OPTIMAL TRANSMIT POWER ALLOCATION IN WIRELESS SENSOR NETWORKS PERFORMING FIELD RECONSTRUCTION

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ABSTRACT

In our previous work, we developed efficient field reconstruction methods in wireless sensor networks. In this paper, we use an amplify-and-forward to transmit the sensor measurements to the fusion center and we derive the mean square error (MSE) of the reconstructed field as a function of the measurement and receive SNR and of the sensor positions. We propose to allocate the sensor node transmit powers such that the sum power is minimized subject to an MSE target and we phrase this approach as a convex optimization problem that can be numerically solved in an efficient manner. For the case of critical sampling we derive a closed-form expression for the optimal power allocation. We illustrate the power savings achieved with the proposed power allocation schemes both for Gaussian and Rayleigh fading channels.

Index Terms— Wireless sensor network, field reconstruction, power allocation

1. INTRODUCTION

Wireless sensor networks (WSN) have recently attracted considerable attention for diverse monitoring applications [1]. With WSN, sensor nodes are deployed in the region to be monitored and communicate wirelessly in order to collect and process information about the phenomenon of interest. In this paper, we consider the problem of power allocation in the context of the system architecture introduced in [2] for distributed sampling and reconstruction of a two-dimensional (2-D) physical fields.

Power scheduling for decentralized estimation in sensor networks based on uncoded quadrature amplitude modulated (QAM) and on analog transmission was studied respectively in [3] and [4]. The optimal power allocation in those papers is similar to water-filling, i.e., sensor nodes with poor channel gains or noisy observations remain inactive to save power. These results were extended to the case of distributed estimation of a random field in [5]. A suboptimal power allocation scheme for the estimation of a random parameter in the presence of noisy links was proposed in [6].

In this paper, we investigate the problem of power allocation in a WSN using amplify-and-forward transmissions to the fusion center (FC) as in [4]. However, contrary to the scalar model in [4], we consider a matrix-vector model that we used for field reconstruction based on shift-invariant spaces in our previous work [7]. Different from [4], the optimal power allocation for our model depends on the sensor node positions. In order to maximize network lifetime, our aim is to minimize the transmit sum power subject to a prescribed estimation accuracy. We formulate this objective as a convex optimization problem that can be solved numerically using standard techniques. For the special case of critical sampling, we derive a closed-form solution for this problem.

Our paper is organized as follows. Section 2 reviews our system model for field reconstruction in WSN. In Section 3, we study the optimal power allocation problem. Section 4 shows numerical results illustrating the performance of our power allocation scheme, and Section 5 provides concluding remarks.

2. SYSTEM MODEL

2.1. Measurement and Transmission Model

We consider a WSN consisting of I sensor nodes deployed over a given region $\mathcal{A}$ to monitor a 2-D physical field $h(x,y)$. Here, x and y are the spatial coordinates. The position of node i is denoted by (x_i, y_i) and its measurement is given by $h_i + v_i$ where $h_i = h(x_i, y_i)$. Here, v_i denotes spatially white measurement noise with (node-dependent) variance $\sigma_{v_i}^2$. We assume that the physical field belongs to a shift-invariant space $V(g)$ (see [2, 8] for details), i.e.,

$$h(x,y) = \sum_{(k,l)\in\mathbb{A}} c_{k,l}\, g(x-k, y-l). \tag{1}$$

Here, $g(x,y)$ is a generator function with compact support $\mathcal{S}$ (e.g., a B-spline function) and $\mathbb{A} = \mathbb{Z}^2 \cap (\mathcal{A} + \mathcal{S})$. We further assume that the field has mean power σ_h^2.

