COOPERATIVE AND DISTRIBUTED LOCALIZATION FOR WIRELESS SENSOR NETWORKS IN MULTIPATH ENVIRONMENTS

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ABSTRACT

We consider the problem of sensor localization in wireless networks in a multipath environment. We propose a distributed and cooperative algorithm based on belief propagation, which allows sensors to cooperatively self-localize with respect to a single anchor node in the network, using range and direction of arrival measurements. In the algorithm, neighboring sensors exchange limited information to update their local mean location estimates and covariance matrices. We show that the covariance matrix for each sensor converges for connected networks, and its mean location estimate converges if all scatters are either parallel or orthogonal to each other. Furthermore, these estimates are asymptotically unbiased. Simulations show that cooperation amongst neighboring nodes significantly improves the localization accuracy.

Index Terms— distributed localization, wireless sensor network, belief propagation, non-line-of-sight errors.

1. INTRODUCTION

A wireless sensor network (WSN) consists of many devices (or nodes) capable of onboard sensing, computing and communications. WSNs are used in industrial and commercial applications, such as environmental monitoring and pollution detection, event detection, and object tracking [1, 2]. In most applications, the data collected by the sensor nodes can only be meaningfully interpreted if it is correlated with the location of the corresponding sensors.

Typical localization techniques are usually studied in line-of-sight (LOS) environments. However, LOS signals do not always exist in urban or cluttered environments, where signals usually experience multiple reflections and diffractions. Such signals are referred to as nonline-of-sight (NLOS) signals and are commonly encountered in both indoor and outdoor environments. Distributed localization algorithms for multipath environments were proposed in [3,4], where NLOS error is modeled as a positive bias in range and angle measurements, and its statistical characteristics are inferred by numerical methods, such as bootstrap sampling in [3], and particle filters in [4]. One of the major disadvantages is that these Bayesian inference techniques require a large number of observations and are computationally expensive.

Instead of modeling NLOS errors in multipath environments as random biases, we can use ray tracing methods to analyze the geometric relationship between range and angle measurements, which produces a more accurate signal model. However, as the number of scatterers increases, the ray-tracing model becomes more complicated, and most current works consider only one-bounce scattering paths [5]. Considering a similar ray tracing model, we propose a distributed localization algorithm, where sensors exchange information to cooperatively perform self-localization relative to a *single* anchor. We give analytical proofs for the convergence of the proposed algorithm, and show through simulation that by exchanging limited information, all the nodes in the network can perform localization to a good accuracy.

The rest of this paper is organized as follows. In Section 2, we briefly describe the system model and the distributed algorithm. We give convergence proofs in Section 3, and provide simulation results in Section 4. In Section 5, we summarize and conclude.

2. COOPERATIVE AND DISTRIBUTED LOCALIZATION

Consider a network of $M + 1$ sensors, $\{S_0, S_1, \cdots, S_M\}$. The position of S_i is $\mathbf{s}_i \triangleq (x_i, y_i)$, where x_i and y_i are its x- and y-coordinates respectively. Without loss of generality, we assume that node S_0 is the anchor with a known location $(0, 0)$. The objective of each S_i is to perform self-localization relative to S_0. In the following, we briefly describe the system model and the distributed localization algorithm. Due to space limits, details for derivation of the algorithm is omitted here and can be found in [6].

Consider two nodes S_i and S_j with R LOS or NLOS paths between them. An example of a single-bounce scattering path is shown in Figure 1. Let d^r_{ji} be the distance measured by S_i along the r^{th} path from S_j, and θ^r_{ji} be the corresponding angle. Given measurements $\{d^r_{ij}, d^r_{ji}, \theta^r_{ij}, \theta^r_{ji}\}^R_{r=1}$, it can be shown that

$$d^r_{ji} = \mathbf{g}(\theta^r_{ij}, \theta^r_{ji})^T (\mathbf{s}_i - \mathbf{s}_j) + \varpi^r_{ji}, \tag{1}$$

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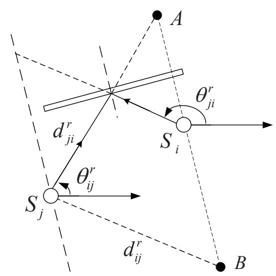

