COST REFERENCE PARTICLE FILTER IN DATA AIDED CHANNEL ESTIMATION AND PHASE NOISE TRACKING FOR OFDM SYSTEMS

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ABSTRACT

In this paper, we present a robust technique based on cost reference particle filter (CRPF) for combined channel impulse response (CIR) estimation and phase noise (PN) tracking in OFDM systems without any priori information regarding the noise in the state and measurement equation. Contrary to previous works, we hold no assumption regarding the magnitude of the PN variance and hence no approximation is made to simplify the model. The algorithm employs CRPF along with a Rao-Blackwellization technique for CIR estimation assuming static channel state over a number of OFDM symbols. Numerical results are given to demonstrate the performance of the algorithm based on mean squared error (MSE).

***Index Terms*—** OFDM, Phase noise (PN), Channel impulse response (CIR), Cost reference particle filter (CRPF).

1. INTRODUCTION

The phase noise (PN) phenomenon and performance of OFDM systems in the presence of PN has been studied extensively, including in [1, 2]. In recent years, various classes of particle filter (PF) based methods have gained considerable interest in data detection, channel estimation and PN tracking applications using approximated PN model by assuming low noise variance [3]–[5]. In [4], a marginalized particle filter algorithm was introduced based on a state space model of the OFDM system assuming no priori knowledge of the channel and PN statistics. An approximation to the optimal importance function, [6], for sampling PN instances was derived based on a linearized PN model.

In this paper, we employ CRPF, introduced in [7], for robust data aided CIR estimation and PN tracking in an OFDM system requiring less computation than previously introduced PF based methods. As in [4], no priori knowledge of the channel and PN statistics is assumed, we furthermore do not regard knowledge of any probability density functions in the observation. Moreover, we refrain from making any assumption regarding the magnitude of the PN variance, therefore no linearization or approximation can be done to simplify the model.

The paper is outlined as follows. In Section 2, we introduce the OFDM received signal model and the PN disturbance commonly described in literatures by the random walk (wiener process). Section 3 will give detailed description of the employed algorithm for channel estimation and PN tracking. In Section 4, we give numerical results showing the performance of the proposed algorithm in mean squared error (MSE) of the CIR estimation. Section 5 concludes the paper.

2. CHANNEL MODEL

Considering the m-th OFDM symbol, N groups of B-arry data bits are encoded into complex symbol depending on the modulation order (2^B-QAM) at the transmitter such that each symbol in turn modulates one of N orthogonal sub-carriers. An IFFT is employed to generate the time domain channel inputs, $s_m(k)$, where k denotes the k-th sample within the m-th OFDM symbol. Assuming perfect frequency and timing synchronization, the equivalent complex baseband signal model at the receiver can be given by the equation below

$$r_m(k) = e^{j\theta_m(k)} \sum_{l=0}^{L-1} h_m(l) s_m(k-l) + w_m(k) \tag{1}$$

where $h_m(l)$ is the channel's CIR with L propagation paths, w_m denotes the additive zero mean complex gaussian channel noise with unknown variance σ_w^2 and $\theta_m(k)$ is the PN sample at time index k of the m-th OFDM symbol. Since the assumption is that the channel remains static for a number of OFDM symbols and the CIR estimation and PN tracking is done using a training sequence which is known at the receiver, we shall henceforth drop the subindex m.

PN in an oscillator perturbed by thermal noise can be modeled by wiener process which is a zero mean Gaussian random process having a variance that grows with time. The variance of the PN process increases linearly with time at a rate c which is an empirical parameter that depends on the quality of the oscillator, [2]. It is related to the single side band -3dB bandwidth, β, of the lorentzian spectrum given by

$$c = 4\pi\beta \tag{2}$$

Based on this assumption, for a sampling period of T_s at the receiver, an equivalent discrete time model of the PN evolution is given by

$$\theta(k) = \theta(k) + \varepsilon(k) \tag{3}$$

where $\varepsilon(k)$ is a Gaussian random variable with zero mean and variance $\sigma_\varepsilon^2 = cT_s$.

