

SPECTRUM SHARING BETWEEN RANDOM GEOMETRIC NETWORKS

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ABSTRACT

In this paper a spectrum sharing scheme is proposed to maximize the successful transmission probability of a single-hop cognitive network coexisting with a random primary network. Both cognitive and primary networks exhibit randomness in topologies and endure imperfect wireless channel conditions. For a given primary outage probability bound, the maximum secondary transmit power is determined and then the maximum transmission capacity of the cognitive user is derived. Numerical results show that the proposed spectrum sharing scheme indeed boosts the transmission capacity of the cognitive network dramatically whilst having little performance loss of the primary network.

Index Terms— Cognitive network, spectrum sharing, random geometric networks, transmission capacity

1. INTRODUCTION

Spectrum sharing has been proposed to enhance spectrum utilization efficiency when a cognitive network (CN) could make use of the spectrum belonging to a primary network (PN), as long as the CN is transparent to the PN. However, hidden primary user (PU) problem makes it difficult for the CN to explore and exploit the spectrum resource. Due to the hidden PU problem, the CN loses precious access opportunities when false alarm happens, and causes the interference to the PU when missed detection occurs.

To limit the harmful interference with the PU, the maximum transmission range of CN is studied in [1], where the locations of PN is assumed to be known. Moreover, it has been verified that primary location information is beneficial to improve opportunities detection and transmission capacity [2]. In practice, however, the location information of primary users may not be easily obtained by the CN. In particular, for the large random geometric wireless network, e.g., ad hoc network, it poses a great challenge in obtaining the location information for the CN. Therefore, spectrum sharing without location information is a highly challenging issue. In [3], the maximum interference-free power of secondary transmitter (ST) without knowing the position of PUs was investigated,

but only one primary transmitter (PT)-primary receiver (PR) pair was considered. Moreover, the PT-PR pair is assumed to be active all the time. In [4], the authors proposed a cooperative spectrum sensing method to detect random PN, but did not indicate how to determine the detection area or how to share the spectrum. Besides, the locations of CN was assumed to be known in [4].

In this paper, we aim to enhance success probability of a single-hop transmission in a random geometric CN while protecting a random geometric PN coexisting in the same band. Location information of both networks is not assumed, but the node densities of the networks are known. A spectrum sharing scheme is proposed to enhance CN transmission capacity by carefully designing optimal ST transmitting power. Numerical results show that CN transmission capacity is significantly improved with minor sacrifice to PN transmission capacity.

The rest of this paper is organized as follows. System model is introduced in Section 2. The spectrum sharing scheme and performance analysis are given in Section 3. Numerical results and discussions are presented in Section 4. Finally, we conclude the paper in Section 5.

2. SYSTEM MODEL

A PN and a CN are assumed to cover randomly the two-dimensional plane \mathbb{R}^2 and share the same spectrum with unit bandwidth.

For any time slot, each PU transmits or receives independently by adopting ALOHA as medium access control protocol. Therefore, the distribution of PUs follows a homogeneous Poisson point process (PPP) of density λ . Let p be the probability that a PU is allowed to transmit, and $1 - p$ be the probability that a PU acts as a PR. Hence, the locations of PTs and PRs form two independent PPPs of density λ and $\lambda(1 - p)$, respectively [5]. Let $\Pi_1 = \{X_i\}$ and $\Pi_2 = \{Y_j\}$ denote the location sets of PTs and PRs, respectively, where $X_i \in \mathbb{R}^2$ is the coordinate of PT i , and $Y_j \in \mathbb{R}^2$ is the coordinate of PR j . Considering practicality and tractability, a PT supports maximum transmission distance r .

One ST and multiple potential secondary receivers (SRs) consist a single-hop transmission CN, i.e. any SR correctly receiving the signals from ST will lead to a successful trans-

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mission. ST locates at O , which is the original of the coordinates. The distribution of SRs follows another independent homogeneous PPP of density λ_s . Let $\Pi_3 = \{Z_k\}$ denote the location set of SRs, where $Z_k \in \mathbb{R}^2$ is the coordinate of SR k .