The reconstruction of $h(x,y)$ from the noisy samples $h_i + v_i$ thus amounts to estimating the coefficients $c_{k,l}$. To this end, we augment the least-squares approach from [2] with an amplify-and-forward (AF) transmission protocol in which sensor node i transmits the scaled measurement $s_i = \sqrt{p_i}\big(h_i + v_i\big)$ to the FC, which requires an average transmit power of

$$P_i \triangleq \mathrm{E}\{s_i^2\} = p_i\big(\sigma_h^2 + \sigma_{v_i}^2\big). \tag{2}$$

The transmissions of the individual nodes are over orthogonal channels and the signals received by the FC are given by

$$r_i = \sqrt{\gamma_i} s_i + w_i = \sqrt{\gamma_i p_i} h_i + z_i.$$

Here, γ_i is the channel gain, w_i is white receiver noise, with variance σ_w^2 and

$$z_i = \sqrt{\gamma_i p_i} v_i + w_i \tag{3}$$

denotes the aggregate noise with variance $\gamma_i p_i \sigma_{v_i}^2 + \sigma_w^2$. Below we will consider Gaussian channels (modeled via $\gamma_i = 1$) and flat Rayleigh fading channels (modeled by exponentially distributed γ_i).

We next formulate the system model using matrices and vectors. To this end, let (k_0, l_0) and (k_1, l_1) denote the smallest and largest

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indices in $\mathbb{A}$, respectively, such that $J = KL$ with $K \triangleq k_1 - k_0 + 1$ and $L \triangleq (l_1 - l_0 + 1)$. We define the block-banded $I \times J$ matrix $\mathbf{G}$ with elements

$$[\mathbf{G}]_{i,p} = g(x_i - k_p,\, y_i - l_p)$$

and the vectors $\mathbf{r}$, $\mathbf{v}$ (length I), and $\mathbf{c}$ (length J) with elements

$$[\mathbf{r}]_i = r_i, \;\; [\mathbf{v}]_i = v_i, \;\; \text{and} \;\; [\mathbf{c}]_p = c_{k_p, l_p},$$

where $k_p = k_0 + ((p-1) \bmod K)$, and $l_p = l_0 + \left\lfloor \frac{p-1}{K} \right\rfloor$. The receive signals can be rewritten in matrix-vector form as

$$\mathbf{r} = \mathbf{A}\mathbf{G}\mathbf{c} + \mathbf{z} = \tilde{\mathbf{G}}\mathbf{c} + \mathbf{z}, \tag{4}$$

where $\mathbf{A} \triangleq \operatorname{diag}\{\sqrt{\gamma_i p_i}\}$, $\tilde{\mathbf{G}} \triangleq \mathbf{A}\mathbf{G}$, and the aggregate noise vector $\mathbf{z}$ has covariance matrix $\mathbf{C}_{\mathbf{z}} = \operatorname{diag}\left\{\gamma_i p_i \sigma_{v_i}^2 + \sigma_w^2\right\}$.

2.2. Field Reconstruction and Reconstruction Performance

We determine the field coefficients $\mathbf{c}$ in the linear system model (4) using the best linear unbiased estimator (BLUE) [9] with the noise covariance matrix $\mathbf{C}_{\mathbf{z}}$ as weight, i.e.,

$$\hat{\mathbf{c}} \triangleq \arg\min_{\mathbf{c}} \left\|\tilde{\mathbf{G}}\mathbf{c} - \mathbf{r}\right\|^2_{\mathbf{C}_{\mathbf{z}}^{-1}} = (\tilde{\mathbf{G}}^T \mathbf{C}_{\mathbf{z}}^{-1} \tilde{\mathbf{G}})^{-1} \tilde{\mathbf{G}}^T \mathbf{C}_{\mathbf{z}}^{-1} \mathbf{r}. \tag{5}$$

Note that the computation of the coefficient estimates $\hat{\mathbf{c}}$ requires that the noise statistics $\mathbf{C}_{\mathbf{z}}$ and the matrix $\tilde{\mathbf{G}}$ (i.e., the sensor node positions and channel gains) be known at the FC. In order for the estimator (5) to exist, the matrix $\tilde{\mathbf{G}}$ must have full rank, which in turn requires $I \geq J$, i.e., that there are at least as many sensors as unknown coefficients (in addition, the sensor nodes need to be sufficiently closely spaced). Technically, the sensor node positions (x_i, y_i), have to form a so-called stable sampling set [8]. The case $I = J$ will be referred to henceforth as critical sampling.