Fig. 1. Example for a single-bounce scattering path.

where $\mathbf{g}(\theta^r_{ij}, \theta^r_{ji}) = \left[\frac{\sin(\theta^r_{ij})+\sin(\theta^r_{ji})}{\sin(\theta^r_{ji}-\theta^r_{ij})}; -\frac{\cos(\theta^r_{ij})+\cos(\theta^r_{ji})}{\sin(\theta^r_{ji}-\theta^r_{ij})}\right]$ for $r = 1, \cdots, R$, and ϖ^r_{ji} is a Gaussian random error with mean 0 and variance σ^2 which approximates total effects of ranging and angle measurement errors. Let $\mathbf{d}_{ji} = [d^1_{ji}, \ldots, d^R_{ji}]^T$, $\mathbf{G}_{ji} = \left[\mathbf{g}\left(\theta^1_{ji}, \theta^1_{ij}\right), \ldots, \mathbf{g}\left(\theta^R_{ji}, \theta^R_{ij}\right)\right]^T$, and $\boldsymbol{\varpi}_{ji} = [\varpi^1_{ji}, \ldots, \varpi^R_{ji}]^T$, we model the position of S_i using

$$\mathbf{s}_i = \mathbf{s}_j + \mathbf{G}^\dagger_{ji}\left(\mathbf{d}_{ji} - \boldsymbol{\varpi}_{ji}\right). \qquad (2)$$

And the MAP estimation for $\mathbf{s}_i$ is found by maximizing the corresponding posterior distribution with respect to $\mathbf{s}_i$.

Generalizing the above idea to network-wide localization, the MAP estimator for sensor locations is obtained by maximizing the joint posterior distribution $p(\{\mathbf{s}_i\}^M_{i=1} \mid \{\mathbf{G}_{ji}, \mathbf{d}_{ji}\}_{i,j})$. However, this is a high dimensional optimization problem and is difficult to solve. To simplify computations and to design a distributed algorithm that localizes every node in the network, we considered the posterior marginal distribution of $\mathbf{s}_i$, denoted as $\{b_i(\mathbf{s}_i)\}^M_{i=1}$. An iterative algorithm based on belief propagation is shown in Algorithm 1 to calculate and maximize these posterior marginal distributions in a distributed fashion.

3. CONVERGENCE ANALYSIS

It is well known that algorithms based on belief propagation converge if the underlying graph is a tree. However, for a general graph topology, convergence is poorly understood and difficult to prove. As observed numerically in [7], when there exists loops, BP algorithms can diverge. Nevertheless, we establish that the covariance matrices $\mathbf{P}^{(l)}_i$ converge. We also show that the means $\boldsymbol{\mu}^{(l)}_i$ converge, and are asymptotically unbiased, when all scatters are either parallel or orthogonal to each other. In Section 4, we show numerically that we still have convergence of the computed means in a general setting.

3.1. Convergence of covariance matrices $\{\mathbf{P}^{(l)}_i\}^M_{i=1}$

We show that the covariance matrices of the local beliefs at each node converges in any matrix norm, by making use of the following elementary results, which we do not prove. The first lemma is from [8]. And Lemma 2 can be found in [9].

Algorithm 1 Cooperative and Distributed Localization in Multi-path Environments

Initialization:
Set the position at the anchor S_0 as $\mathbf{s}_0 = (0, 0)$.
Set $\boldsymbol{\mu}^{(0)}_i = (0, 0)$ and $[\mathbf{P}^{(0)}_i]^{-1} = \mathbf{0}$.
Iteration until convergence:
for the l^{th} iteration **do**
  sensors S_i with $i = 1 : M$ **in parallel**
  broadcast current belief $b^{(l-1)}_i(\mathbf{s}_i)$ to neighbors;
  receive $b^{(l-1)}_j(\mathbf{s}_j)$ from neighbors S_j, where $j \in \mathcal{B}_i$;
  update its belief as $b^{(l)}_i(\mathbf{s}_i) \sim \mathcal{N}(\boldsymbol{\mu}^{(l)}_i, \mathbf{P}^{(l)}_i)$ with

