OFDM CARRIER FREQUENCY OFFSET CORRECTION BASED ON TYPE-2 CONTROL LOOP

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ABSTRACT

In this paper, we propose a robust data-aided carrier frequency offset (CFO) tracking algorithm for orthogonal frequency division multiplexing (OFDM) system. The problem is solved as a sequence of estimation and correction steps. We find the major contributors to the CFO measurement uncertainty are the additive white noise together with the inter-carrier interference (ICI). Our derivation results show the noise variance can be reduced via averaging while the ICI introduced uncertainty can be decreased by iterative CFO compensations. These considerations lead us to a Type-2 correction loop. Theoretical analysis and simulation results show our proposed algorithm is robust and able to compensate and track comparably large CFO.

***Index Terms*—** OFDM, Carrier Frequency Offset, Inter-carrier Interference, Pilot Tone Tracking, Type-2 Loop.

1. INTRODUCTION

OFDM occupies a unique position in modern communication systems. It appears in both wireless and wired systems. Wireless systems include the ubiquitous IEEE-802.11a Wireless Local Area Network (WLAN), Digital Audio Broadcasting (DAB), and Terrestrial Digital Video Broadcasting (DVB). Wired systems include Asymmetric Digital Subscriber Lines (ADSL) phone line and HomePlug power line communications. OFDM based next generation wireless network systems include Worldwide Interoperability for Microwave Access (WiMax) and Long Term Evolution (LTE).

OFDM offers many advantages to the communications task. The primary one is ease of combating severe distortion due to multipath channels and the ability to maximize signal throughput for severely impaired channels. OFDM systems exhibit high sensitivity to certain signal modulation characteristics and errors. The one we address here is carrier frequency offset (CFO). CFO is the frequency difference between the up and down conversion processes at the transmitter (Tx) and receiver (Rx). In this paper, we address the problem of optimum acquisition and tracking of CFO required for robust OFDM system performance. CFO estimation and compensation is commonly accomplished in two phases: a coarse CFO estimate and compensation is often performed in the preamble processing stage; and fine CFO tracking is performed by data aided or non-data aided methods. This paper deals with the second phase, fine CFO estimation and correction.

Early researchers [1] observed that CFO is the cause of Inter Channel Interference (ICI). The author in [2] considered the ICI as the primary interference source and proposed alternative pilot tone structures aimed at minimizing ICI. We show that performance degradation due to additive noise must be included in CFO related system analysis. The author in [3] showed the variance of the CFO estimate can be reduced by de-rotating the received signal with a search over the range of frequency offset. In practice, the searching process can be computationally unattractive and difficult to be implemented in real time communication systems. Also, our ultimate goal is to drive CFO to zero rather than making the best CFO measurement. In this context, because we have the ability to actively effect measurements, we will show that an iterative CFO compensation approach is more appropriate than a one-time estimate and correct approach. The author in [4], [5] proposed a non-data aided, loop based CFO correction scheme. Their CFO estimator is based on a phase detector (slicer), which can only handle very small CFO. This limitation is because the CFO information resides in successive OFDM blocks rather than a single OFDM block. In this paper, we propose a real time CFO tracking and compensation scheme which allows the compensation be updated on a block by block basis while still enjoying the benefits of high quality estimation.

2. SIGNAL MODEL

Let $X_{m,k}$ denote the data symbol for the k^{th} OFDM block, $m = 0,1,2,\dots N-1$, where N is the size of DFT/IDFT. Then, the k^{th} transmitted OFDM block can be written as:

$$x_k(n) = \frac{1}{N}\sum_{m=0}^{N-1} X_{m,k}\, e^{j\frac{2\pi}{N}mn}, n = 0,1,2,\dots,N-1 \tag{1}$$

Let the cyclic prefix (CP) size be γN, $0 < \gamma < 1$, thus the block length is $(1+\gamma)N$. Denote $\underline{x}_k^c$ as the CP added k^{th} transmitted OFDM block, the transmitted signal $s(\hat{n})$ can be expressed as:

$$s(n) = [\underline{x}_0^c \;\; \underline{x}_1^c \;\; \dots\dots \;\; \underline{x}_\infty^c], n = 0,1,\dots.\infty \tag{2}$$

