serial data sequence but the serial data sequence is not modulated by IFFT at the transmitter, which enables transmissions with low peak-to-average power ratio (PAPR) as compared to OFDM [1].

Due to the presence of CP, OFDM, CP-OFDM and SC-FDMA fall in block transmissions. Let us assume that the length N_{cp} of CP satisfies $N_{cp} \geq L$. This means that there are no inter block interferences (IBI) between data blocks. We denote $\bar{N} = N + N_{cp}$, where N is the block size. For simplicity of presentation, let us assume that N is a power of 2.

For clarity of exposition, we use two arguments n and m to describe the serial index $i = m\bar{N} + n$ for $n \in [-N_{cp}, N-1]$, where n is the index inside a block and m is the block index . We collect samples $\{y(i)\}$ into $N \times 1$ vectors defined as

$$\boldsymbol{y}^{(m)} = [y^{(m)}(0), \cdots, y^{(m)}(N-1)]^T \tag{2}$$

where the $(n+1)$st entry of the mth block of the received signal in the time domain is denoted as $y^{(m)}(n) = y(m\bar{N}+n)$ and its corresponding channel coefficient

$$h^{(m)}(n;l) := h(m\bar{N}+n;l). \tag{3}$$

Similarly, the transmitted signal $u(i)$ and the noise $w(i)$ are expressed as $\boldsymbol{u}^{(m)} = [u^{(m)}(0), \cdots, u^{(m)}(N-1)]^T$ and $\boldsymbol{w}^{(m)} = [w^{(m)}(0), \cdots, w^{(m)}(N-1)]^T$ respectively.

After the removal of the received portion corresponding to CP at the receiver, (1) can be rewritten as

$$\boldsymbol{y}^{(m)} = \boldsymbol{H}^{(m)}\boldsymbol{u}^{(m)} + \boldsymbol{w}^{(m)}, \tag{4}$$

where $\boldsymbol{H}^{(m)}$ is an $N \times N$ "pseudo-circulant" matrix composed of the channel coefficients as

$$\boldsymbol{H}^{(m)} = \begin{bmatrix} h^{(m)}(0;0) & \mathbf{0} & h^{(m)}(0;L)\cdots & h^{(m)}(0;1) \\ & \ddots \quad \ddots & \ddots & \vdots \\ \vdots & \ddots & & h^{(m)}(L-1;L) \\ h^{(m)}(L;L) & & \ddots \quad \ddots & \\ & \ddots & \ddots & \mathbf{0} \\ & \mathbf{0} \quad h^{(m)}(N-1;L)\cdots & h^{(m)}(N-1;0) & \end{bmatrix}. \tag{5}$$

If the channel coefficients in (4) are time-invariant within a block, i.e., block-constant fading, $\boldsymbol{H}^{(m)}$ becomes circulant. Since circulant $\boldsymbol{H}^{(m)}$ can be diagonalized by FFT and IFFT matrices, computationally efficient one-tap frequency domain equalization for OFDM works well when the channel state information (CSI) is available.

Similarly, thanks to the diagonalization, zero forcing (ZF) equalization for SC transmissions requires only $O(2N\log_2 N)$ computations (complex multiplications and additions), while the direct computation of the inverse of $\boldsymbol{H}^{(m)}$ consumes $O(N^3)$ computations. However, channels are often time-varying due to relative motion between the transmitter and the receiver. The change of channels degrades performances of ZF equalizers.

In OFDM, the composite channel between the data vector and the received vector after FFT is given by $\boldsymbol{F}\boldsymbol{H}^{(m)}\boldsymbol{F}^{\mathcal{H}}$. For doubly selective channels, $\boldsymbol{F}\boldsymbol{H}^{(m)}\boldsymbol{F}^{\mathcal{H}}$ is well approximated as a (circularly) banded matrix [2, 3]. It has been shown in [2] that using fast LDU decomposition of the banded matrix, an (approximate) MMSE equalization can be performed by $O(B^2N)$ computations, where B is the upper and the lower bandwidth of the banded matrix. An iterative soft equalization with $O(B^2N)$ computations per iteration has been also developed in [3].