With the optimal coefficient estimates (5), the field can be reconstructed for at any position $(x, y) \in \mathcal{A}$ according to (cf. (1))

$$\hat{h}(x,y) \;=\; \sum_{(k,l)\in\mathbb{A}} \hat{c}_{k,l}\, g(x-k, y-l) \tag{6}$$

To assess accuracy of (6), we next derive the mean-square field reconstruction error within $\mathcal{A}$. To this end, we define the length-J vector

$$[\mathbf{g}]_p(x,y) = g(x - k_p, y - l_p),$$

with k_p and l_p as in Section 2.1, and the associated Gramian $\mathbf{G}_{\mathbf{g}} = \iint_{\mathcal{A}} \mathbf{g}(x,y)\, \mathbf{g}^T(x,y)\, dx\, dy$. Taking the expectation with respect to the noise, with the sensor positions and channel gains fixed, we obtain

$$\begin{aligned}
\varepsilon &\triangleq \mathrm{E}\left\{ \iint_{\mathcal{A}} \left(\hat{h}(x,y) - h(x,y)\right)^2 dx\, dy \right\} \\
&= \iint_{\mathcal{A}} \mathbf{g}^T(x,y)\, \mathrm{E}\left\{(\hat{\mathbf{c}} - \mathbf{c})\,(\hat{\mathbf{c}} - \mathbf{c})^T\right\} \mathbf{g}(x,y) dx\, dy \\
&= \iint_{\mathcal{A}} \operatorname{tr}\left\{\mathbf{C}_{\hat{\mathbf{c}}}\, \mathbf{g}(x,y)\, \mathbf{g}^T(x,y)\right\} dx\, dy = \operatorname{tr}\left\{\mathbf{C}_{\hat{\mathbf{c}}} \mathbf{G}_{\mathbf{g}}\right\}.
\end{aligned} \tag{7}$$

Here, $\mathbf{C}_{\hat{\mathbf{c}}} = \operatorname{cov}\left\{\hat{\mathbf{c}}\right\} \triangleq \mathrm{E}\left\{(\hat{\mathbf{c}} - \mathbf{c})(\hat{\mathbf{c}} - \mathbf{c})^T\right\}$ denotes the covariance matrix of the (unbiased) coefficient estimates $\hat{\mathbf{c}}$. For compactly supported generator functions, the Gramian $\mathbf{G}_{\mathbf{g}}$ can be shown to be a symmetric block-banded Toeplitz matrix. The covariance matrix of the coefficient estimates can be further developed as

$$\begin{aligned}
\mathbf{C}_{\hat{\mathbf{c}}-\mathbf{c}} &= \operatorname{cov}\left\{(\tilde{\mathbf{G}}^T \mathbf{C}_{\mathbf{z}}^{-1} \tilde{\mathbf{G}})^{-1} \tilde{\mathbf{G}}^T \mathbf{C}_{\mathbf{z}}^{-1} \mathbf{z}\right\} \\
&= (\tilde{\mathbf{G}}^T \mathbf{C}_{\mathbf{z}}^{-1} \tilde{\mathbf{G}})^{-1} = (\mathbf{G}^T \mathbf{A}^T \mathbf{C}_{\mathbf{z}}^{-1} \mathbf{A}\mathbf{G})^{-1} \\
&= (\mathbf{G}^T \mathbf{D} \mathbf{G})^{-1}.
\end{aligned} \tag{8}$$

Here, we used the diagonal matrix

$$\mathbf{D} \triangleq \mathbf{A}^T \mathbf{C}_{\mathbf{z}}^{-1} \mathbf{A} = \operatorname{diag}\{d_i\}, \quad \text{with} \quad d_i \triangleq \frac{1}{\sigma_{v_i}^2 + \frac{\sigma_w^2}{\gamma_i p_i}}. \tag{9}$$

Inserting (8) and (9) into (7), we finally obtain the MSE as

$$\varepsilon = \operatorname{tr}\left\{\mathbf{C}_{\hat{\mathbf{c}}} \mathbf{G}_{\mathbf{g}}\right\} = \operatorname{tr}\left\{(\mathbf{G}^T \mathbf{D} \mathbf{G})^{-1} \mathbf{G}_{\mathbf{g}}\right\}. \tag{10}$$

This expression captures the dependence of the reconstruction MSE on the channel gains γ_i, the sensor node positions (x_i, y_i), the AF factors p_i, and the noise variances $\sigma_{v_i}^2$ and σ_w^2.