3. COST REFERENCE PARTICLE FILTERING

A standard dynamic state (DSS) space model is formulated as

$$\begin{aligned}\mathbf{x}_k &= g(\mathbf{x}_{k-1}) + \mathbf{v}_k,\\ \mathbf{y}_k &= f(\mathbf{x}_k) + \mathbf{w}_k\end{aligned} \tag{4}$$

where for sample index k, $\mathbf{x}_k$ and $\mathbf{y}_k$ are the state and observation vectors of the system respectively while $\mathbf{v}_k$ and $\mathbf{w}_k$ denote process and measurement noise of the system. The functions $g(\cdot)$ and $f(\cdot)$ are the state transition and the measurement functions respectively. Typically, the distribution of $\mathbf{v}_k$, $\mathbf{w}_k$ and the functions $g(\cdot)$ and $f(\cdot)$ determine the choice of methods employed for recursive estimation of the unknown state vector $\mathbf{x}_k$.

3.1. Introducing CRPF

CRPF is within the family of PF introduced to work in the Bayesian framework without requiring the noise statistics in the state and observation equation [7]. In standard PF algorithm, the posterior distribution associated with the DSS model, $p(\mathbf{x}_{0:k}|\mathbf{y}_{0:k})$ such that $\mathbf{y}_{0:k} = (\mathbf{y}_0, \mathbf{y}_1, \cdots, \mathbf{y}_k)$ and $\mathbf{x}_{0:k} = (\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_k)$, at time index k is represented by samples (particles) with corresponding weights, $\{\mathbf{x}^i_{0:k}, w^i_k\}^M_{i=1}$ where M is the number of particles [6]. The particles in CRPFs, on the other hand are associated with the cost and risk functions. The cost associated with the i-th particle given the current measurement, $\mathbf{x}^i_{0:k}|\mathbf{y}_{0:k}$, is determined by the recursive convex sum

$$\mathcal{C}(\mathbf{x}^i_{0:k}|\mathbf{y}_{0:k}, \lambda) = \lambda\mathcal{C}(\mathbf{x}^i_{0:k-1}|\mathbf{y}_{0:k-1}, \lambda) + \Delta\mathcal{C}(\mathbf{x}^i_k|\mathbf{y}_k) \tag{5}$$

where $\lambda \in (0, 1)$ is a forgetting factor which controls the contribution of past particles while evaluating the cost function and $\Delta\mathcal{C}(\mathbf{x}^i_{0:k}|\mathbf{x}_k)$ is the incremental cost function which indicates the accuracy of $\mathbf{x}^i_{0:k}|\mathbf{y}_{0:k}$. A high value or low value $\mathcal{C}(\mathbf{x}^i_{0:k}|\mathbf{y}_{0:k}, \lambda)$ respectively indicates that the current estimate $\mathbf{x}^i_k$ given the past and current measurement $\mathbf{y}_{0:k}$ is far from or close to the true value of the state $\mathbf{x}_k$. Moreover, the risk function which measures the adequacy of the state at the instant $k-1$ given the new measurement is given by

$$\mathcal{R}(\mathbf{x}^i_{k-1}|\mathbf{y}_k) = \Delta\mathcal{C}(g(\mathbf{x}^i_{k-1})|\mathbf{y}_k). \tag{6}$$