The equalizers developed for OFDM using the banded approximation can be modified to apply to SC transmissions using CP. Although the equalizers may have low computational complexity, additional $O(2N^2\log_2 N)$ computations are required to construct an equalizer for SC transmissions, since they utilize the values of entries of $\boldsymbol{F}\boldsymbol{H}^{(m)}\boldsymbol{F}^{\mathcal{H}}$. Iterative equalizations without computations $\boldsymbol{F}\boldsymbol{H}^{(m)}\boldsymbol{F}^{\mathcal{H}}$ have been proposed in [5] based on basis expansion models (BEM) for channels [6]. Each iteration of the iterative algorithm has a small complexity of $O(BN\log_2 N)$ but it also inherits disadvantages of iterative methods.

In this paper, we deal with SC transmission with CP, focusing on SC-OFDM, and develop a non-iterative equalizer that requires only $O(5N\log_2 N)$ computations. The low complexity is achieved at the expense of performance loss due to approximation. Since we process the received vector block-wise, we omit the block index $(\cdot)^{(m)}$ in the following.

3. LOW COMPLEXITY EQUALIZATION USING LINEAR APPROXIMATION

Let us introduce the parallel FFT equalization [7], based on which our equalizer will be developed.

Define the DFT of the channel at time i and frequency $2\pi k/N$ as

$$H_k(n) = \sum_{l=0}^{L-1} h(n;l) e^{-j\frac{2\pi kl}{N}} \tag{6}$$

and denote the $(n+1)$st row of FFT matrix $\boldsymbol{F}$ as $\boldsymbol{f}_n$. Then, the output of the parallel FFT equalizer for $u(n)$ is given by

$$\boldsymbol{f}_n \boldsymbol{D}_n^{-1} \boldsymbol{F}^{\mathcal{H}} \boldsymbol{y} \tag{7}$$

where $\boldsymbol{D}_n$ is a diagonal matrix whose diagonal entries are the DFT of the channel at time n defined as

$$\boldsymbol{D}_n = \text{diag}\left(H_0(n), \ldots, H_{N-1}(n)\right). \tag{8}$$

It has been shown in [7] by numerical simulations that the parallel FFT equalizer exhibits good performance. However, since $O(2N \log_2 N)$ computations are required to equalize each $u(n)$ for $n = 0, 1, \ldots, N-1$, at least $O(2N^2 \log_2 N)$ computations are required in total. To reduce the computational complexity, we exploit the linear interpolation of channel coefficients as follows.

Suppose that $\{h(0;l), h(N-1;l)\}_{l=0,\ldots,L}$ are available. Let us assume that each channel coefficient can be described by

$$h(n;l) = h(0;l) + n\Delta h_l \tag{9}$$

where Δh_l is the unit of the variation given by

$$\Delta h_l = \frac{h(N-1;l) - h(0;l)}{N-1}. \tag{10}$$

This means that channel coefficients $h(n;l)$ for $n \in [1, N-2]$ can be obtained by the linear interpolation of $h(0;l)$ and $h(N-1;l)$. Actually, (9) approximately holds true for slowly changing channels.

Taking DFT of both sides of (9) with respect to l results in

$$H_k(n) = H_k(0) + n\Delta H_k \tag{11}$$

where

$$\Delta H_k = \sum_{l=0}^{L-1} \Delta h_l e^{-j\frac{2\pi kl}{N}} \tag{12}$$

If ΔH_k is small enough, then we can approximate $H_k^{-1}(n)$ as

$$\frac{1}{H_k(n)} = \frac{1}{H_k(0) + n\Delta H_k} \approx \frac{1}{H_k(0)}\left(1 - n\frac{\Delta H_k}{H_k(0)}\right). \tag{13}$$