3. OPTIMAL POWER ALLOCATION

In the context of sensor networks, one of the main concerns is network lifetime, which in turn is directly related to energy efficiency. For that reason, we propose to keep the total transmit power of the sensor nodes as low as possible while satisfying prescribed requirements for the reconstruction quality. Hence, we aim at allocating the power scaling factors p_i such that the transmit sum power

$$P \triangleq \sum_{i=1}^{I} P_i = \sum_{i=1}^{I} p_i \left(\sigma_h^2 + \sigma_{v_i}^2\right)$$

is minimized subject to a given MSE target $\varepsilon_{\max}$. Defining the length-I vectors $\mathbf{p} = (p_1 \, \ldots \, p_I)^T$ and $\mathbf{q} = (q_1 \, \ldots \, q_I)^T$ with $q_i = \sigma_h^2 + \sigma_{v_i}^2$, we have $P = \mathbf{p}^T \mathbf{q}$ and the optimal power allocation problem is given by (cf. (10))

$$\begin{aligned}
&\underset{\mathbf{p}\in\mathbb{R}_+^I}{\text{minimize}} \quad && \mathbf{p}^T \mathbf{q} \\
&\text{subject to} \quad && \operatorname{tr}\left\{(\mathbf{G}^T \mathbf{D}(\mathbf{p}) \mathbf{G})^{-1} \mathbf{G}_{\mathbf{g}}\right\} \leq \varepsilon_{\max}.
\end{aligned} \tag{11}$$

Here, we made the dependence of $\mathbf{D}$ on $\mathbf{p}$ explicit by writing $\mathbf{D}(\mathbf{p})$. The optimization problem (11) can be shown to be convex (see [10]). Note that the MSE target $\varepsilon_{\max}$ cannot be chosen arbitrarily small. In fact, even with infinite transmit powers, the MSE is lower bounded by a strictly positive number due to the presence of measurement noise, i.e.,

$$\varepsilon \geq \varepsilon_{\min} \triangleq \lim_{p_i \to \infty} \varepsilon = \operatorname{tr}\left\{(\mathbf{G}^T \operatorname{diag}\{\sigma_{v_i}^{-2}\} \mathbf{G})^{-1} \mathbf{G}_{\mathbf{g}}\right\}.$$

Hence the MSE target has to be chosen larger than $\varepsilon_{\min}$ in order for (11) to have a solution. While the problem in general has no closed-form solution, it can be solved numerically in an efficient manner using standard algorithms [11]. Clearly, the power allocated to node i depends on the local measurement noise variance $\sigma_{v_i}^2$, on the channel gain γ_i, and (through the matrix $\mathbf{G}$) on the sensor node positions.

3.1. Critical Sampling

We next show that for the special case of critical sampling the power allocation problem (11) has a closed-form solution. In this case, there are as many sensor nodes as unknown coefficients, i.e., $I = J$. We furthermore assume that the node positions form a stable sampling set such that $\mathbf{G}$ is a square invertible matrix. In this case, (10) can be specialized as

$$\begin{aligned}\varepsilon &= \operatorname{tr}\left\{(\mathbf{G}^T\mathbf{D}\mathbf{G})^{-1}\mathbf{G_g}\right\} = \operatorname{tr}\left\{\mathbf{D}^{-1}(\mathbf{G}^T)^{-1}\mathbf{G_g}\mathbf{G}^{-1}\right\} \\ &= \sum_{i=1}^{I}\frac{1}{d_i}g_i = \sum_{i=1}^{I}\sigma_{v_i}^2 g_i + \sum_{i=1}^{I}\frac{\sigma_w^2 g_i}{\gamma_i p_i},\end{aligned} \tag{12}$$

where we defined $g_i \triangleq \left[(\mathbf{G}^T)^{-1}\mathbf{G_g}\mathbf{G}^{-1}\right]_{ii}$ and used that $\mathbf{D}^{-1} = \operatorname{diag}\{d_i^{-1}\}$ with $d_i^{-1} = \left(\sigma_{v_i}^2 + \frac{\sigma_w^2}{\gamma_i p_i}\right)$ (cf. (9)). Then the side-constraint in the optimization problem in (11) simplifies to