$$\left[\mathbf{P}^{(l)}_i\right]^{-1} = \sum_{j\in\mathcal{B}_i}\left[\mathbf{W}^{(l-1)}_{ji}\right]^{-1}, \qquad (3)$$

$$\boldsymbol{\mu}^{(l)}_i = \mathbf{P}^{(l)}_i \sum_{j\in\mathcal{B}_i}\left[\mathbf{W}^{(l-1)}_{ji}\right]^{-1}\boldsymbol{\nu}^{(l-1)}_{ji}, \qquad (4)$$

  where $\boldsymbol{\nu}^{(l-1)}_{ji} = \boldsymbol{\mu}^{(l-1)}_j + \mathbf{G}^\dagger_{ji}\mathbf{d}_{ji}$, and $\mathbf{W}^{(l-1)}_{ji} = \sigma^2\boldsymbol{\Sigma}_{ji} + \mathbf{P}^{(l-1)}_j$ with $\boldsymbol{\Sigma}_{ji} = \mathbf{G}^\dagger_{ji}(\mathbf{G}^\dagger_{ji})^T$.
  estimate its position as $\hat{\mathbf{s}}^{(l)}_i = \boldsymbol{\mu}^{(l)}_i$.
  end parallel
end for

Lemma 1. *If the sequence $\{\mathbf{A}^{(l)}\}^{+\infty}_{l=1}$ of positive definite matrices is non-increasing, i.e., $\mathbf{A}^{(l)} \succeq \mathbf{A}^{(l+1)}$ for $l = 1, 2, \cdots$, this sequence converges to a positive semi-definite matrix.*

Lemma 2. *If the matrices $\mathbf{A}$ and $\mathbf{B}$ are positive definite, then $\mathbf{A} \succeq \mathbf{B}$ iff $\mathbf{B}^{-1} \succeq \mathbf{A}^{-1}$.*

The following result shows that the covariance matrices of the beliefs at each variable node in the factor graph converges.

Theorem 1. *The covariance matrices $\{\mathbf{P}^{(l)}_i\}^M_{i=1}$ of beliefs at sensors $\{S_i\}^M_{i=1}$ in Algorithm 1 converges for connected networks, i.e., there exists unique positive semi-definite matrices $\{\mathbf{P}^*_i\}^M_{i=1}$ such that $\lim_{l\to+\infty}\mathbf{P}^{(l)}_i = \mathbf{P}^*_i$ for all $i = 1, \cdots, M$.*

Proof. Let $\sigma^2\boldsymbol{\Sigma}_{ji} = \mathbf{U}^T_{ji}\mathbf{D}_{ji}\mathbf{U}_{ji}$, where $\mathbf{U}_{ji}$ is a unitary matrix, and $\mathbf{D}_{ji}$ is a diagonal matrix with non-negative entries. Define $\mathbf{L}^{(l)}_{ji} = \mathbf{U}_{ji}\mathbf{P}^{(l)}_j\mathbf{U}^T_{ji}$, and let $\mathbf{K}^{(l)}_i = \left[\mathbf{P}^{(l)}_i\right]^{-1}$. From (3), we then have

$$\mathbf{K}^{(l)}_i = \sum_{j\in\mathcal{B}_i}\mathbf{U}_{ji}\left(\mathbf{D}_{ji} + \mathbf{L}^{(l-1)}_{ji}\right)^{-1}\mathbf{U}^T_{ji}. \qquad (5)$$

We show by induction on l that $\mathbf{K}^{(l)}_i$ is non-decreasing for all $i = 1, \ldots, M$. The proof for $\mathbf{K}^{(1)}_i \succeq \mathbf{K}^{(0)}_i$ is trivial. Suppose $\mathbf{K}^{(l)}_i \succeq \mathbf{K}^{(l-1)}_i$ for all j. From Lemma 2, we have