For every new observation $\mathbf{y}_{k+1}$, particles are made to randomly propagate updating their associated cost,

$$\Xi_{k+1} = \{\mathbf{x}^i_{k+1}, \mathcal{C}^i_{k+1}\}^M_{i=1} \tag{7}$$

where $\mathcal{C}^i_{k+1} = \mathcal{C}(\mathbf{x}^i_{0:k+1}|\mathbf{y}_{0:k+1}, \lambda)$ and $\mathcal{R}^i_k = \mathcal{R}(\mathbf{x}^i_k|\mathbf{y}_{k+1})$. Given the risk, $\mathcal{R}^i_k$, and cost, $\mathcal{C}^i_k$, of the i-th particle, a probability mass function (pmf), $\Pi^i_k \propto \mu(\mathcal{R}^i_k)$, is defined such that $\mu : \mathbb{R} \to [0, +\infty)$ is a real valued monotonically decreasing function. The concept of re-sampling which exists in standard PF algorithm is revived in CRPF. The particles are resampled according to Π^i_k in such a way that $\tilde{x}^i_k = x^j_k$ with probability Π^j_k and a new particle set is formed with $\tilde{\Xi}_k = \{\tilde{\mathbf{x}}^i_k, \tilde{\mathcal{C}}^i_k\}^M_{i=1}$ where $\tilde{\mathcal{C}}^i_k = \mathcal{C}^j_k$ for $\tilde{\mathbf{x}}^i_k = \mathbf{x}^j_k$. The steps in CRPF algorithm is outlined in Table 1. Within the standard DSS model provided in (4), the state space representation of the problem at hand is put in a matrix form as

$$\begin{aligned}\mathbf{x}_k &= \mathbf{x}_{k-1} + \mathbf{v}_k\\ r_k &= \mathbf{h}_k\mathbf{S}^H_k e^{j\theta_k} + w_k\end{aligned} \tag{8}$$

where the state vector $\mathbf{x}_k = [\theta_k, \mathbf{h}_k]$ such that $\mathbf{h}_k = [h_0, h_1, \ldots, h_{L-1}]$ is a vector with L channel taps, $\mathbf{S}_k = [s_k, s_{k-1}, \ldots, s_{k-L+1}]$ is the corresponding vector of transmitted symbols and H denotes the Hermitian operator. Since the channel is assumed static, the process noise vector is $\mathbf{v}_k = [\varepsilon_k, \mathbf{0}_{L\text{x}1}]$ where $\varepsilon_k \sim \mathcal{N}(\mathbf{0}; \sigma^2_\varepsilon)$ and $\mathbf{0}_{L\text{x}1}$ is an Lx1 vector of zeros . In the proceeding, the variances σ^2_ε and σ^2_w are assumed to be unknown.

Table 1. Standard CRPF algorithm

Initialization: for k=0 and draw M initial particles according to $\{\mathbf{x}^i_0\}^M_{i=1} \sim p(\mathbf{x}_0)$ and setting the cost for each to be $\mathcal{C}^i_0 = 0$.

Then the recursive update for each instant k goes by the steps

1. Compute the risk and normalized pmf (function of $\mathcal{R}^i_k$) associated with each particle $\mathbf{x}^i_k$ by
$\mathcal{R}^i_k = \lambda\mathcal{C}^i_{k-1} + ||\mathbf{y}_k - f(g(\mathbf{x}^i_{k-1}))||^q$.
and
$\pi(\mathcal{R}^i_k) = \frac{(\mathcal{R}^i_k - min[\mathcal{R}^j_k]^M_{j=1} + \delta)^{-\beta}}{\sum^M_{l=0}(\mathcal{R}^l_k - min[\mathcal{R}^j_k]^M_{j=1} + \delta)^{-\beta}}$
where $q \geq 1$ and $\beta, \delta > 0$ with δ being a very small positive real number which ensures numerical stability.

2. Resample particles based on the normalized pmf, Π^i_k such that a new particle set is formed
$\tilde{\Xi}_{k-1} = \{\tilde{\mathbf{x}}^i_{k-1}, \tilde{\mathcal{C}}^i_{k-1}\}^M_{i=1}$.

