TIME-VARYING CLOCK OFFSET ESTIMATION IN TWO-WAY TIMING MESSAGE EXCHANGE IN WIRELESS SENSOR NETWORKS USING FACTOR GRAPHS[‡]

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ABSTRACT

The problem of clock offset estimation in a two-way timing exchange regime is considered when the likelihood function of the observation time stamps is exponentially distributed. In order to capture the imperfections in node oscillators, which render a time-varying nature to the clock offset, a novel Bayesian approach to the clock offset estimation is proposed using a factor graph representation of the posterior density. Message passing using the max-product algorithm yields a closed form expression for the Bayesian inference problem.

Index Terms— Clock offset, factor graphs, message passing, max-product algorithm

1. INTRODUCTION

Clock synchronization in wireless sensor networks (WSN) is a critical component in data fusion and duty cycling, and has gained widespread interest in recent years [1]. Most of the current methods consider sensor networks exchanging time stamps based on the time at their respective clocks [2]. In a *two-way* timing exchange process, adjacent nodes aim to achieve pairwise synchronization by communicating their timing information with each other. After a round of N messages, each node tries to estimate its own clock parameters. A representative protocol of this class is the timing-sync protocol for sensor networks (TPSNs) which uses this strategy in two phases to synchronize clocks in a network [3].

The clock synchronization problem in a WSN offers a natural statistical signal processing framework [4]. Assuming an exponential delay distribution, several estimators were proposed in [5]. It was argued that when the propagation delay d is unknown, the maximum likelihood (ML) estimator for the clock offset θ is not unique. However, it was shown in [6] that the ML estimator of θ does exist uniquely for the case of unknown d. The performance of these estimators was compared with benchmark estimation bounds in [7]. A common theme in the aforementioned contributions is that the effect of possible time variations in clock offset, arising from imperfect oscillators, is not incorporated and hence, they entail frequent re-synchronization requirements.

In this work, assuming an exponential distribution for the network delays, a closed form solution to clock offset estimation is presented by considering the clock offset as a random Gauss-Markov process. Bayesian inference is performed using factor graphs and the max-product algorithm.

2. SYSTEM MODEL

By assuming that the respective clocks of a sender node S and a receiver node R are related by $C_R(t) = \theta + C_S(t)$ at real time t, the two-way timing message exchange model at the kth instant can be represented as [5] [6]

$$U_k = d + \theta + X_k, \quad V_k = d - \theta + Y_k \tag{1}$$

where d represents the propagation delay, assumed symmetric in both directions, and θ is offset of the clock at node R relative to the clock at node S. The network delays, X_k and Y_k, are the independent exponential random variables. By further defining $\xi \triangleq d + \theta$ and $\psi \triangleq d - \theta$, the model in (1) can be written as

$$U_k = \xi + X_k, \quad V_k = \psi + Y_k \tag{2}$$

for $k = 1, \ldots, N$. The imperfections introduced by environmental conditions in the quartz oscillator in sensor nodes result in a time-varying clock offset between nodes in a WSN. In order to sufficiently capture these temporal variations, the parameters ξ and ψ are assumed to evolve through a Gauss-Markov process given by

$$\xi_k = \xi_{k-1} + w_k, \quad \psi_k = \psi_{k-1} + v_k \quad \text{for } k = 1, \ldots, N$$

where w_k and v_k are $i.i.d$ such that $w_k, v_k \sim \mathcal{N}(0, \sigma^2)$. The goal is to determine precise estimates of ξ and ψ in the Bayesian framework using observations $\{U_k, V_k\}_{k=1}^N$. An estimate of θ can, in turn, be obtained as

$$\hat{\theta} = \frac{\hat{\xi} - \hat{\psi}}{2} \,. \tag{3}$$

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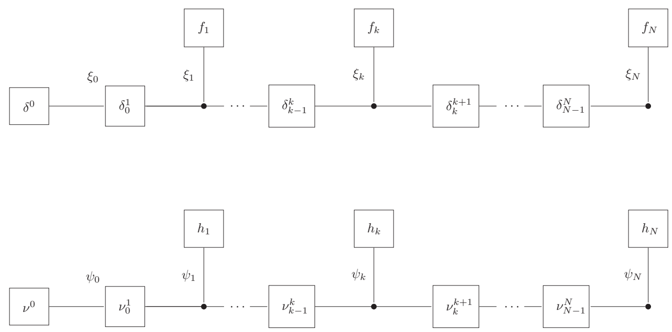