All the transmissions are omnidirectional. Considering path-loss effect and Rayleigh fading, the received power $P_{a,b}$ at node b from node a is given by

$$P_{a,b} = P_a \cdot H_{a,b} \cdot (D_{a,b})^{-\alpha} \quad (1)$$

where P_a is the transmitting power from node a . All the PTs employ the same given transmitting power P_0 . ST transmitting power P_1 will be designed adaptively. $H_{a,b}$ is Rayleigh fading component from node a to node b , which is exponentially distributed with normalized mean. All the Rayleigh fading components are independent and identically distributed (i.i.d.). $D_{a,b}$ is the distance between node a and node b . α is the path-loss exponent, which is assumed to be constant.

3. SPECTRUM SHARING BETWEEN WIRELESS NETWORKS

3.1. Spectrum Sharing

The goal of the proposed spectrum sharing scheme is to maximize the success probability p_s of the CN single-hop transmission while guaranteeing the success probability p_p of the PN transmission to be above a level. It can be formulated as the following optimization problem.

$$\begin{aligned} \max_{P_1} \quad & p_s \\ \text{s.t.} \quad & p_p \geq 1 - \varepsilon_p \end{aligned} \quad (2)$$

where ε_p is a predefined PN outage probability bound.

The success probability p_p of PN transmission is the probability that the achievable rate is no less than the target transmission rate $b_p = \log(1 + \beta)$, where β is the predefined signal-to-interference and noise-ratio (SINR) threshold. Equivalently, p_p is the probability that the SINR at the reference PR is larger than or equal to β . We set this reference PR as the closest PR to ST, termed as Y_0 in Fig. 1. We also set the corresponding reference PT at X_0 , which is r -away from the reference PR. Let random distance $D_{O,Y_0} = L$, we have

$$p_p = \mathbb{P}\left[\frac{P_{X_0,Y_0}}{I_1 + I_2 + N_0} \geq \beta\right] \quad (3)$$

where N_0 denotes the noise power, the received signal power is

$$P_{X_0,Y_0} = P_0 \cdot H_{X_0,Y_0} \cdot r^{-\alpha}, \quad (4)$$

the internal interference from other transmitting PTs to the reference PR is

$$I_1 = \sum_{X_i \in \Pi_1 \setminus X_0} P_0 \cdot H_{X_i,Y_0} \cdot D_{X_i,Y_0}^{-\alpha}, \quad (5)$$

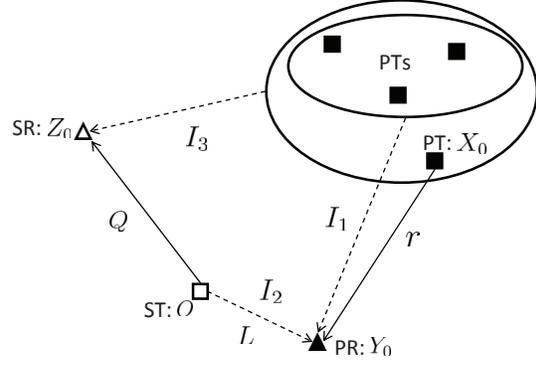


Fig. 1: Illustration of spectrum sharing problem between random geometric single-hop CN and random geometric PN

and the external interference from ST to the reference PR is

$$I_2 = P_1 \cdot H_{O,Y_0} \cdot L^{-\alpha}. \quad (6)$$

Note that the reference X_0 - Y_0 pair is typical of the overall PN due to its worst SINR condition. Firstly, the received power from X_0 to Y_0 is the lowest considering r is the maximum transmission distance. Secondly, the internal interference I_1 from all the other transmitting PTs is equal throughout the whole PN. This will be verified in section 3.2. Thirdly, the external interference I_2 from ST is the largest.