Substituting the R.H.S. of (13) into $\boldsymbol{D}_n^{-1}$, one can express $\boldsymbol{f}_n \boldsymbol{D}_n^{-1} \boldsymbol{F}$ in (7) as

$$\boldsymbol{f}_n \left(\boldsymbol{D}_0^{-1} - n\boldsymbol{\Delta}\right) \boldsymbol{F}^{\mathcal{H}} \tag{14}$$

where

$$\boldsymbol{\Delta} = \text{diag}\left(\frac{\Delta H_0}{H_0^2(0)}, \frac{\Delta H_1}{H_1^2(0)} \ldots, \frac{\Delta H_{N-1}}{H_{N-1}^2(0)}\right) \tag{15}$$

Thus, the output of the parallel FFT equalizer for $u(n)$ is approximately given by

$$\begin{aligned} \hat{u}(n) &= \boldsymbol{f}_n \left(\boldsymbol{D}_0^{-1} - n\boldsymbol{\Delta}\right) \boldsymbol{F}^{\mathcal{H}} \boldsymbol{y} && (16) \\ &= \boldsymbol{f}_n \boldsymbol{D}_0^{-1} \boldsymbol{F}^{\mathcal{H}} \boldsymbol{y} - n\boldsymbol{f}_n \boldsymbol{\Delta} \boldsymbol{F}^{\mathcal{H}} \boldsymbol{y}. && (17) \end{aligned}$$

Let us define the ZF equalizer for the channel at time 0 as

$$\boldsymbol{E} = \boldsymbol{F} \boldsymbol{D}_0^{-1} \boldsymbol{F}^{\mathcal{H}} \tag{18}$$

and an auxiliary matrix $\boldsymbol{\Delta}_E$ as

$$\boldsymbol{\Delta}_E = \boldsymbol{F} \boldsymbol{\Delta} \boldsymbol{F}^{\mathcal{H}}. \tag{19}$$

If the channel is not time-varying, then the ZF equalized received vector is given by

$$\hat{\boldsymbol{y}}_0 = \boldsymbol{E}\boldsymbol{y}. \tag{20}$$

For the time-varying channel, we compute a *bias* vector defined as

$$\hat{\boldsymbol{y}}_b = \boldsymbol{\Delta}_E \boldsymbol{y}. \tag{21}$$

Then, $\hat{u}(n)$ is given by

$$\hat{u}(n) = [\hat{\boldsymbol{y}}_0]_{n+1} - n[\hat{\boldsymbol{y}}_b]_{n+1} \tag{22}$$

where $[\boldsymbol{a}]_m$ is the mth entry of the vector $\boldsymbol{a}$.

Now let us evaluate computational complexity of our equalization method, assuming that the channel coefficients $\{h(0;l), h(N-1;l)\}_{l=0,\ldots,L}$ are given.

We can obtain $\{\Delta h_l\}$ in (10) with $O(L)$ multiplications and additions. Then, two FFT are utilized to compute $\{H_n(0)\}_{n=0,\ldots,N-1}$ and $\{\Delta H_k\}_{k=0,\ldots,N-1}$. We take the inverse of $\boldsymbol{D}_0$ with $O(N)$ multiplications. Similarly, $\boldsymbol{\Delta}$ with $O(2N)$ multiplications.

One IFFT is performed to obtain

$$\tilde{\boldsymbol{y}} = \boldsymbol{F}^{\mathcal{H}} \boldsymbol{y} \tag{23}$$

then, $\hat{\boldsymbol{y}}_0 = \boldsymbol{F}(\boldsymbol{\Delta}\tilde{\boldsymbol{y}})$ is given by one FFT after multiplication of the $N \times N$ diagonal matrix $\boldsymbol{\Delta}$ and the vector $\tilde{\boldsymbol{y}}$ of size N, which requires $O(2N \log_2 N + N)$ multiplications and additions in total.

Similarly, $\hat{\boldsymbol{y}}_b$ is obtained by $O(N \log_2 N + N)$ multiplications and additions. Finally, $\hat{u}(n)$ in (22) is computed with $O(N)$ multiplications and additions.

In summary, our equalization requires $O(5N \log_2 N + 3N)$ multiplications and additions, which is quite smaller than $O(2N^2 \log_2 N)$ for equalizers in [2, 7] and is comparable with the multiplications and additions of just one iterations of the iterative equalizer [5].