Fig. 1. Factor graph representation of posterior density (4)

3. A FACTOR GRAPH APPROACH

The posterior pdf can be expressed as

$$f(\boldsymbol{\xi},\boldsymbol{\psi}|\boldsymbol{U},\boldsymbol{V}) \propto f(\boldsymbol{\xi},\boldsymbol{\psi})f(\boldsymbol{U},\boldsymbol{V}|\boldsymbol{\xi},\boldsymbol{\psi}) = f(\xi_0)\prod_{k=1}^{N} f(\xi_k|\xi_{k-1})f(\psi_0)\prod_{k=1}^{N} f(\psi_k|\psi_{k-1}) \cdot \prod_{k=1}^{N} f(U_k|\xi_k)f(V_k|\psi_k) \quad (4)$$

where uniform priors $f(\xi_0)$ and $f(\psi_0)$ are assumed. Define $\delta_{k-1}^{k} \triangleq f(\xi_k|\xi_{k-1}) \sim \mathcal{N}(\xi_{k-1},\sigma^2)$, $\nu_{k-1}^{k} \triangleq f(\psi_k|\psi_{k-1}) \sim \mathcal{N}(\psi_{k-1},\sigma^2)$, $f_k \triangleq f(U_k|\xi_k)$, $h_k \triangleq f(V_k|\psi_k)$, where the likelihood functions are given by

$$f(U_k|\xi_k) = \lambda_\xi \exp\left(-\lambda_\xi(U_k-\xi_k)\right)\mathbb{I}(U_k-\xi_k)$$
$$f(V_k|\psi_k) = \lambda_\psi \exp\left(-\lambda_\psi(V_k-\psi_k)\right)\mathbb{I}(V_k-\psi_k) \ . \quad (5)$$

The factor graph representation of the posterior pdf is shown in Fig. 1. The factor graph is cycle-free and inference by message passing is indeed optimal. In addition, the two factor graphs shown in Fig. 1 have a similar structure and hence, message computations will only be shown for the estimate $\hat{\xi}_N$. Clearly, similar expressions will apply to $\hat{\psi}_N$.

The message updates in factor graph using max-product can be computed as follows

$$m_{f_N\to\xi_N} = f_N, \quad m_{\xi_N\to\delta_{N-1}^{N}} = f_N$$
$$m_{\delta_{N-1}^{N}\to\xi_{N-1}} \propto \max_{\xi_N} \delta_{N-1}^{N}\cdot m_{\xi_N\to\delta_{N-1}^{N}} = \max_{\xi_N}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-(\xi_N-\xi_{N-1})^2}{2\sigma^2}\right) \cdot \exp\left(\lambda_\xi\xi_N\right)\mathbb{I}(U_N-\xi_N)$$

which can be rearranged as

$$m_{\delta_{N-1}^{N}\to\xi_{N-1}} \propto \max_{\xi_N\le U_N} \exp\big(A_{\xi,N}\xi_N^2 + B_{\xi,N}\xi_{N-1}^2 + C_{\xi,N}\xi_N\xi_{N-1} + D_{\xi,N}\xi_N\big) \quad (6)$$

where

$$A_{\xi,N} \triangleq -\frac{1}{2\sigma^2}, \quad B_{\xi,N} \triangleq -\frac{1}{2\sigma^2}$$
$$C_{\xi,N} \triangleq \frac{1}{\sigma^2}, \quad D_{\xi,N} \triangleq \lambda_\xi \ . \quad (7)$$

