MATCHED FILTERING ASSISTED ENERGY DETECTION FOR SENSING WEAK PRIMARY USER SIGNALS

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ABSTRACT

Energy detection is widely used by cognitive radios for spectrum sensing. During a silent period, secondary users (SUs) are kept silent so that the energy detector does not confuse SU signals for primary user (PU) signals. Due to imperfect coordination, an SU may transmit during a silent period and cause possible false alarms. We propose to leverage matched filters that already exist in many SUs to alleviate the impact of such SU interference by combining the matched filtering result and the energy detection result. The analysis shows that for practical purposes, our algorithm virtually eliminates all of the negative impact of SU interference with only negligible penalty in delay and energy consumption.

***Index Terms*—** Cognitive radio, spectrum sensing, matched filter, energy detection.

1. INTRODUCTION

Spectrum sensing is an important technique to allow a cognitive radio [1] to learn the radio environment. For Secondary Users (SUs) that are cognitive radios and operate in the TV broadcast channels, although the latest FCC ruling [2] does not require spectrum sensing, it does encourage further research on spectrum sensing because spectrum sensing has a number of advantages over the alternative geo-location database approach. In the database approach, a cognitive radio needs to know its geo-location before using the database. However, such information may not always be available (e.g., in certain indoor environments), and database access may not always be possible, e.g., in some remote areas. Moreover, the database approach creates a single point of failure, leading to security concerns. Spectrum sensing may be an integral part of future cognitive radios.

Energy detection is widely used for spectrum sensing because of its simplicity and cost effectiveness – no need for a dedicated detector for each possible target signal, despite its inferior performance compared to matched filtering [3]. Since an energy detector does not differentiate the target signal from interfering signals, SUs must be silenced. Due to reasons such as loss of control messages and time synchronization errors, the coordination of silent periods may be imperfect. As a result, during a silent period, an SU may transmit, causing a false alarm at the energy detector and unnecessarily reducing spectrum access opportunities. Many radios, such as IEEE 802.11 (WiFi), use matched filtering to detect an incoming packet (also known as packet synchronization). These built-in matched filters can be leveraged to improve the performance of spectrum sensing.

In this paper, we focus on the low SNR regime, a major challenge to spectrum sensing, and propose an algorithm to leverage such matched filters to improve the performance of spectrum sensing. The algorithm can be applied to a vast range of cognitive radios and requires minimal hardware change. The remainder of this paper is organized as follows. Section 2 describes the system model, Section 3 presents the algorithms with analysis, and Section 4 concludes the paper.

2. SYSTEM MODEL

The system architecture is shown in Fig. 1. There are a number of Primary Users (PUs): a PU transmitter and N_p passive PU receivers. The value of N_p is irrelevant since passive PU receivers do not leak significant energy. There are two Secondary Users (SUs): an SU transmitter and an SU receiver. The two SUs may represent a WiFi network at home, where the SU receiver is an Access Point and the SU transmitter is a Station. The SU receiver is equipped with an energy detection (ED) based Spectrum Sensor (SS) and a Sensing Processor (SP). The SU receiver uses matched filtering (MF) to detect incoming SU packets. The matched filtering result and the energy detection result are combined at the SP to make a decision. The channel gains c_m, c_p, c_e are shown in Fig. 1, where we normalize the gain of the channel from the SU transmitter to the Spectrum Sensor. If the MF and the SS share the same receive antenna, then $c_m = c_e$ and $c_p = 1$. They may not share the same receive antenna, e.g., the SS is a standalone module connected to the rest of the SU receiver.