Assuming the channel $h(n)$ is slow fading and the coarse CFO correction limits the residue CFO within one DFT bin, i.e., the residual CFO is in the range of $[-f_s/2N, +f_s/2N]$, where f_s is the Nyquist sampling frequency. Thus, the received signal $y(n)$, after preamble processing (before DFT), can be written as:

$$y(n) = s'(n)e^{j\phi n} + Z(n) \tag{3}$$

where $s'(n) = s(n) * h(n)$. And, ϕ is the residue CFO which can also be expressed as $\phi = \varepsilon\frac{2\pi}{N}$ with $|\varepsilon| < 0.5$. $Z(n)$ is Gaussian noise with zero mean and variance σ_{n}^2. Let $Y_{l,k}$ be the DFT result for the k^{th} OFDM block on the l^{th} bin and it can be written as:

$$Y_{l,k} = \sum_{n=0}^{N-1} s'[n+\gamma N+(1+\gamma)Nk]e^{j\phi[n+\gamma N+(1+\gamma)Nk]}e^{-j\frac{2\pi}{N}nl} + Z_{l,k} \tag{4}$$

where, $Z_{l,k} = \sum_{n=0}^{N-1} Z(n)e^{-j\frac{2\pi}{N}nl}$ is the DFT of the additive noise. We can treat it as band limited white noise with zero mean and variance σ_{n}^2. We also assume the CP length is longer than the channel impulse response, thus each received OFDM block experiences circular convolution with the channel. After some manipulation, Eq.(4) can be re-written as:

$$Y_{l,k} = \frac{1}{N} e^{j\phi[\gamma N+(1+\gamma)Nk]} \sum_{m=0}^{N-1} H_m X_{m,k} \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n[(m-l)+\varepsilon]} + Z_{l,k} \quad (5)$$

where H_m is the channel gain at the m^{th} subcarrier. Now, let us define $c_{m-l} = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n[(m-l)+\varepsilon]}$, which describes the ICI effect, i.e. when $\varepsilon = 0$, we find $c_0 = 1$ and $c_v = 0, \forall v \neq 0$, then Eq.(5) can be rewritten as:

$$\begin{aligned} Y_{l,k} = &\underbrace{e^{j\phi[\gamma N+(1+\gamma)Nk]} c_0 H_l X_{l,k}}_{Signal} \\ &\underbrace{+ e^{j\phi[\gamma N+(1+\gamma)Nk]} \sum_{\substack{m=0 \\ m\neq l}}^{N-1} c_{m-l} H_m X_{m,k}}_{ICI} + \underbrace{Z_{l,k}}_{Noise} \end{aligned} \quad (6)$$

3. CFO ESTIMATION

It has been mentioned in many places, i.e. [2], [3], that a CFO estimate can be made via Eq.(7) if one ignores ICI effects. This method is called pulse-pair method, which first appeared in the radar community [6], [7]. The author in [7] proved the estimator shown in Eq.(7) is unbiased, i.e., $E[\hat{\phi}] = \phi$. Yet, both authors in [2], [3] mentioned the loss of estimation accuracy of Eq.(7) due to ignoring the ICI term and proposed alternative estimators based on it.

$$\hat{\phi} = ATAN\left[\frac{Im\left(\frac{1}{M}\sum_{k=1}^{M} Y_{l,k-1}^{*} Y_{l,k}\right)}{Re\left(\frac{1}{M}\sum_{k=1}^{M} Y_{l,k-1}^{*} Y_{l,k}\right)}\right] \quad (7)$$

Let us revisit the estimator shown in Eq.(7). Notice that, $\hat{\phi}$ is obtained from $\frac{1}{M}\sum_{k=1}^{M} Y_{l,k-1}^{*} Y_{l,k}$. Thus the variance of $\hat{\phi}$ is directly related with the variance of $\frac{1}{M}\sum_{k=1}^{M} Y_{l,k-1}^{*} Y_{l,k}$. To quantitively analyze the estimator's performance, we compute the variance of $Y_{l,k-1}^{*} Y_{l,k}$. Let the l^{th} FFT bin be the pilot tone bin, and since the pilot tone has a fixed value like defined in 802.11a, i.e. $X_{l,k} = +1, \forall k$, $Y_{l,k-1}^{*} Y_{l,k}$ can be expressed in Eq.(8).