Let $\bar{\xi}_N$ be the unconstrained maximizer of the exponent in the objective function above, i.e.,

$$\bar{\xi}_N = \arg\max_{\xi_N}\left(A_{\xi,N}\xi_N^2 + B_{\xi,N}\xi_{N-1}^2 + C_{\xi,N}\xi_N\xi_{N-1} + D_{\xi,N}\xi_N\right) .$$

This implies that

$$\bar{\xi}_N = -\frac{C_{\xi,N}\xi_{N-1}+D_{\xi,N}}{2A_{\xi,N}} \ . \quad (8)$$

If $\bar{\xi}_N > U_N$, then the estimation problem is solved, since $\hat{\xi}_N = U_N$. However, if $\bar{\xi}_N \le U_N$, the solution is $\hat{\xi}_N = \bar{\xi}_N$. Therefore, in general, we can write

$$\hat{\xi}_N = \min\left(\bar{\xi}_N, U_N\right) \ .$$

Notice that $\bar{\xi}_N$ depends on ξ_{N-1}, which is undetermined at this stage. Hence, we need to further traverse the chain backwards. Assuming that $\bar{\xi}_N \le U_N$, $\bar{\xi}_N$ from (8) can be plugged back in (6) which after some simplification yields

$$m_{\delta_{N-1}^{N}\to\xi_{N-1}} \propto \exp\left\{\left(B_{\xi,N}-\frac{C_{\xi,N}^2}{4A_{\xi,N}}\right)\xi_{N-1}^2 - \frac{C_{\xi,N}D_{\xi,N}}{2A_{\xi,N}}\xi_{N-1}\right\} . \quad (9)$$

Similarly the message from the factor δ_{N-2}^{N-1} to the variable node ξ_{N-2} can be expressed as

$$m_{\delta_{N-2}^{N-1}\to\xi_{N-2}} \propto \max_{\xi_{N-1}\le U_{N-1}} \delta_{N-2}^{N-1}\cdot m_{\xi_{N-1}\to\delta_{N-2}^{N-1}}$$
$$= \max_{\xi_{N-1}}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(\xi_{N-1}-\xi_{N-2})^2}{2\sigma^2}\right) \cdot \exp\left\{\left(B_{\xi,N}-\frac{C_{\xi,N}^2}{4A_{\xi,N}}\right)\xi_{N-1}^2 - \frac{C_{\xi,N}D_{\xi,N}}{2A_{\xi,N}}\xi_{N-1}\right\} \cdot \exp\left(\lambda_\xi\xi_{N-1}\right)\mathbb{I}(U_{N-1}-\xi_{N-1}) \ .$$

The message above can be compactly represented as

$$m_{\delta_{N-2}^{N-1}\to\xi_{N-2}} \propto \max_{\xi_{N-1}\le U_{N-1}} \exp(A_{\xi,N-1}\xi_{N-1}^2 + B_{\xi,N-1}\xi_{N-2}^2 + C_{\xi,N-1}\xi_{N-1}\xi_{N-2} + D_{\xi,N-1}\xi_{N-1}) \quad (10)$$

where

$$A_{\xi,N-1} \triangleq -\frac{1}{2\sigma^2} + B_{\xi,N} - \frac{C_{\xi,N}^2}{4A_{\xi,N}},$$
$$B_{\xi,N-1} \triangleq -\frac{1}{2\sigma^2}, \quad C_{\xi,N-1} \triangleq \frac{1}{\sigma^2}$$
$$D_{\xi,N-1} \triangleq \lambda_\xi - \frac{C_{\xi,N}D_{\xi,N}}{2A_{\xi,N}} \ .$$