In the absence of the CN, the success probability of PN is

$$p'_p = \mathbb{P}\left[\frac{P_{X_0,Y_0}}{I_1 + N_0} \geq \beta\right] \quad (7)$$

The success probability p_s of CN single-hop transmission is the probability that the achievable rate is no less than the target transmission rate $b_s = \log(1 + \gamma)$, where γ is the predetermined SINR threshold. Equivalently, p_s is the probability that the SINR at the reference SR is larger than or equal to γ . We set this reference SR as the nearest SR to ST, represented as Z_0 in Fig. 1. Let random distance $D_{O,Z_0} = Q$, we have

$$p_s = \mathbb{P}\left[\frac{P_{O,Z_0}}{I_3 + N_0} \geq \gamma\right], \quad (8)$$

where the received power is

$$P_{O,Z_0} = P_1 \cdot H_{O,Z_0} \cdot Q^{-\alpha} \quad (9)$$

and the interference is

$$I_3 = \sum_{X_i \in \Pi_1} P_0 \cdot H_{X_i,Z_0} \cdot D_{X_i,Z_0}^{-\alpha}. \quad (10)$$

3.2. Maximum ST Power

Next, we show the existence of the maximum ST power P_1 under the constraint of PN outage probability bound ε_p .

From (3) and (4), we get

$$\begin{aligned} p_p &= \mathbb{P}[H_{X_0, Y_0} \geq ur^\alpha (I_1 + I_2 + N_0)] \\ &= \mathbb{E}_{I_1}[\exp(-ur^\alpha I_1)] \cdot \mathbb{E}_{I_2}[\exp(-ur^\alpha I_2)] \\ &\quad \cdot \exp(-uN_0r^\alpha) \end{aligned} \quad (11)$$

where $u = \beta/P_0$. According to (3.4) in [5] and (61) in [7], we further get

$$\mathbb{E}_{I_1}[\exp(-ur^\alpha I_1)] = \exp[-2k\lambda p\beta^{\frac{2}{\alpha}}r^2] \quad (12)$$

where $k = \pi^2\alpha^{-1}/\sin(2\pi/\alpha)$. It can be seen that the expected internal interference I_1 is irrespective to the location of the reference PR. Also, we have

$$\begin{aligned} &\mathbb{E}_{I_2}[\exp(-ur^\alpha I_2)] \\ &= \mathbb{E}_L\{\mathbb{E}_{H_{O, Y_0}}[\exp(-ur^\alpha P_1 L^{-\alpha} \cdot H_{O, Y_0})]\} \\ &= \mathbb{E}_L[1/(ur^\alpha P_1 L^{-\alpha} + 1)] \end{aligned} \quad (13)$$

Since L is the distance between the ST and its closest PR, the probability density function (pdf) of L is [6] $f_L(l) = 2\pi\lambda(1-p)l \exp[-\lambda(1-p)\pi l^2]$. Therefore,

$$\begin{aligned} p_p &= 2\pi\lambda(1-p) \exp[-2k\lambda p\beta^{\frac{2}{\alpha}}r^2] \cdot \exp(-uN_0r^\alpha) \\ &\quad \cdot \int_0^\infty \frac{l \cdot \exp[-\lambda(1-p)\pi l^2]}{(ur^\alpha P_1 l^{-\alpha} + 1)} dl \end{aligned} \quad (14)$$

Note that p_p reduces with increasing P_1 . Rewrite (14) as $p_p = g_1(P_1)$. Then, for a given PN outage upper bound ε_p , the maximum ST power is given by

$$\max(P_1) = g_1^{-1}(\varepsilon_p) \quad (15)$$

In the absence of CN, from Eq. (7), the success probability of PN is equal to

$$p'_p = \exp[-2k\lambda p\beta^{\frac{2}{\alpha}}r^2] \cdot \exp(-uN_0r^\alpha) \quad (16)$$

3.3. Spectrum Sharing Performance Gain

In this subsection, we show the maximum success probability and transmission capacity obtained by the spectrum sharing.