3. Particles are made to propagate based on,
$\mathbf{x}^i_k \sim p(\mathbf{x}_k|\tilde{\mathbf{x}}_{k-1})$
according to the distribution of the state process noise

4. Corresponding cost of each particle is computed by
$\mathcal{C}^i_k = \lambda\mathcal{C}^i_{k-1} + ||\mathbf{y}_k - f(\mathbf{x}^i_{k-1})||^q$
and the normalized pmf as a function of $\mathcal{C}^i_{k+1}$ is given as
$\pi(\mathcal{C}^i_k) = \frac{(\mathcal{C}^i_k - min[\mathcal{C}^j_k]^M_{j=1} + \delta)^{-\beta}}{\sum^M_{l=0}(\mathcal{C}^l_k - min[\mathcal{C}^j_k]^M_{j=1} + \delta)^{-\beta}}$

5. Estimate of the current state is finally obtained by taking the average with respect to Π^i_k
$\mathbf{x}^{mean}_k = \sum^M_{i=1} \pi(\mathcal{C}^i_k)\mathbf{x}^i_k$.

3.2. Rao-blackwellization

Although the state equation in (8) is linear in both state variables $\{\theta_k, \mathbf{h}_k\}$, the observation however is nonlinear in θ_k. Given the state variables at the time index k, the idea behind rao-blackwellization in PFs is to integrate out and marginalize some of the states in the posterior distribution analytically in order to improve the accuracy in the approximation. Moreover, it results in reduced dimension of states and thus reduced number of particles employed in PF algorithm. The marginalization follows from Baye's rule

$$p(\theta_k, \mathbf{h}_k | r_{0:k}) = p(\mathbf{h}_k | \theta_k, r_{0:k}) p(\theta_k | r_{0:k}) \tag{9}$$

in which case, $p(\mathbf{h}_k|\theta_k, r_k)$ is analytically tractable being circular gaussian distribution with linear state variable $\mathbf{h}_k$. In this paper, standard kalman filtering is employed for optimal estimation of the channel taps $\mathbf{h}_k$ given the PN estimates obtained by CRPF algorithm.

3.3. CIR estimation and PN tracking

Having separated the state variable into linear and non-linear parts, the proposed method relies on the available observations $r_{0:k}$ without a priori assumption of the probability distribution of the measurement noise, $p(w_k)$. The channel is static and conditionally linear in the measurement model for which the posterior distribution is given by $p(\mathbf{h}_k|\theta_k, r_{0:k}) \sim \mathcal{N}(0, \sigma_w^2)$. Sequential updating of the CIR, $\mathbf{h}_k$, and the covariance matrix, $\mathbf{P}_{k|k-1}$, of estimation error corresponding to each particle is given by

$$\begin{aligned} \mathbf{h}^i_{k|k-1} &= \mathbf{h}^i_{k-1|k-1} \\ \mathbf{P}^i_{k|k-1} &= \mathbf{P}^i_{k-1|k-1} \end{aligned} \tag{10}$$

for $i = 1, \ldots, M$. Following which the kalman gain $\mathbf{K}^i_k$ is computed by

$$\mathbf{K}^i_k = \mathbf{P}^i_{k|k-1} (\mathbf{S}_k e^{j\theta^i_k})^H (\mathbf{S}_k \mathbf{P}^i_{k|k-1} \mathbf{S}_k + \sigma^2_{w_k})^{-1} \tag{11}$$

An unbiased numerical estimator of $\sigma^2_{w_k}$ was introduced in [8] and is computed for each particle

$$\hat{\sigma}^{2,(i)}_{w_k} = \sum_{n=0}^{k} \left(\frac{\|r_k - (\mathbf{h}^i_k \mathbf{S}_k e^{j\theta^i_k} + \chi^i_k)\|^2}{k-1} - \frac{\mathbf{S}_k \mathbf{P}^i_{k|k-1} \mathbf{S}_k}{k} \right) \tag{12}$$

where $\chi^i_k = \frac{1}{k} \sum_{n=0}^{k} (r_k - \mathbf{h}^i_{k|k-1} \mathbf{S}_k e^{j\theta^i_k})$. The state update follows in the standard form

$$\begin{aligned} \mathbf{h}^i_{k|k} &= \mathbf{h}^i_{k|k-1} + \mathbf{K}^i_k (r_k - \mathbf{h}^i_{k|k-1} \mathbf{S}_k e^{j\theta^i_k}) \\ \mathbf{P}^i_{k|k} &= (\mathbf{I} - \mathbf{K}^i_k \mathbf{S}_k e^{j\theta^i_k}) \mathbf{P}^i_{k|k-1} \end{aligned} \tag{13}$$