Proceeding as before, the unconstrained maximizer $\bar{\xi}_{N-1}$ of the objective function above is given by

$$\bar{\xi}_{N-1} = -\frac{C_{\xi,N-1}\xi_{N-2} + D_{\xi,N-1}}{2A_{\xi,N-1}}$$

and the solution to the maximization problem (10) is expressed as

$$\hat{\xi}_{N-1} = \min\left(\bar{\xi}_{N-1}, U_{N-1}\right) .$$

Again, $\bar{\xi}_{N-1}$ depends on ξ_{N-2} and therefore, the solution demands another traversal backwards on the factor graph representation in Fig. 1. By plugging $\bar{\xi}_{N-1}$ back in (10), it follows that

$$m_{\delta_{N-2}^{N-1}\to\xi_{N-2}} \propto \exp\left\{\left(B_{\xi,N-1} - \frac{C_{\xi,N-1}^2}{4A_{\xi,N-1}}\right)\xi_{N-2}^2 - \frac{C_{\xi,N-1}D_{\xi,N-1}}{2A_{\xi,N-1}}\xi_{N-2}\right\} \tag{11}$$

which has a form similar to (9). It is clear that one can keep traversing back in the graph yielding messages similar to (9) and (11). In general, for $i = 1, \ldots, N-1$, we can write

$$\begin{aligned} A_{\xi,N-i} &\triangleq -\frac{1}{2\sigma^2} + B_{\xi,N-i+1} - \frac{C_{\xi,N-i+1}^2}{4A_{\xi,N-i+1}} \\ B_{\xi,N-i} &\triangleq -\frac{1}{2\sigma^2}, \quad C_{\xi,N-i} \triangleq \frac{1}{\sigma^2} \\ D_{\xi,N-i} &\triangleq \lambda_\xi - \frac{C_{\xi,N-i+1}D_{\xi,N-i+1}}{2A_{\xi,N-i+1}} \end{aligned} \tag{12}$$

and

$$\bar{\xi}_{N-i} = -\frac{C_{\xi,N-i}\xi_{N-i-1} + D_{\xi,N-i}}{2A_{\xi,N-i}} \tag{13}$$

$$\hat{\xi}_{N-i} = \min\left(\bar{\xi}_{N-i}, U_{N-i}\right) . \tag{14}$$

Using (13) and (14) with $i = N-1$, it follows that

$$\bar{\xi}_1 = -\frac{C_{\xi,1}\xi_0 + D_{\xi,1}}{2A_{\xi,1}}, \quad \hat{\xi}_1 = \min\left(\bar{\xi}_1, U_1\right) . \tag{15}$$

Similarly, by observing the form of (9) and (11), it follows that

$$m_{\delta_0^1\to\xi_0} \propto \exp\left\{\left(B_{\xi,1} - \frac{C_{\xi,1}^2}{4A_{\xi,1}}\right)\xi_0^2 - \frac{C_{\xi,1}D_{\xi,1}}{2A_{\xi,1}}\xi_0\right\} . \tag{16}$$

The estimate $\hat{\xi}_0$ can be obtained by maximizing (16).

$$\hat{\xi}_0 = \bar{\xi}_0 = \max_{\xi_0} m_{\delta_0^1\to\xi_0} = \frac{C_{\xi,1}D_{\xi,1}}{4A_{\xi,1}B_{\xi,1} - C_{\xi,1}^2} . \tag{17}$$

The estimate in (17) can now be substituted in (15) to yield $\bar{\xi}_1$, which can then be used to solve for $\hat{\xi}_1$. Clearly, this chain of calculations can be continued using recursions (13) and (14). Define

$$g_{\xi,k}(x) \triangleq -\frac{C_{\xi,k}x + D_{\xi,k}}{2A_{\xi,k}} . \tag{18}$$

Lemma 1 *For real numbers a and b, the function $g_{\xi,k}(.)$ defined in* (18) *satisfies*

$$g_{\xi,k}\left(\min(a,b)\right) = \min\left(g_{\xi,k}(a), g_{\xi,k}(b)\right) .$$

Proof: The constants $A_{\xi,k}$, $C_{\xi,k}$ and $D_{\xi,k}$ are defined in (7) and (12). The proof follows by noting that $\frac{-C_{\xi,k}}{2A_{\xi,k}} > 0$ which implies that $g_{\xi,k}(.)$ is a monotonically increasing function.