$$Y^{*}_{l,k-1} Y_{l,k} = \left\{ e^{j\phi[\gamma N+(1+\gamma)N(k-1)]} c_0 H_l + e^{j\phi[\gamma N+(1+\gamma)N(k-1)]} \sum_{\substack{m=0 \\ m\neq l}}^{N-1} c_{m-l} H_m X_{m,k-1} + Z_{l,k-1} \right\}^{*} \left\{ e^{j\phi[\gamma N+(1+\gamma)Nk]} c_0 H_l + e^{j\phi[\gamma N+(1+\gamma)Nk]} \sum_{\substack{m=0 \\ m\neq l}}^{N-1} c_{m-l} H_m X_{m,k} + Z_{l,k} \right\} \quad (8)$$

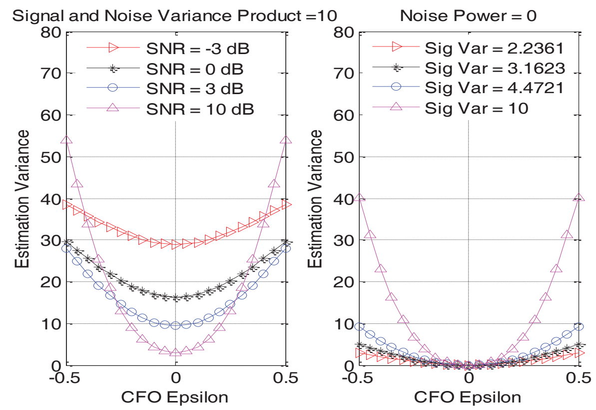
Assuming the transmitted data are i.i.d, zero mean with variance σ_s^2, and the data symbols are uncorrelated with noise. Thus, the expected value of $Y_{l,k-1}^{*} Y_{l,k}$ can be written as:

$$E\left[Y_{l,k-1}^{*} Y_{l,k}\right] = |c_0|^2 |H_l|^2 e^{j\phi(1+\gamma)N} \quad (9)$$

And the variance of $Y_{l,k-1}^{*} Y_{l,k}$ can be computed as:

$$\begin{aligned} Var\left[Y_{l,k-1}^{*} Y_{l,k}\right] = &2|c_0|^2|H_l|^2\sigma_s^2 C + \sigma_s^4 C^2 + 2\sigma_s^2\sigma_n^2 C \\ &+ 2|c_0|^2|H_l|^2\sigma_n^2 + \sigma_n^4 \end{aligned} \quad (10)$$

where C is defined as $C = \sum_{\substack{m=0 \\ m\neq l}}^{N-1} |c_{m-l}|^2 |H_l|^2$. It can be seen that $Var[Y_{l,k-1}^{*} Y_{l,k}]$ is composed by four sources: signal, ICI, noise and channel gain. We observed that for a moderate frequency selective channel, the channel gain coefficients H_m have very limited influence to Eq.(10). For analysis simplicity and to gain insight, we assume flat fading for this moment and set H_m to unity. Under a given signal and noise power, we plot the variance of $Y_{l,k-1}^{*} Y_{l,k}$ as a function of ICI parameter ε. Figure 1 (a) shows the variance curve when $\sigma_s^2\sigma_n^2 = 10$. As the SNR decreases, the curve becomes flatten across the horizontal axis, and shifts up along the vertical axis. The curves shown in Figure 1 (b) are obtained by setting noise variance to zero as compared with LHS. We find the signal introduced ICI decreases as ε approaches to zero. From Eq(10) and Figure 1, we conclude that the additive white noise is an important contributor for estimation variance. And, the estimation variance can be reduced by using the combination of two approaches: 1. Making M independent measurements as shown in Eq (7), which reduce the estimation variance by a factor of M; 2. Iteratively correct the CFO, since smaller ε gives lower estimation variance.