CRPF algorithm is employed to approximate the marginal distribution $p(\theta_k|r_{0:k})$ using particles with their associated cost. The implementation has an advantage over standard PF since expression of the expected posterior distribution is not required and thus computationally less demanding. In (8), the state and observation noise are zero mean with unknown variances. The algorithm is therefore set to run with initial parameters and states

$$\lambda, q, \delta, \beta, M, \left\{ \sigma^{2,(i)}_{\varepsilon_0}, \sigma^{2,(i)}_{w_0}, \theta^i_0 \sim p(\theta_0), \mathbf{h}^i_{0|0}, \mathbf{P}^i_{0|0}, \mathcal{C}^i_0 = 0 \right\} \tag{14}$$

where $\{\cdot\}$ contains the parameters and states that are sequentially updated with time and $p(\theta_0)$ is a distribution from which particles are initially drawn. Given the set of initial particles, $\{\theta^i_0, \mathcal{C}^i_0 = 0\}^M_{i=1}$, the corresponding risk function is sequentially computed for time index k

$$\mathcal{R}^i_k = \lambda \mathcal{C}^i_{k-1} + \|r_k - \mathbf{h}^i_{k|k-1} \mathbf{S}_k e^{j\theta^i_{k-1}}\|^q \tag{15}$$

In order to avoid the re-sampling stage and computation of $\pi^i_k(\mathcal{R}^i_k)$, once again a technique employed in [8] and showed to have similar performance as the re-sampling technique in the standard CRPF algorithm (Table 1) is employed. Particles are sorted in ascending order of the corresponding risks and the first M/N particles are chosen and replicated N times forming a new particle set, $\{\tilde{\theta}^i_{k-1}, \tilde{\mathcal{C}}^i_{k-1}\}^M_{i=1}$.

Particles are then propagated based on the state transition distribution, $\theta^i_k \sim p(\theta_k|\tilde{\theta}^i_{k-1}) = \mathcal{N}(0, \sigma^{2,(i)}_{\varepsilon_k})$, for which the unknown variance $\sigma^{2,(i)}_{\varepsilon_k}$ is computed in time adaptive form by

$$\begin{aligned} \sigma^{2,(i)}_{\varepsilon_k} &= \sigma^{2,(i)}_{\varepsilon_{k-1}}, && \text{for } k \le 10 \\ \sigma^{2,(i)}_{\varepsilon_k} &= \frac{k-1}{k} \sigma^{2,(i)}_{\varepsilon_{k-1}} + \frac{\|\theta^i_k - \tilde{\theta}^i_{k-1}\|^2}{k} && \text{for } k > 10 \end{aligned} \tag{16}$$

and the associated cost of each particle is

$$\mathcal{C}^i_k = \lambda \mathcal{C}^i_{k-1} + \|r_k - \mathbf{h}^i_{k|k} \mathbf{S}_k e^{j\theta^i_k}\|^q \tag{17}$$

Having computed the normalized pmf as a function of the cost, $\pi^i_k(\mathcal{C}^i_k)$, the final estimates of the CIR and the PN sample at time index k is given by taking the average as

$$\hat{\mathbf{h}}_k = \sum_{i=1}^{M} \pi^i_k(\mathcal{C}^i_k) \mathbf{h}^i_{k|k} \ , \ \hat{\theta}_k = \sum_{i=1}^{M} \pi^i_k(\mathcal{C}^i_k) \theta^i_k \tag{18}$$

The proposed algorithm for CIR estimation and PN tracking is outlined by the steps in Table. 2.