$\mathbf{P}_j^{(l-1)} \succeq \mathbf{P}_j^{(l)}$. Since $\mathbf{U}_{ji}$ is unitary for any $j \in \mathcal{B}_i$, we have $\mathbf{L}_{ji}^{(l-1)} \succeq \mathbf{L}_{ji}^{(l)}$, from which we obtain $\mathbf{D}_{ji} + \mathbf{L}_{ji}^{(l-1)} \succeq \mathbf{D}_{ji} + \mathbf{L}_{ji}^{(l)}$. Applying Lemma 2, we have $\left(\mathbf{D}_{ji} + \mathbf{L}_{ji}^{(l)}\right)^{-1} - \left(\mathbf{D}_{ji} + \mathbf{L}_{ji}^{(l-1)}\right)^{-1} \succeq \mathbf{0}$, which together with (5) yields

$$\begin{aligned}&\mathbf{K}_i^{(l+1)} - \mathbf{K}_i^{(l)}\\ &= \sum_{j\in\mathcal{B}_i} \mathbf{U}_{ji}\left[\left(\mathbf{D}_{ji} + \mathbf{L}_{ji}^{(l)}\right)^{-1} - \left(\mathbf{D}_{ji} + \mathbf{L}_{ji}^{(l-1)}\right)^{-1}\right]\mathbf{U}_{ji}^T.\end{aligned}$$

And hence $\mathbf{K}_i^{(l+1)} - \mathbf{K}_i^{(l)} \succeq \mathbf{0}$. This completes the induction, and the claim that $\mathbf{K}_i^{(l)}$ is non-decreasing in l is now proved. This implies that $\mathbf{P}_i^{(l)}$ is non-increasing in l. The theorem now follows from Lemma 1, and the proof is complete. □

3.2. Convergence of belief means $\{\boldsymbol{\mu}_i^{(l)}\}_{i=1}^M$

Suppose that all scatters in the environment where the sensor nodes are positioned are such that any two scatters are either parallel or orthogonal to each other. Without loss of generality, we assume that all scatters are either horizontal or vertical. We show that under this constraint, the mean of the belief at each node S_i converges.

Theorem 2. *Suppose that all scatters are either horizontal or vertical. Then, $\boldsymbol{\mu}_i^{(l)}$ converges for all $i = 1, \ldots, M$. In addition, if $\{\mathbf{s}_i\}_{i=1}^M$ are non-random parameters, then $\boldsymbol{\mu}_i^{(l)}$ is an asymptotically unbiased estimator of $\mathbf{s}_i$.*

Proof. Stack $\{\boldsymbol{\mu}_i^{(l)}\}_{i=1}^M$ in (4) into a vector and write

$$\underbrace{\begin{bmatrix}\boldsymbol{\mu}_1^{(l)}\\ \vdots \\ \boldsymbol{\mu}_M^{(l)}\end{bmatrix}}_{\triangleq \mathbf{U}^{(l)}} = \mathbf{Q}^{(l-1)} \underbrace{\begin{bmatrix}\boldsymbol{\mu}_1^{(l-1)}\\ \vdots \\ \boldsymbol{\mu}_M^{(l-1)}\end{bmatrix}}_{\triangleq \mathbf{U}^{(l-1)}} + \underbrace{\mathbf{H}\left(\mathbf{I}_M \otimes \mathbf{Q}^{(l-1)}\right)\mathbf{Z}}_{\triangleq \mathbf{A}^{(l-1)}}, \quad (6)$$

where $\mathbf{Q}^{(l)}$ is a $2M$-by-$2M$ matrix consisting of $M \times M$ blocks of 2-by-2 submatrices, with the $(i,j)^{\text{th}}$ block being

$$\mathbf{Q}^{(l)}(i,j) = \begin{cases}\left[\sum_{k\in\mathcal{B}_i}\mathbf{W}_{ki}^{(l)}\right]^{-1}\mathbf{W}_{ki}^{(l)}, & 0 \notin \mathcal{B}_i, j \in \mathcal{B}_i, i \neq j,\\ \left[\mathbf{W}_{0i} + \sum_{k\in\mathcal{B}_i, k\neq 0}\mathbf{W}_{ki}^{(l)}\right]^{-1}\mathbf{W}_{ji}^{(l)}, & 0 \in \mathcal{B}_i, j \in \mathcal{B}_i, i \neq j,\\ \mathbf{0}, & \text{otherwise.}\end{cases}$$