$$\sum_{i=1}^{I}\frac{\sigma_w^2 g_i}{\gamma_i p_i} \le \varepsilon'_{\max} \triangleq \varepsilon_{\max} - \varepsilon_{\min},$$

where $\varepsilon_{\min} = \sum_{i=1}^{I}\sigma_{v_i}^2 g_i$ and $\varepsilon_{\max} \ge \varepsilon_{\min}$ ensures a nonempty feasible set. The Lagrangian associated to the optimum power allocation problem (11) in this case equals

$$\begin{aligned}L(\mathbf{p},\lambda) &= \mathbf{p}^T\mathbf{q} + \lambda\left(\sum_{i=1}^{I}\frac{\sigma_w^2 g_i}{\gamma_i p_i} - \varepsilon'_{\max}\right) \\ &= \sum_{i=1}^{I}\left(p_i q_i + \lambda\frac{\sigma_w^2 g_i}{\gamma_i p_i}\right) - \lambda\varepsilon'_{\max}\end{aligned}$$

and the Lagrangian dual function reads

$$\begin{aligned}g(\lambda) = L(\mathbf{p}^\star,\lambda) = \inf_{\mathbf{p}} L(\mathbf{p},\lambda) &= \sum_{i=1}^{I}\left(p_i^\star q_i + \lambda\frac{\sigma_w^2 g_i}{\gamma_i p_i^\star}\right) - \lambda\varepsilon'_{\max} \\ &= 2\sigma_w\sqrt{\lambda}\sum_{i=1}^{I}\sqrt{\frac{g_i q_i}{\gamma_i}} - \lambda\varepsilon'_{\max},\end{aligned}$$

where

$$p_i^\star = \arg\inf_{p_i}\left(p_i q_i + \lambda\frac{\sigma_w^2 g_i}{\gamma_i p_i}\right) = \sqrt{\frac{\lambda\sigma_w^2 g_i}{\gamma_i q_i}}.$$

We therefore have the Lagrange dual problem

$$\begin{aligned}&\underset{\lambda}{\text{maximize}} \quad g(\lambda) = 2\sigma_w\sqrt{\lambda}\sum_{i=1}^{I}\sqrt{\frac{g_i q_i}{\gamma_i}} - \lambda\varepsilon'_{\max} \\ &\text{subject to} \quad \lambda \ge 0,\end{aligned}$$

whose solution is given by

$$\lambda^\star = \left(\frac{\sigma_w\sum_{i=1}^{I}\sqrt{\frac{g_i q_i}{\gamma_i}}}{\varepsilon'_{\max}}\right)^2.$$

It can be shown that the Karush-Kuhn-Tucker conditions [11, Chapter 5] hold, so that strong duality is fulfilled. With the dual optimal solution $\lambda^\star$, the optimal solution for the primal problem equals $L(\mathbf{p}^\star,\lambda^\star) = g(\lambda^\star)$ with the primal feasible minimizer

$$p_i^\star = \frac{\sigma_w^2}{\varepsilon'_{\max}}\sqrt{\frac{g_i}{\gamma_i q_i}}\sum_{j=1}^{I}\sqrt{\frac{g_j q_j}{\gamma_j}}. \tag{13}$$

The optimal sensor node transmit powers can then be written as

$$P_i^\star = p_i^\star q_i = \frac{\sigma_w^2}{\varepsilon'_{\max}}\sqrt{\frac{g_i q_i}{\gamma_i}}\sum_{j=1}^{I}\sqrt{\frac{g_j q_j}{\gamma_j}}.$$

Clearly, a smaller MSE target entails larger transmit powers. Defining $\beta_i \triangleq \sqrt{\frac{g_i q_i}{\gamma_i}}$ and $\alpha_i \triangleq \frac{\beta_i}{\sum_{j=1}^{I}\beta_j}$ and inserting (13) into (12), we obtain for the MSE

$$\begin{aligned}\varepsilon^\star &= \sum_{i=1}^{I}\sigma_{v_i}^2 g_i + \sum_{i=1}^{I}\frac{\sigma_w^2 g_i}{\gamma_i p_i^\star} = \sum_{i=1}^{I}\left(\sigma_{v_i}^2 g_i + \alpha_i\,\varepsilon'_{\max}\right) \\ &= \sum_{i=1}^{I}\left(\sigma_{v_i}^2 g_i + \alpha_i\left(\varepsilon_{\max} - \sum_{j=1}^{I}\sigma_{v_j}^2 g_j\right)\right) = \varepsilon_{\max}.\end{aligned} \tag{14}$$

Hence, the optimal power allocation scheme meets the MSE target $\varepsilon_{\max}$ with equality.