From (8) and (9), the success probability of CN single-hop transmission can be expressed as

$$\begin{aligned} p_s &= \mathbb{P}[H_{O, Z_0} \geq vQ^\alpha (I_3 + N_0)] \\ &= \mathbb{E}_Q\{\mathbb{E}_{I_3}[\exp(-vQ^\alpha I_3)] \cdot \exp[-vN_0Q^\alpha]\} \end{aligned} \quad (17)$$

where $v = \gamma/P_1$. According to [5] and [7], we have

$$\mathbb{E}_{I_3}[\exp(-vQ^\alpha I_3)] = \exp[-2k\lambda p(vP_0)^{\frac{2}{\alpha}}Q^2] \quad (18)$$

where k is given in (12). The pdf of the distance Q between ST at O and its nearest SR at Z_0 is [6] $f_Q(q) = 2\pi\lambda_s q \exp(-\lambda_s\pi q^2)$. Consequently,

$$\begin{aligned} p_s &= 2\pi\lambda_s \int_0^\infty \{\exp\{[-2k\lambda p(vP_0)^{\frac{2}{\alpha}} - \pi\lambda_s]q^2 \\ &\quad - vN_0q^\alpha\} \cdot q\} dq \end{aligned} \quad (19)$$

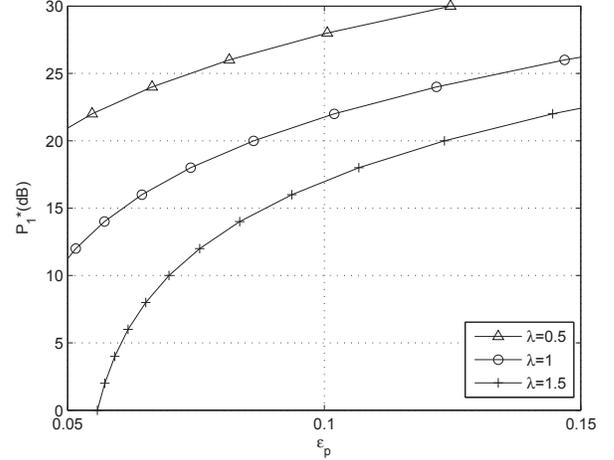


Fig. 2: The optimal ST power P_1^* against the PN outage probability bound ε_p with different PN densities λ .

Note that p_s increases with larger P_1 . Rewrite (19) into $p_s = g_2(P_1)$. Then, the solution to the proposed optimization problem in Eq. (2) is

$$p_s^*(\varepsilon_p) = g_2(P_1^*) = g_2(\max(P_1)) = g_2(g_1^{-1}(\varepsilon_p)) \quad (20)$$

Transmission capacity represents spectrum efficiency per unit area [7]. The transmission capacity of CN c_s is defined as the product of CN density λ_s , transmission data rate b_s and the maximum success probability of CN single-hop transmission under the constraint of PN outage probability bound ε_p .

$$c_s(\varepsilon_p) = \lambda_s \cdot b_s \cdot p_s^*(\varepsilon_p) = \lambda_s \cdot \log(1 + \gamma) \cdot p_s^*(\varepsilon_p) \quad (21)$$

The corresponding transmission capacity of PN c_p is defined as the product of PN density λ , transmission data rate b_p and success probability p_p .

$$c_p(\varepsilon_p) = \lambda \cdot b_p \cdot p_p = \lambda \cdot \log(1 + \beta) \cdot (1 - \varepsilon_p) \quad (22)$$

In the absence of the CN, the transmission capacity of PN is

$$c'_p = \lambda \cdot \log(1 + \beta) \cdot p'_p \quad (23)$$

where p'_p is given in (16).

4. NUMERICAL RESULTS

In this section, some numerical results are presented. The following parameters are used: the probability of allowed PT transmission $p = 0.1$, path-loss component $\alpha = 3$, target SINR threshold for PN $\beta = 10$ dB, target SINR threshold for CN $\gamma = 10$ dB, PT power $P_0 = 30$ dB, noise power $N_0 = 0$ dB, PN transmission distance $r = 0.1$ m.