(a) (b)

Figure 1. Var[$Y_{l,k-1}^{*} Y_{l,k}$] as a Function of CFO Parameter ε

4. CFO CORRECTION

In this section, we introduce the CFO correction scheme based on Type-2 loop. The proposed CFO correction structure is shown in Figure 2. We can see that the correction loop is indeed a DSP based Phase Locked Loop (PLL). The 'Conjugate Product' block and 'Leaky Integrator', together with 'ATAN' form the PLL's phase detector, which is also the CFO estimator. The rest of the components are the standard PLL components.

4.1. CFO Estimator / Phase Detector

The conjugate product performs the operation described by Eq.(8). However, we now ignore the ICI term, because the estimation accuracy is largely dominated by the additive noise. Moreover, as the loop gradually corrects the CFO, the ICI effect also decreases. Notice that, scalar $c_0 H_l$ in the signal term of Eq.(6) introduces a gain and phase shift to the pilot tone which has no influence on CFO estimation, thus we can replace it with a constant. Similarly, we can also ignore the constant initial phase $e^{j\phi\gamma N}$ from the signal term in Eq.(6). The simplified expression $\hat{Y}_{l,k}$ for the k^{th} OFDM block at the output of DFT for the l^{th} bin can be rewritten as:

$$\hat{Y}_{l,k} = A e^{jk\theta_k} + Z_{l,k} \quad (11)$$

where A is the signal amplitude; θ_k is the angular frequency of the pilot tone at the k^{th} block. If we do not correct CFO (open loop), then θ_k is a constant for all k. And the CFO estimation problem can be viewed as estimating the frequency of a complex sinusoid. If we choose to correct CFO (closed loop), the problem then becomes driving θ_k to zero and this is the preferred approach. Since our goal is to drive CFO to zero, the estimator proposed in Eq.(7) has to be

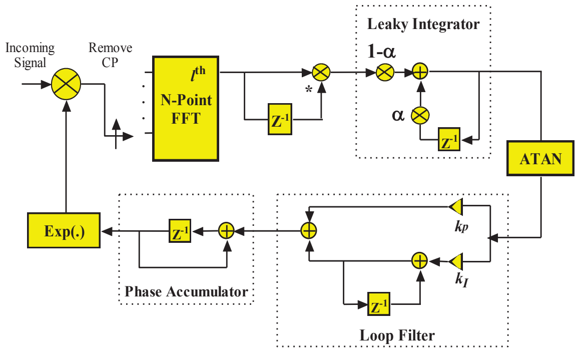


Figure 2. Proposed OFDM CFO Estimation and Correction Block Diagram

modified to accommodate the loop dynamics. The leaky integrator allows the CFO estimator to forget the old estimates while still conducting averaging in the sense of reducing the estimation variance. Note that, a single estimate of CFO requires at least two consecutive OFDM blocks. Thus, we restrict the CFO estimator to be updated once for every two OFDM blocks. Under this restriction, we have the relationship $\theta_k = \theta_{k-1}$. We now re-write the leaky integrator input signal $\hat{Y}^*_{l,k-1}\hat{Y}_{l,k}$, denoted as $P(k)$ for $k = 2i$, and $i = 1, 2, 3, \ldots \infty$.

$$P(2i) = A^2 e^{j\theta_{2i}} + Ae^{j2i\theta_{2i}} Z^*_{l,2i-1} + Ae^{-j(2i-1)\theta_{2i}} Z_{l,2i} + Z_{l,2i} Z^*_{l,2i-1} \quad (12)$$

The leaky integrator output signal $I(i)$ can then be expressed in Eq.(13), which contains four terms. The term T_1 is a deterministic signal term containing the measured angular frequency. All other terms are zero mean random interference. The SNR of $I(i)$ is written in Eq.(14). And, in high SNR, Eq.(14) can be approximated as Eq.(15). The parameter α is the forgetting factor of the leaky integrator, which is a design constant close to unity. A bigger α causes larger integrator memory and provides higher output SNR but sacrifices the tracking ability. We show the tradeoff relationship between α and SNR in Figure 3.