Due to imperfect coordination, when the SU receiver is in a silent period performing energy detection, the SU transmitter may transmit, potentially causing a false alarm. Let the PU

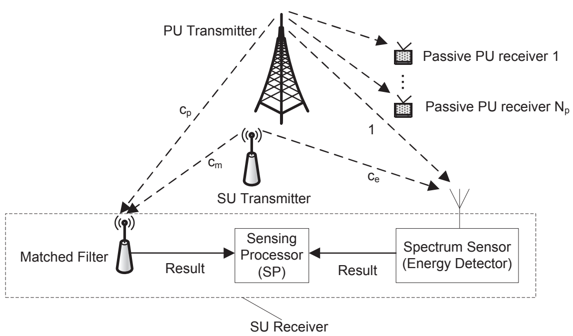

Fig. 1. The system architecture.

signal be $x_p(n)$ for $n = 1, ..., N$ with mean zero and variance σ_p^2, and the SU signal be $x_s(n)$ with mean zero and variance σ_s^2 for $n = 1, ..., M$ and with $x_s(n) = 0$ for $n > M$, which captures the scenario where the SU signal may not span the entire observation window N of energy detection. Among the M samples of the SU signal, $L \gg 1$ are used for matched filtering, which could implement packet synchronization in practice for systems such as IEEE 802.11. We assume that the SU transmitter mistakenly transmits during a silent period with probability $q \ll 1$, and such transmission is independent of the PU transmission. Note that $L \leq M \leq N$.

We focus on the case where the PU signal is weak, i.e., $Mc_m^2\sigma_s^2 \geq N((1+\delta)\sigma_p^2 + \delta\sigma_w^2)$ where $\delta > 0$ and σ_w^2 is the noise power at the energy detector.

3. ALGORITHMS

We first characterize energy detection and matched filtering algorithms, and then propose two algorithms leveraging matched filtering. We define

$$\alpha_i(\lambda) := P(\text{MF decides } A_1|h_i) \text{ with threshold } \lambda) \quad (1)$$
$$\beta_i(\lambda) := P(\text{ED decides } H_1|h_i) \text{ with threshold } \lambda) \quad (2)$$

where $i = 0, ..., 3$, and h_i, A_1 and H_1 will be defined later.

3.1. Energy Detection at the SU Receiver

There are two possible cases: (1) in the ideal case, there is no SU signal, and this is the case usually considered in the literature; (2) in the practical case, SU signals may be present.

Ideal Case: The spectrum sensor may receive one of the following two possible signals:

$$H_0 : z(n) = w(n), \quad n = 1, ..., N \quad (3)$$
$$H_1 : z(n) = x_p(n) + w(n), \quad n = 1, ..., N \quad (4)$$

where $w(n)$ are i.i.d Gaussian noise with mean zero and variance σ_w^2. The optimal energy detector is

$$r := \sum_{n=1}^{N} |z(n)|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \quad (5)$$

where λ is the test threshold.

For large N, we have the probability of false alarm and the probability of detection [4]

$$P_F^e(\lambda) = \int_\lambda^\infty P(r|H_0)dr = Q\left(\frac{\lambda - N\sigma_w^2}{\sqrt{2N}\sigma_w^2}\right) = \beta_0(\lambda) \quad (6)$$

$$P_D^e(\lambda) = \int_\lambda^\infty P(r|H_1)dr = Q\left(\frac{\lambda - N(\sigma_w^2+\sigma_p^2)}{\sqrt{2N}(\sigma_w^2+\sigma_p^2)}\right) = \beta_1(\lambda), \quad (7)$$

respectively, where $\gamma_p := \sigma_p^2/\sigma_w^2$ is the PU SNR at the energy detector, and $Q(\cdot)$ is the Q function [4].

Non-ideal Case: The SU transmitter may or may not transmit during a silent period. Therefore, hypotheses H_0 and H_1 each splits into two hypotheses. The energy detector may receive one of the following four possible signals:

$$h_0 : z(n) = w(n) \quad (8)$$
$$h_1 : z(n) = x_p(n) + w(n) \quad (9)$$
$$h_2 : z(n) = c_e x_s(n) + w(n) \quad (10)$$
$$h_3 : z(n) = x_p(n) + c_e x_s(n) + w(n) \quad (11)$$

where $n = 1, ..., N$ and h_i are the hypotheses. As an example, (8) says that, under h_0, i.e., if neither SU signal nor PU signal is transmitted, the SS will receive $z(n) = w(n)$.