Using the notation $g_{\xi,k}(.)$, it follows that

$$\begin{aligned} \bar{\xi}_1 &= g_{\xi,1}\left(\hat{\xi}_0\right), \quad \hat{\xi}_1 = \min\left(U_1, g_{\xi,1}\left(\hat{\xi}_0\right)\right) \\ \bar{\xi}_2 &= g_{\xi,2}\left(\hat{\xi}_1\right), \quad \hat{\xi}_2 = \min\left(U_2, g_{\xi,2}\left(\hat{\xi}_1\right)\right) \end{aligned}$$

where

$$\begin{aligned} g_{\xi,2}\left(\hat{\xi}_1\right) &= g_{\xi,2}\left(\min\left(U_1, g_{\xi,1}\left(\hat{\xi}_0\right)\right)\right) \\ &= \min\left(g_{\xi,2}\left(U_1\right), g_{\xi,2}\left(g_{\xi,1}\left(\hat{\xi}_0\right)\right)\right) \end{aligned} \tag{19}$$

where (19) follows from Lemma 1. The estimate $\hat{\xi}_2$ can be expressed as

$$\begin{aligned} \hat{\xi}_2 &= \min\left(U_2, \min\left(g_{\xi,2}\left(U_1\right), g_{\xi,2}\left(g_{\xi,1}\left(\hat{\xi}_0\right)\right)\right)\right) \\ &= \min\left(U_2, g_{\xi,2}\left(U_1\right), g_{\xi,2}\left(g_{\xi,1}\left(\hat{\xi}_0\right)\right)\right) . \end{aligned}$$

Hence, one can keep estimating $\hat{\xi}_k$ at each stage using this strategy. Note that the estimator only depends on functions of data and can be readily evaluated. For $m \geq k$, define

$$G_{\xi,k}^m(.) \triangleq g_{\xi,m}\left(g_{\xi,m-1}\ldots g_{\xi,k}\left(.\right)\right) . \tag{20}$$

The estimate $\hat{\xi}_N$ can, therefore, be compactly represented as

$$\hat{\xi}_N = \min\left(U_N, G_{\xi,N}^N\left(U_{N-1}\right), \ldots, G_{\xi,2}^N\left(U_1\right), G_{\xi,1}^N\left(\hat{\xi}_0\right)\right) . \tag{21}$$

By a similar reasoning, the estimate $\hat{\psi}_N$ can be analogously expressed as

$$\hat{\psi}_N = \min\left(V_N, G_{\psi,N}^N\left(V_{N-1}\right), \ldots, G_{\xi,2}^N\left(V_1\right), G_{\xi,1}^N\left(\hat{\psi}_0\right)\right)$$

and the factor graph based clock offset estimate (FGE) $\hat{\theta}_N$ is given by

$$\hat{\theta}_N = \frac{\hat{\xi}_N - \hat{\psi}_N}{2} . \tag{22}$$

It only remains to calculate the functions of data $G(.)$ in the expressions for $\hat{\xi}_N$ and $\hat{\psi}_N$ to determine the FGE estimate $\hat{\theta}_N$. With the constants defined in (7), it follows that

$$G_{\xi,N}^N(U_{N-1}) = -\frac{C_{\xi,N}U_{N-1} + D_{\xi,N}}{2A_{\xi,N}} = U_{N-1} + \lambda_\xi\sigma^2 .$$

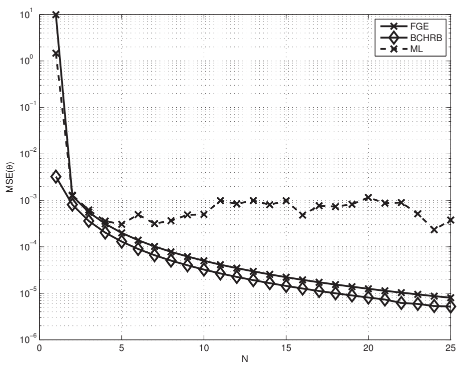

Fig. 2. Comparison of MSE of $\hat{\theta}_N$ and $\hat{\theta}_{\text{ML}}$.