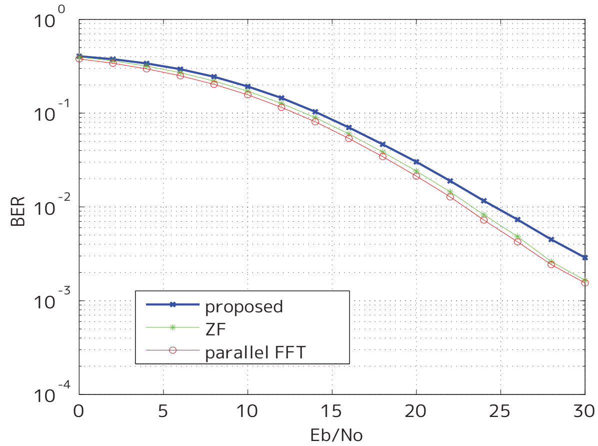

Fig. 1. Averaged BER of the proposed (with $\times$), ZF (with $*$), and the parallel FFT (with $\circ$) equalizer at $\bar{f}_D = 10^{-4}$.

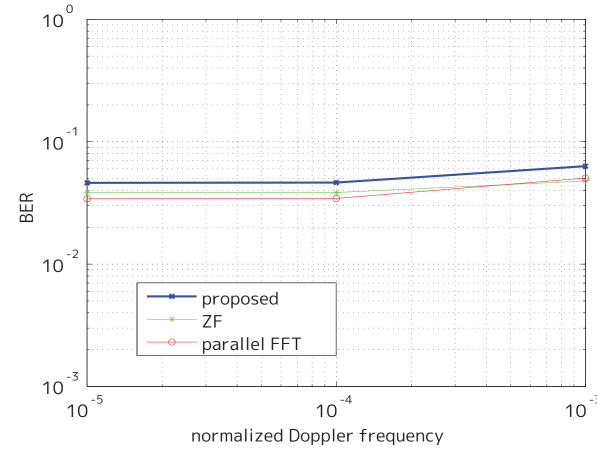

Fig. 2. Averaged BER of the proposed (with $\times$), ZF (with $*$), and the parallel FFT (with $\circ$) equalizer at 20dB.

4. SIMULATION RESULTS

We compare our equalizer with ZF equalizer and the parallel FFT equalizer [7]. It should be noted that for the same CSI, the BER of ZF equalizer is the performance limit of corresponding equalizers in [2, 5]. In the simulations, each transmitted vector has $N = 64$ and the length of the cyclic prefix is $N_{cp} = 16$. The average BER of QPSK symbols are computed by averaging the results for $5 \cdot 10^5$ Rayleigh channels with 10 complex zero-mean Gaussian taps of identical power profile, where channel taps are independent of each other and fade according to the Jakes fading model [8].

Using pilot symbols having the same power as information symbols, we estimate $\{h(0; l), h(N-1; l)\}_{l=0,\ldots,L}$ as detailed in [1] and linearly interpolate them as in (9) to estimate $h(n; l)$ for $n \in [1, N-2]$. The same estimates are used to construct three equalizers.

Fig. 1 shows BERs of three equalizers at the normalized Doppler frequency $\bar{f}_D = 10^{-4}$, where $\bar{f}_D$ is defined as the maximum Doppler frequency divided by the sampling frequency f_s. If $f_s = 10^7$, i.e., the bandwidth of our transmission is about 10 MHz, and the carrier frequency is assigned as 5GHz, then $\bar{f}_D = 10^{-4}$ corresponds to mobile terminal's velocity $v = 216$km/h.

Our equalizer exhibits slightly worse performance than other equalizers. This is due to the approximation, which enables a large reduction of the computations. It should be remarked that the BERs of three equalizers may be enhanced by optimizing power allocation between pilot and data symbols and by resorting to iterative channel estimation and equalization, but is beyond the scope of this paper.

For different $\bar{f}_D$. Fig. (2) depicts BERs at a fixed SNR $E_b/N_0 = 20$dB. For the practical range of $\bar{f}_D$, our equalizer attains comparable performance with other equalizers.

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