Since we have assumed that the SU mistakenly transmits during a silent period with probability q and such transmission is independent of the PU transmission, we have that $P(h_0) = (1-q)P(\text{PU does not transmit})$ and $P(h_2) = qP(\text{PU does not transmit})$. The SS is unaware of the existence of the SU signal and continues to use the decision regions used in the ideal case. A false alarm occurs if the SS decides H_1 when h_0 or h_2 has occurred, with probability

$$P'_F(\lambda) = P(\text{ED decides } H_1|h_0 \cup h_2) = (1-q)\beta_0(\lambda) + q\beta_2(\lambda) \quad (12)$$

where $\beta_0(\lambda)$ defined in (2) is given in (6), and it can be shown

$$\beta_2(\lambda) = Q\left(\frac{\lambda - Mc_e^2\sigma_s^2 - N\sigma_w^2}{\sqrt{2(Mc_e^4\sigma_s^4 + 2Mc_e^2\sigma_s^2\sigma_w^2 + N\sigma_w^4)}}\right). \quad (13)$$

Similarly, we have the probability of detection

$$P'_D(\lambda) = P(\text{SS decides } H_1|h_1 \cup h_3) = (1-q)\beta_1(\lambda) + q\beta_3(\lambda) \quad (14)$$

where $\beta_1(\lambda)$ defined in (2) is given in (7) and

$$\beta_3(\lambda) = Q\left(\frac{(\lambda - Mc_e^2\sigma_s^2 - N(\sigma_w^2+\sigma_p^2))/\sqrt{2}}{\sqrt{Mc_e^4\sigma_s^4 + 2Mc_e^2\sigma_s^2(\sigma_w^2+\sigma_p^2) + N(\sigma_w^2+\sigma_p^2)^2}}\right) \tag{15}$$

3.2. Matched Filtering at the SU Receiver

There are two possible cases: (1) in the ideal case, there is no PU signal; (2) in the non-ideal case, PU signals may be present.

Ideal Case: The matched filter receives

$$A_0 : y(n) = v(n) \tag{16}$$
$$A_1 : y(n) = c_m x_s(n) + v(n) \tag{17}$$

where $n = 1, ..., L \leq M$, $v(n)$ is i.i.d noise following a Gaussian distribution $\mathcal{N}(0, \sigma_v^2)$ and $c_m > 0$ is the channel gain. The optimal detector is a matched filter [4]:

$$T = \sum_{i=1}^{L} y(i)c_m x_s(i) \underset{A_0}{\overset{A_1}{\gtrless}} \eta, \tag{18}$$

where η is the test threshold. Since c_m and $x_s(i)$ are known to the SU receiver and $y(i)$ are Gaussian under both hypotheses, T is also Gaussian. We have the probability of false alarm and the probability of detection [4]

$$P_F^m(\eta) = \int_\eta^\infty P(T|A_0)dT = Q\left(\frac{\eta}{\sqrt{L}\sigma_v c_m \sigma_s}\right) = \alpha_0(\eta) \tag{19}$$

$$P_D^m(\eta) = \int_\eta^\infty P(T|A_1)dT = Q\left(\frac{\eta - Lc_m^2\sigma_s^2}{\sqrt{L}\sigma_v c_m \sigma_s}\right) \alpha_2(\eta) \tag{20}$$

respectively, where $\gamma_s := c_m^2\sigma_s^2/\sigma_v^2$.

Non-ideal Case: The matched filter receives one of the following four possible signals:

$$h_0 : y(n) = v(n) \tag{21}$$
$$h_1 : y(n) = c_p x_p(n) + v(n) \tag{22}$$
$$h_2 : y(n) = c_m x_s(n) + v(n) \tag{23}$$
$$h_3 : y(n) = c_p x_p(n) + c_m x_s(n) + v(n) \tag{24}$$

where $n = 1, ..., L$, $v(n)$ is the noise, $x_p(n)$ is the PU signal, $x_s(n)$ is the SU signal, and $c_p > 0$ satisfying $c_p^2\sigma_p^2/\sigma_v^2 \ll 1$ is the channel gain. Note that h_i in (21)-(24) and h_i in (8)-(11) refer to the same event. Invoking the definitions in (1) and (2), we have for large L

$$\alpha_1(\eta) = Q\left(\frac{\eta}{\sqrt{L(\sigma_v^2 + c_p^2\sigma_p^2)}c_m\sigma_s}\right) \tag{25}$$

$$\alpha_3(\eta) = Q\left(\frac{\eta - Lc_m^2\sigma_s^2}{\sqrt{L(\sigma_v^2 + c_p^2\sigma_p^2)}c_m\sigma_s}\right) \tag{26}$$