The vector $\mathbf{Z} \triangleq [\mathbf{Z}_1, \cdots, \mathbf{Z}_M]^T$ with $\mathbf{Z}_i \triangleq \left[\left(\mathbf{G}_{1i}^\dagger\mathbf{d}_{1i}\right)^T, \cdots, \left(\mathbf{G}_{Mi}^\dagger\mathbf{d}_{Mi}\right)^T\right]$, and $\mathbf{H}$ is a selection matrix defined as $\mathbf{H} \triangleq \left[(\mathbf{e}_1^T \otimes \mathbf{I}_2)^T, \cdots, (\mathbf{e}_{(i-1)M+i}^T \otimes \mathbf{I}_2)^T, \cdots, (\mathbf{e}_{M^2}^T \otimes \mathbf{I}_2)^T\right]^T$ The vector $\mathbf{e}_i$ is a $M^2 \times 1$ vector with all entries 0, except a 1 at the i^{th} entry. We show that the sequence $\{\mathbf{U}^{(l)}\}_{l=0}^{+\infty}$ converges, relying on the following properties, whose proofs are omitted.

Lemma 3. *(i) For all $l \geq 0$, we have $\mathbf{Q}^{(l)}$ is strictly substochastic with spectral radius $\rho(\mathbf{Q}^{(l)}) < 1$.*

(ii) There exists $\mathbf{Q}^*$ with spectral radius $\rho(\mathbf{Q}^*) < 1$, and such that $\lim_{l\to+\infty}\mathbf{Q}^{(l)} = \mathbf{Q}^*$. Furthermore, there exists an induced matrix norm $\|\cdot\|$ such that $\|Q^*\| < 1$.

(iii) There exists a constant $r < 1$ such that for all $l \geq 0$, $\left\|\mathbf{Q}^{(l)}\right\| \leq r$.

(iv) There exists a constant c such that for all $l \geq 0$, $\left\|\mathbf{A}^{(l)}\right\| \leq c$.

Using induction with (6), we have

$$\mathbf{U}^{(l)} = \prod_{k=1}^{l}\mathbf{Q}^{(l-k)}\mathbf{U}^{(0)} + \sum_{m=1}^{l}\prod_{k=1}^{m-1}\mathbf{Q}^{(l-k)}\mathbf{A}^{(l-m)}.$$

From Lemma 3(iii), we have $\left\|\prod_{k=1}^{l}\mathbf{Q}^{(l-k)}\right\| \leq r^l$, which together with Lemma 3(iv), yields

$$\left\|\mathbf{U}^{(l)} - \mathbf{U}^{(p)}\right\| \leq 2r^l\left\|\mathbf{U}^{(0)}\right\| + c\sum_{m=l}^{p} r^{m-1}, \text{for } l \leq p.$$

Therefore, $\{\mathbf{U}^{(l)}\}_{l\geq 0}$ is a Cauchy sequence, and it converges.

Suppose $\{\mathbf{s}_i\}_{i=1}^M$ are nonrandom parameters. Substituting $\mathbf{G}_{ji}^\dagger\mathbf{d}_{ji} = \mathbf{s}_i - \mathbf{s}_j + \mathbf{G}_{ji}^\dagger\boldsymbol{\varpi}_{ji}$ into (4), and letting $\tilde{\boldsymbol{\mu}}_i^{(l)} \triangleq \boldsymbol{\mu}_i^{(l)} - \mathbf{s}_i$, we obtain $\mathbb{E}\left[\tilde{\boldsymbol{\mu}}_i^{(l)}\right] = \left[\sum_{j\in\mathcal{B}_i}\mathbf{W}_{ji}^{(l-1)}\right]^{-1}\sum_{j\in\mathcal{B}_i}\mathbf{W}_{ji}^{(l-1)}\left\{\mathbb{E}\left[\tilde{\boldsymbol{\mu}}_j^{(l-1)}\right]\right\}$, and the same argument as above shows that $\mathbb{E}\left[\tilde{\boldsymbol{\mu}}_i^{(l)}\right] \to 0$ as $l \to \infty$. This completes the proof. □