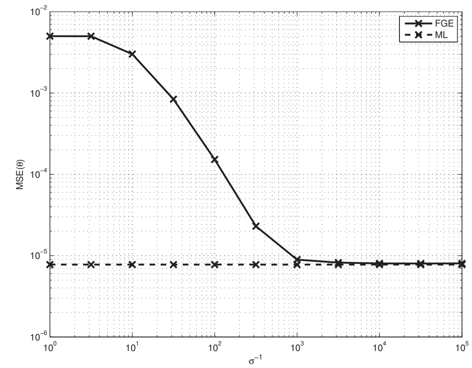

Fig. 3. MSE in estimation of θ_N vs σ.

Similarly it can be shown that

$$G^N_{\xi,N-1}(U_{N-2}) = U_{N-2} + 2\lambda_\xi \sigma^2$$

and so on. Using the constants defined in (12) for $i = N-1$, it can be shown that $\hat{\xi}_0 = \frac{C_{\xi,1}D_{\xi,1}}{4A_{\xi,1}B_{\xi,1}-C^2_{\xi,1}} = +\infty$. This implies that $G^N_{\xi,1}(\hat{\xi}_0) = +\infty$. Plugging this in (21) yields

$$\hat{\xi}_N = \min(U_N, U_{N-1} + \lambda_\xi\sigma^2, \ldots, U_1 + (N-1)\lambda_\xi\sigma^2)\ .$$

Similarly, the estimate $\hat{\psi}_N$ is given by

$$\hat{\psi}_N = \min(V_N, V_{N-1} + \lambda_\psi\sigma^2, \ldots, V_1 + (N-1)\lambda_\psi\sigma^2)$$

and the estimate $\hat{\theta}_N$ can be obtained using (22) as

$$\begin{aligned}\hat{\theta}_N = \frac{1}{2}&\min(U_N, U_{N-1} + \lambda_\xi\sigma^2, U_{N-2} + 2\lambda_\xi\sigma^2, \\ &\qquad \ldots, U_1 + (N-1)\lambda_\xi\sigma^2) - \\ &\frac{1}{2}\min(V_N, V_{N-1} + \lambda_\psi\sigma^2, V_{N-2} + 2\lambda_\psi\sigma^2, \\ &\qquad \ldots, V_1 + (N-1)\lambda_\psi\sigma^2)\ . \quad (23)\end{aligned}$$

As the Gauss-Markov system noise $\sigma^2 \to 0$, (23) yields

$$\hat{\theta}_N \to \hat{\theta}_{\text{ML}} = \frac{\min(U_N, \ldots, U_1) - \min(V_N, \ldots, V_1)}{2} \quad (24)$$

which is the ML estimator proposed in [6].

4. SIMULATION RESULTS

With $\lambda_\xi = \lambda_\psi = 10$ and $\sigma = 10^{-2}$, Fig. 2 shows the MSE performance of $\hat{\theta}_N$ and $\hat{\theta}_{\text{ML}}$, compared with the Bayesian Chapman-Robbins bound (BCHRB). It is clear that $\hat{\theta}_N$ exhibits a better performance than $\hat{\theta}_{\text{ML}}$ by incorporating the effects of time variations in clock offset. As the variance of the Gauss-Markov model accumulates with the addition of more samples, the MSE of $\hat{\theta}_{\text{ML}}$ gets worse. Fig. 3 depicts the MSE of $\hat{\theta}_N$ in (23) with $N = 25$. The horizontal line represents the MSE of the ML estimator (24). It can be observed that the MSE obtained by using the FGE for estimating θ approaches the MSE of the ML as $\sigma < 10^{-3}$.

5. CONCLUSION

The estimation of a possibly time-varying clock offset is studied using factor graphs. A closed form solution to the clock offset estimation problem is presented using a novel message passing strategy based on the max-product algorithm. This estimator shows a performance superior to the ML estimator proposed in [6] by capturing the effects of time variations in the clock offset efficiently.

6. REFERENCES

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