4. NUMERICAL RESULTS

Performance of the proposed method was analyzed from simulations using a known OFDM symbol with $N = 80$ sub-carriers employing 4-QAM to modulate each sub-carrier. Moreover, an $L = 10$ tap rayleigh channel is considered and $M = 100$ particles are drawn from a uniform distribution $\theta^i_0 \sim p(\theta_0) = [-0.5, 0.5]$ to form initial cloud of PN samples. Corresponding to each particle, state and error covariance matrix in the kalman filter are initialized by $\mathbf{h}^i_{0|0} = \mathbf{0}_{L\text{x}1}$ and $\mathbf{P}^i_{0|0} = \frac{1}{L}\mathbf{I}_L$ respectively, where $\mathbf{0}_{L\text{x}1}$ is an Lx1 vector of

Table 2. CRPF algorithm for CIR estimation and PN tracking

1. Initialize parameters and states in (14).
2. Proceed with the linear state (CIR) propagation stage of the Kalman filter in (10).
3. Employ CRPF algorithm as detailed in (15)-(17) and obtain final estimate of the PN sample at time index k by taking the average with respect to $\pi_k^i(\mathcal{C}_k^i)$, (18).
4. Kalman state update of the CIR follows using (11)-(13).
5. Once again final estimate of the CIR is obtained by taking the mean with respect to $\pi_k^i(\mathcal{C}_k^i)$ as in (18).

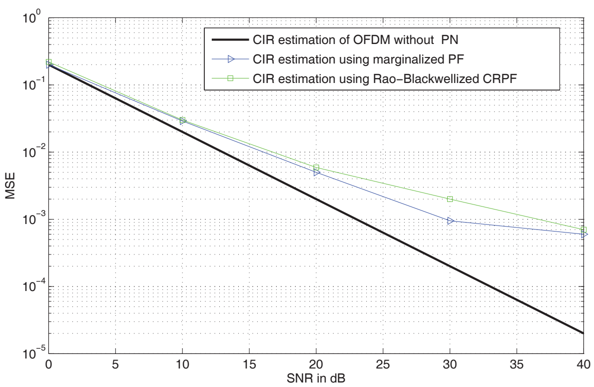

Fig. 1. MSE of CIR estimate as a function of SNR for a PN variance $\sigma_\varepsilon = 1 \times 10^{-3}$

zeros and $\mathbf{I}_L$ is the identity matrix. Furthermore, the channel noise and PN variances are initialized by values drawn from the uniform distribution $\sigma_{w_0}^{2,(i)} \sim [0,1]$ and $\sigma_{\varepsilon_0}^{2,(i)} \sim [0,0.1]$ respectively. The remaining parameters in (14) were set to be $\lambda = 0.1, q = 2, \delta = 0.01$ and $\beta = 2$.

The mean squared error (MSE) of the CIR estimate was taken as performance measure based on 1000 OFDM symbols and was computed

$$MSE_{\mathbf{h}} = \sum_{n=1}^{1000} \sum_{l=1}^{L} (h_l^n - \hat{h}_l^n)^2. \tag{19}$$

It can be seen from Fig. 4 that for lower SNR values, having no priori knowledge of the measurement distribution will have minimal relevance to the estimation performance. For higher SNRs, the MSE of the proposed method has a slightly higher but comparable value compared with the MSE of the CIR estimate employed using marginalized PF in [4]. Given that we made no assumption regarding the probability distribution of the measurement noise when tracking the PN noise, the impact due to the absence of the information on the performance is felt at higher SNRs.

5. CONCLUSIONS

In this paper, we presented CRPF algorithm combined with rao-blackwellization technique for robust CIR estimation and PN tracking, making no assumptions regarding the noise statistics. The outlined algorithm required no linearization or approximation of the measurement model. Moreover, it does not require computation of the optimal importance function and employs a simplified particle re-sampling technique. Therefore compared to previously developed PF based techniques [3, 4], the presented algorithm has less computational requirement with comparative performance.

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