In the critical sampling case, the overall MSE can be split into contributions from the individual sensor nodes. Specifically, node i contributes $\varepsilon_i = \sigma_{v_i}^2 g_i + \alpha_i\varepsilon'_{\max}$ to the overall MSE, where the fraction α_i of the excess MSE $\varepsilon'_{\max}$ attributed to node i is large if g_i is large (which means that sensor node i is rather isolated) or if the associated channel gain γ_i is small. As a result, sensor nodes whose measurements are already received with high accuracy (i.e., low $\sigma_{v_i}^2$ and/or large γ_i) are assigned a smaller transmit power P_i than those whose measurement would be received with poor quality. We emphasize that—in contrast to existing work—the placement of the sensor node (reflected by the factor g_i) plays an important role in the power allocation. Indeed, the MSE contribution of those nodes that are necessary to maintain a stable sampling set (i.e., the nodes with large g_i) may be higher than that of nodes with small g_i.

4. NUMERICAL SIMULATIONS

We next present numerical results to illustrate the performance of our power allocation scheme. We model the field via a shift-invariant space according to (1), with the generator function chosen as third-order B-spline [2]. The field model was normalized such that $\sigma_h^2 = 1$. The region being monitored is given by $\mathcal{A} = [0,5] \times [0,5]$; thus, there are $J = 36$ field coefficients to be estimated. We used a WSN with $I \ge 36$ sensor nodes, where the first 36 nodes (the minimum number) were located on the square grid $\mathbb{Z}^2 \cap \mathcal{A}$ and the remaining $I - 36$ nodes were placed randomly according to a spatially uniform distribution over the region $\mathcal{A}$. Since the first 36 nodes can be shown to form a stable sampling set for the critically sampled case, all I nodes form a stable sampling set, too. For the case of Gaussian channels, all channel gains γ_i were set to 1, whereas for the Rayleigh fading case the channel gains were generated i.i.d. using an exponential distribution with mean $\mu_\gamma = 1$. The measurement noise variances $\sigma_{v_i}^2$ of the different sensors and the receiver noise variance σ_w^2 were chosen i.i.d. according to a uniform distribution Unif $\{0.01, 0.1\}$ (this corresponds to measurement SNRs and receive SNRs between 10 dB and 20 dB).

Using the parameters specified above, we simulated three power allocation schemes: (A) the numerically evaluated optimal power allocation scheme according to (11); (B) power allocation according to the closed-form expression (13) for the first 36 sensor nodes and zero power for the remaining sensor nodes; (C) a scheme with uniform power allocation for the first 36 sensor nodes and zero power for the remaining sensor nodes (uniform power allocation over *all* nodes tends to perform even poorer in the scenarios considered).