Algorithm 1 Immediate Decision

Wait till the next silent period
Receive decisions from matched filter and energy detector
IF matched filter decides A_1 *// presence of SU signals*
  Decision of SP $\leftarrow H_0$ (absence of PU signals)
ELSE
  Decision of SP $\leftarrow$ decision of energy detector
END

3.3. Proposed Algorithms for the Sensing Processor

We propose to leverage the matched filter at the SU receiver to identify mistaken SU transmissions occurring during a silent period. The matched filter reports the result (presence or absence of an SU signal) to the SP at the end of each silent period. The SP can take two approaches to leveraging the matched filtering result. In the first approach, the SP makes a decision immediately (hence Immediate Decision) after receiving the results from both the energy detector and the matched filter. In the second approach, the SP delays its decision (hence Delayed Decision) until the next round of spectrum sensing if the matched filter reports the presence of an SU signal, and makes a decision immediately otherwise.

3.3.1. Algorithm 1: Immediate Decision

In a dynamic spectrum access environment, it is often reasonable to assume that PU under utilizes the spectrum, i.e., $P(H_0) > P(H_1)$. If the matched filter reports a detection, the optimal SP decision is to declare the absence of PU signals. To see this, let the probability that the SP decides H_0 be p_0. The average probability of a decision error $(1-p_0)P(H_0) + p_0P(H_1)$ is minimized at $p_0 = 1$. With this decision, the probability of false alarm is

$$\begin{aligned} P_F^{(1)}(\zeta) &= P(\text{SP decides } H_1 | h_0 \cup h_2) \\ &= (1-q)(1-\alpha_0(\zeta))\beta_0(\zeta) + q(1-\alpha_2(\zeta))\beta_2(\zeta) \end{aligned} \tag{27}$$

where ζ is the threshold used by the SS, and $\alpha_0(\zeta)$, $\alpha_2(\zeta)$, $\beta_0(\zeta)$ and $\beta_2(\zeta)$ are given in (19), (20), (6) and (13), respectively. The probability of detection is

$$\begin{aligned} P_D^{(1)}(\zeta) &= P(\text{SP decides } H_1 | h_1 \cup h_3) \\ &= (1-q)(1-\alpha_1(\zeta))\beta_1(\zeta) + q(1-\alpha_3(\zeta))\beta_3(\zeta) \end{aligned} \tag{28}$$

3.3.2. Algorithm 2: Delayed Decision

If the matched filter detects a PU signal and if the number of rounds of spectrum sensing is less than threshold K, the SP delays its decision and requests another round of spectrum sensing. Otherwise, the SP makes a decision immediately. As

Algorithm 2 Delayed Decision

```
1:  k = 1  //initialize counter
2:  LOOP
3:    Wait till the next silent period
4:    Receive results from matched filter and energy detector
5:    IF matched filter decides A_1   // presence of SU signals
6:      IF k ≤ K − 1
7:        Ignore decision of energy detector
8:        k ← k + 1  // go to Line 2 for the next round
9:      ELSE  // reach maximum round, i.e.,  k = K
10:       Decision of SP ← H_0 (absence of PU signals)
11:       Break LOOP
12:     END
13:   ELSE
14:     Decision of SP ← decision of energy detector
15:     Break LOOP
16:   END
```

to be shown in Section 3.4, the penalty of delay by Algorithm 2, for practical purposes, is negligible, only proportional to q.