4. SIMULATION RESULTS AND DISCUSSION

Numerical simulations are conducted to validate the effectiveness of our proposed algorithm. We consider a network with 5 nodes randomly distributed in a $10\text{m} \times 10\text{m}$ square area. We set S_0 to be the anchor with a fixed location at $(0, 0)$. Sensors S_1 and S_2 have NLOS paths to S_0. Sensors S_3 and S_4 do not have any paths to S_0, but each has a NLOS path to S_1 and S_2 respectively, and a NLOS path between themselves. The ranging measurement errors are i.i.d. Gaussian random variables with zero mean and standard variance 3. The measurement error for AOA is assumed to be uniformly distributed in $[-5°, 5°]$.

Scatters are horizontal or at angle $45°$ to the horizontal. The performances of cooperative and pairwise localization are compared. In pairwise localization, S_3 localizes using only measurements from S_1, and S_4 localizes with respect to

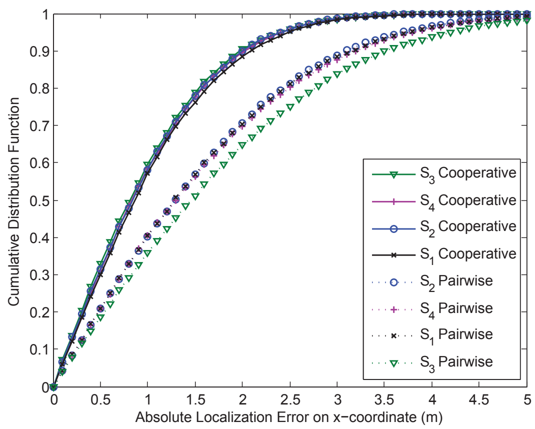

Fig. 2. CDF of absolute errors on x-coordinates when scatters are horizontal or at $45°$.

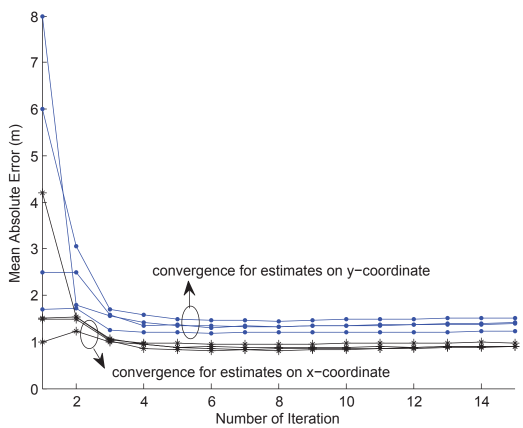

Fig. 3. Convergence of the mean absolute error when scatters are horizontal or at $45°$.

S_2. It can be seen from Figure 2 that S_3 and S_4 are localized with larger errors than S_1 and S_2, and this is because errors are accumulated over hops. In cooperative localization, S_3 and S_4 exchange information and incorporate measurements from the NLOS path between themselves. As shown in Figure 2, the proposed algorithm achieves better performances with more than 90% of the localization errors less than 2m and all errors smaller than 3m. Similar results can be obtained for estimation on y-coordinates.

Simulations are also conducted when scatters are at $10°$, $20°$ and $30°$ to the horizontal. Similar results as in Figure 3 are obtained and hence omitted here. These numerical results suggest the mean of the belief at each sensor converges in general.

5. CONCLUSION

In this paper, we propose a distributed algorithm based on belief propagation for network-wide localization in multipath environments. The proposed algorithm requires communications only between neighboring sensors, and has low overhead. By utilizing both range and direction of arrival information of the single-bounce scattering paths, we require only one anchor in the whole network, and sensors that do not have either LOS or NLOS paths to the anchor can be localized by cooperating with its neighboring sensors. The convergence of our proposed algorithm is analytically proved. Simulation results show that our proposed algorithm has better localization accuracy compared with the non-cooperative pairwise localization.

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