$$I(i) = (1-\alpha)\sum_{u=1}^{i} \alpha^{i-u} P(2u) \quad (13)$$

$$= (1-\alpha)\Bigg[\underbrace{\sum_{u=1}^{i} \alpha^{i-u} A^2 e^{j\theta_{2u}}}_{T1} + \underbrace{\sum_{u=1}^{i} \alpha^{i-u} A e^{j2u\theta_{2u}} Z^*_{l,2u-1}}_{T2} + \underbrace{\sum_{u=1}^{i} \alpha^{i-u} A e^{-j(2u-1)\theta_{2u}} Z_{l,2u}}_{T3} + \underbrace{\sum_{u=1}^{i} \alpha^{i-u} Z_{l,2u} Z^*_{l,2u-1}}_{T4}\Bigg]$$

$$SNR_{I(i)} = \frac{|T_1|^2}{Var\left[\frac{I(i)}{1-\alpha}\right]} = \frac{A^2}{2\sigma_n^2\left(1+\frac{\sigma_n^2}{2A^2}\right)} \frac{\left(1-\alpha^{-i}\right)^2\left(1-\alpha^{-2}\right)}{\left(1-\alpha^{-2i}\right)\left(1-\alpha^{-1}\right)^2} \quad (14)$$

$$SNR_{I(i)} \approx \frac{A^2}{2\sigma_n^2} \frac{\left(1-\alpha^{-i}\right)^2\left(1-\alpha^{-2}\right)}{\left(1-\alpha^{-2i}\right)\left(1-\alpha^{-1}\right)^2} = SNR_{Input}\,\frac{1}{2}\,\frac{\left(1-\alpha^{-i}\right)^2\left(1-\alpha^{-2}\right)}{\left(1-\alpha^{-2i}\right)\left(1-\alpha^{-1}\right)^2} \quad (15)$$

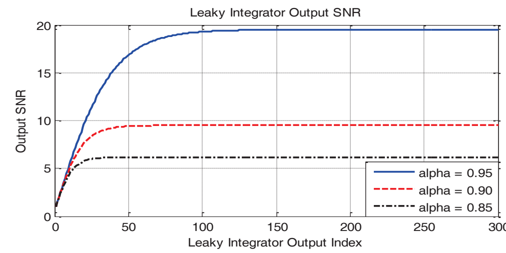


Figure 3. Leaky Integrator Output SNR (SNR_{Input}=1)

The last component of the phase detector is the ATAN operator, which computes the angle of the leaky integrator output $I(i)$. The ATAN output can be expressed as:

$$\hat{\theta}_{2i} = ATAN[I(i)] = \theta_{2i} + N(i), \; for\; i = 1, 2, \ldots \infty \quad (16)$$

where $N(i)$ is the phase noise term caused by T_2, T_3 and T_4 from Eq.(14). A rule of thumb is if the SNR is large, then the noise variance does not change between the input and output of the ATAN operation. This is because the noise terms introduced angles are in the linear region of the ATAN operator.

4.2. CFO Compensation Loop

In this section, we briefly introduce the correction loop and comment on the design of the loop parameters. Figure 4 presents the block diagram of the linearized phase locked loop. It is well known in control theory that a Type-2 loop is able to drive both step error and ramp error to zero. The variables used in the small signal model are the two input phase terms presented to the mixer $\theta(i)$ and $\tilde{\theta}(i)$. $\theta(i)$ can be thought as the input CFO; while $\tilde{\theta}(i)$ is the compensation value from the DDS. The transfer function in the absence of noise can be expressed:

$$H(z) = \frac{(k_P+k_I)\left(z - \frac{k_P}{k_P+k_I}\right)}{z^2 - 2\left(1-\frac{k_P+k_I}{2}\right)z + (1-k_P)} \quad (17)$$

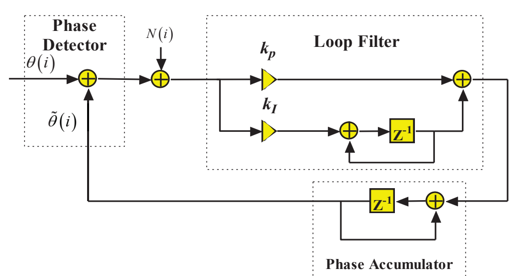


Figure 4. Linearized Discrete PLL Phase Model

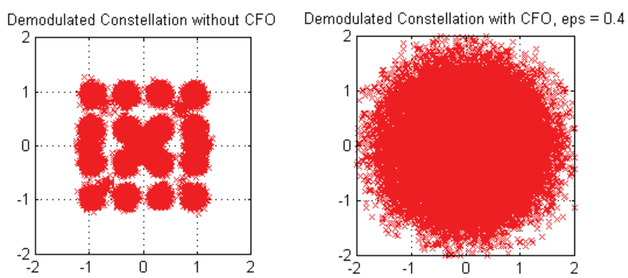
This transfer function can then be mapped into standard 2nd order continuous system via bilinear transform. The parameters can be computed through the following equations.