We first consider the probability of detection. We assume that the occurrences of SU transmissions in different silent periods are independent. To make a decision, the SP goes through a sequence of hypotheses consisting of h_1 (with probability $1-q$) and h_3 (with probability q) and ending at a hypothesis for which the matched filter declares the absence of an SU signal. This is a particularly good model for certain PU signals (e.g., TV signals) whose presence/absence do not change quickly over time. An example of such sequence is: in the first silent period the hypothesis is h_3 and the matched filter decides A_1, and in the second silent period the hypothesis is h_1 and the matched filter decides A_0. For this sequence, the SP must make a decision in the second silent period. The contribution of sequence $h_3 h_1$ to the probability of detection is $q\alpha_3(1-q)(1-\alpha_1)\beta_1$. Considering all possible sequences and defining $\bar{\alpha}_i := 1-\alpha_i$, we have the probability of detection and similarly the probability of false alarm

$$P_D^{(2)} = \frac{((1-q)\bar{\alpha}_1\beta_1 + q\bar{\alpha}_3\beta_3)(1-(1-\bar{\alpha}_1+q(\bar{\alpha}_1-\bar{\alpha}_3))^K)}{(1-q)\bar{\alpha}_1 + q\bar{\alpha}_3} \tag{29}$$

$$P_F^{(2)} = \frac{((1-q)\bar{\alpha}_0\beta_0 + q\bar{\alpha}_2\beta_2)(1-(1-\bar{\alpha}_0+q(\bar{\alpha}_0-\bar{\alpha}_2))^K)}{(1-q)\bar{\alpha}_0 + q\bar{\alpha}_2} \tag{30}$$

3.4. Analysis

As mentioned earlier, we focus on the low SNR regime for the PU signal, i.e., $\gamma_p \ll 1$. We plot the Receiver Operating Characteristic (ROC) [4] curves in Fig. 2 by setting the parameters: $\gamma_p = -10$dB, $\gamma_s = 10$dB, $N = 3000$, $M = 1000$, $L = 100$, $c_p = 1$, $c_e = 1$, $q = 0.05$ and $K = 10$. The threshold η for the matched filter in (19) is set such that $P_F^m = 0.01$. We can show that the energy detector in the non-ideal case (solid blue line) achieves poor false alarm performance for a wide range of probability of detection. Algorithm 1 (red solid line with dots) achieves poor probability of detection, which is upper bounded at $1-q$. Algorithm 2 (red dashed line) achieves almost ideal performance. In fact, it follows from (29) and (30) that for relatively large K and a properly configured matched filter (i.e., $\alpha_0 \approx 0$, $\alpha_1 \approx 0$, $\alpha_2 \approx 1$, $\alpha_3 \approx 1$), we have $P_D^{(2)} \approx \beta_1$, and $P_F^{(2)} \approx \beta_0$. It can be shown that the average number of rounds of sensing $N_{\text{rounds}} = 1+q+o(q)$. That is, the penalty in average delay is negligible, only proportional to q, so is the penalty in energy consumption.

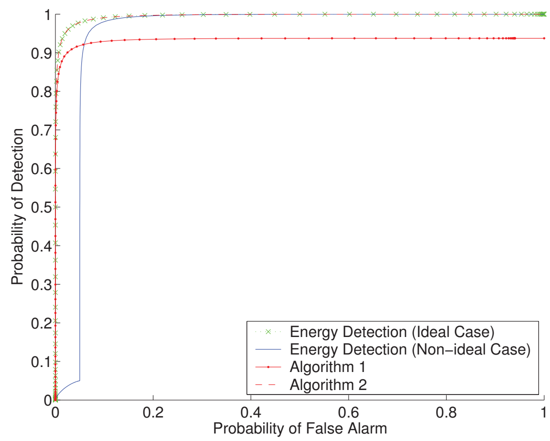

Fig. 2. ROC performance.

4. CONCLUSION

We leverage the existence of a matched filter in a typical SU to improve the performance of energy-detection based spectrum sensing with imperfect silent period coordination. The analysis shows that our algorithm (Delayed Decision), for practical purposes, virtually eliminates all of the negative impact of mistaken SU transmissions with only negligible penalty in delay and energy consumption.

5. REFERENCES

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