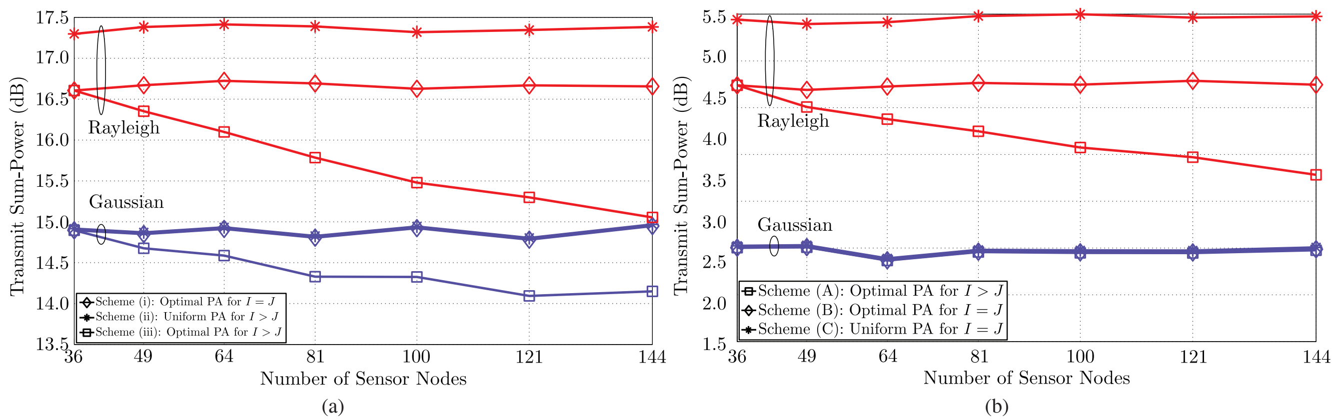

Fig. 1. *Performance comparison of different transmit power allocation schemes in terms of transmit sum power versus number of sensor nodes in Gaussian and Rayleigh fading channels: (a) normalized MSE target of -20 dB (b) normalized MSE target of -10 dB.*

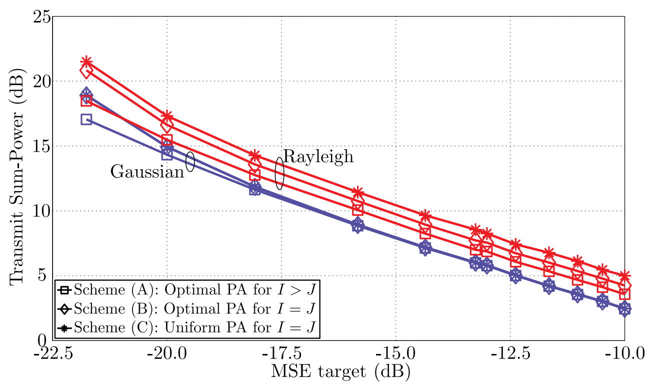

Fig. 2. *Comparison of different power allocation schemes in terms of transmit sum power versus normalized MSE target for 100 sensor nodes and Gaussian and Rayleigh fading channels.*

We first compare a setup with a stringent (normalized) MSE target of $\frac{\varepsilon_{\max}}{J|\mathcal{A}|} = 0.01$ and another setup with a relaxed MSE target of $\frac{\varepsilon_{\max}}{J|\mathcal{A}|} = 0.1$. Figure 1 displays the required transmit sum power versus the number of sensor nodes I for both MSE targets. Note that schemes B and C use only the first 36 nodes and hence their sum power is independent of I. Clearly, the optimal power allocation (scheme A) requires the lowest power in all cases; furthermore, its performance advantage over schemes B and C grows with increasing I. For Gaussian channels, schemes B and C perform identically and the scheme A offers power savings only in the case of small MSE target. For Rayleigh fading, scheme B performs consistently better than scheme C and the optimal scheme A saves further transmit power also in the case of large MSE target. The gains of scheme A are as high as 2.5 dB, which roughly translates into an 80% increase of network lifetime.

Figure 2 shows how the required transmit sum power P depends on the MSE target $\varepsilon_{\max}$ for the case of $I = 100$ nodes. With all schemes, achieving a smaller MSE requires to increase the transmit sum power. For Gaussian channels and large MSE targets, all power allocation schemes perform virtually identical. However, for decreasing MSE targets, the performance advantage of the optimal power allocation schemes (A and B) increases. For Rayleigh fading, scheme A has a non-vanishing performance advantage over scheme B at all MSE targets, and scheme C performs strictly poorer than both A and B.

5. CONCLUSIONS

We considered field reconstruction in WSN based on shift-invariant spaces and an AF protocol for the transmission of sensor node measurements to the fusion center. We derived the MSE achieved by this scheme and developed optimal schemes for allocating transmit powers to the sensor nodes in order to minimize the sum power while maintaining a desired MSE performance. For the case of critical sampling, we derived closed-form expressions for the optimal power allocation. Numerical simulations for Gaussian and Rayleigh fading channels showed that the proposed power allocation schemes have the potential for several dB of power savings compared to uniform power allocation.

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