In Fig. 2, we show the optimal ST power P_1^* in terms of PN outage probability bound ε_p . As shown in the figure,

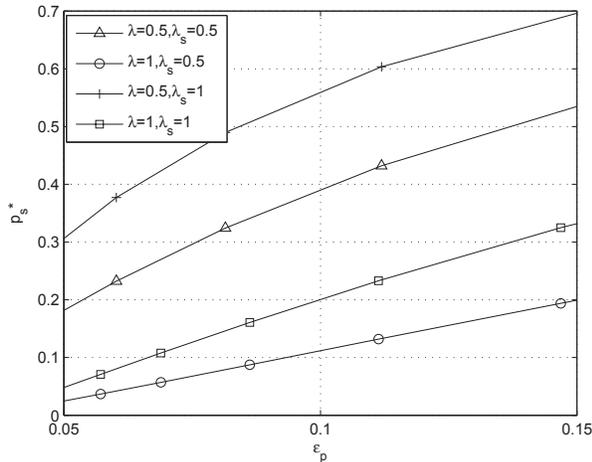


Fig. 3: The optimal success probability p_s^* of CN single-hop transmission against the PN outage probability bound ε_p with different PN densities λ and CN densities λ_s .

P_1^* increases monotonically over ε_p . This is because PN with larger outage probability tolerates more interference from C-N. It can also be observed that larger PN density λ leads to smaller P_1^* . It shows us that for large density PN, the internal interference I_1 is already sufficiently large. Then, the external interference I_2 must be reduced accordingly to ensure the outage not exceeding the given bound ε_p . Consequently, P_1^* is smaller for larger λ .

In Fig. 3, the maximum success probability of CN single-hop transmission p_s^* is shown in terms of of PN outage probability bound ε_p . It can be seen from the figure that p_s^* increases monotonically over ε_p . This is because higher ε_p allows ST to employ higher transmitting power and then improve the success probability of CN. Moreover, it can be found that denser CN increases p_s^* , because SR at Z_0 is closer to ST. However, denser PN will decrease p_s^* , because interference I_3 is larger.

In Fig. 4, we show the CN transmission capacity c_s , the PN transmission capacity c_p as functions of PN outage probability bound ε_p . As shown in the figure, PN transmission capacity is reduced by less than 0.25. However, CN transmission capacity is significantly improved from 1.1 to 2.4, for PN outage probability varying from 0.05 to 0.15.

5. CONCLUSIONS

We proposed a spectrum sharing scheme for a random geometric single-hop CN coexisting with a random geometric PN. No location information were assumed for both networks. To maximize the success probability of single-hop transmission, the ST uses the maximum power under the primary outage probability constraint. Numerical results show that the CN transmission capacity is greatly improved with marginal

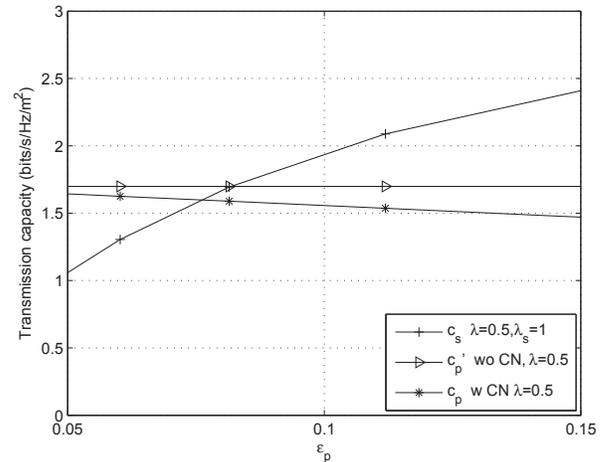


Fig. 4: The CN transmission capacity c_s , the PN transmission capacity c_p' without CN and the PN transmission capacity c_p with CN against the PN outage probability bound ε_p .

loss of the PN transmission capacity.

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