$$k_p = \frac{4\zeta\theta_n}{1+2\zeta\theta_n+\theta_n^2}\ ,\ \ k_I = \frac{4\theta_n^2}{1+2\zeta\theta_n+\theta_n^2} \tag{18}$$

where θ_n is the loop bandwidth and the parameter ζ is the damping factor selected to obtain a reasonable transient response with acceptable spectral peaking. Standard design practice sets the damping factor to values between 0.5 and 1.0. These values limit the transient time domain overshoot to less than 20% and the frequency domain spectral peaking to less than 1.25 dB (15%). Another common practice is to set the damping factor to 0.707 which results in 6% overshoot and zero spectral peaking. Based on the past experience and private correspondence [8], the minimum equivalent noise bandwidth occurs when $\zeta=0.5$, but with little penalty for setting it to 0.707.

5. SIMULATION RESULTS

Consider an OFDM system with 54 subcarriers and 64-Point DFT. Let the CFO parameter ε be 0.4, which is 40% of the DFT bin width. The SNR is set to be 15 dB and the loop bandwidth $\theta_n = 2\pi/200$. The channel we used is a near flat fading multipath channel. Figure 5 (a), (b) show the demodulated constellations of the received signal without and with CFO. We see from Figure 5(b) that the residue CFO causes the spin of all the subcarriers.



(a) (b)

Figure 5. Demodulated Constellations With and Without CFO

Figure 6 shows the results once we engage the proposed correction loop. The steady state constellation (upper left corner) shows the ICI has been significantly reduced. Notice that, although the CFO has been taken off, all the subcarriers from different OFDM blocks may have a phase shift. This residue phase shift can be corrected trivially by computing the offset angle of the pilot tone and complex rotate each subcarrier with that offset angle. The phase corrected constellation is shown in the upper right corner of Figure 6, where we can see that the constellation has been nicely de-spun and phase corrected. Another meaningful fact to examine is the phase accumulator and the CFO detector (after ATAN) time profile. We can see from Figure 6 that the phase accumulator reaches it steady state at ε = -0.4, which is the desired de-spin value. And, the CFO detector output oscillates around zero at around 75th OFDM block indicating the CFO has been taken off.

6. CONLUSION

We proposed a robust algorithm for estimating and correcting residue CFO for OFDM system. We showed that the CFO estimation problem ought to be combined with the compensation. Comparing with the existing methods, our algorithm mitigates the estimate impact from both additive noise and ICI. Moreover, the proposed algorithm is capable of correcting large CFO.

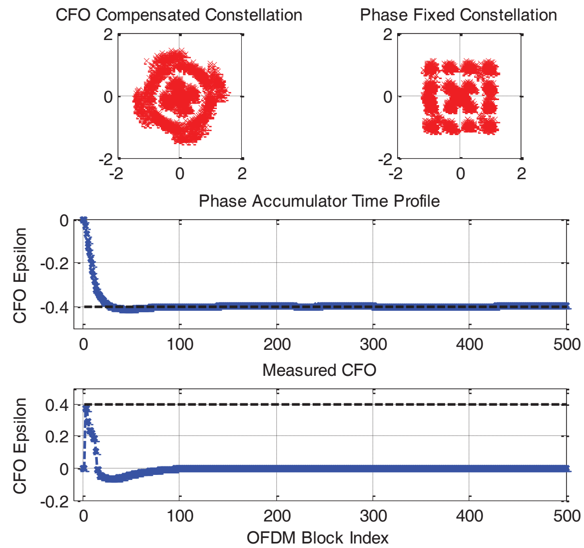


Figure 6. CFO Compensation Results

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