# ON THE IMPACT OF DUTY CYCLE ON THE ESTIMATION OF SPATIAL FIELDS WITH COMPRESSED OBSERVATIONS IN M2M CAPILLARY NETWORKS

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**ABSTRACT**

In this paper, we focus on the use capillary M2M (Machine-to-Machine) networks for the estimation of spatial random fields. The observations (samples) collected by the sensors are spatially correlated and, for this reason, we propose a distributed pre-coding scheme based on the Karhunen-Loève (KL) transform. This allows us to obtain an over-the-air *compressed* representation of such set of observations. We assume that sensors operate with (independent) duty-cycles and we derive a closed-form expression of the optimal power allocation strategy which minimizes the estimation error for a given power constraint. For benchmarking purposes, we also assess the performance of another scheme based on a particularization of the *partial* KL transform.

## I. INTRODUCTION

In recent years, we have witnessed the emergence of the paradigm of Machine-to-Machine (M2M) communications. M2M devices such as automatic meter readers are characterized by very low data rates, low mobility requirements and, in the coming years, they are expected to significantly outnumber voice and data terminals. All this entails a major re-design of future cellular networks. In an attempt to make such transition smoother, ETSIs (European Telecommunication Standards Institute) M2M Technical Committee has proposed a hybrid architecture whereby cellular-enabled gateways (GW) act as traffic aggregation and protocol translation points for their capillary (i.e. wireless sensor) networks.

Our goal is, thus, to accurately reconstruct a spatial random field (e.g. temperature, concentration of air pollutants) from the samples collected by sensors. In [1], the authors analyze the impact of *random* sampling patterns on the resulting estimation distortion. As for transmission aspects, [2] proved that cooperative beamforming (CBF) is optimal when sensors convey a *common* message (observation) to a remote destination. Otherwise, sensors must disseminate their observations to the rest of nodes prior to the beamforming stage. The signalling overhead and the fact that the exchanged information is partly known to the recipients (due to correlation) render such approaches inefficient.

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**Contribution**: In this paper, we focus on the estimation of spatial random fields from a set of correlated observations. As in [3], we allow sensors to *simultaneously* transmit their observations (since this reverts into lower transmission latency) but, unlike in [3], we make no assumption on signal sparsity. Instead, we leverage on the spatial correlation properties of the field. Hence, we propose a distributed pre-coding scheme based on the Karhunen-Loève (KL) transform [4] which allows us to obtain an over-the-air *compressed* representation of the observations. This can actually be regarded as an extension of the cooperative beamforming scheme of [5] for the case of correlated observations. We further consider that sensor nodes operate with independent duty cycles and derive the optimal power allocation for the minimization of distortion subject to a sum-power constraint.

## II. SIGNAL AND COMMUNICATION MODEL

Let $X(\mathbf{s})$ be a spatial field defined over the two-dimensional space $\mathbb{R}^2$. We assume that $X(\mathbf{s})$ is stationary, zero-mean and Gaussian-distributed. The spatial field is sampled by a set $\mathcal{S}$ of $N$ sensors located at $\mathbf{s}_1, \ldots, \mathbf{s}_N$ (locations are assumed to be known), this yielding

$$x_j \triangleq X(\mathbf{s}_j) \quad ; \quad j = 1, \ldots, N. \tag{1}$$

Consequently, the vector of observations $\mathbf{x} = [x_1, \ldots, x_N]^T$ is jointly Gaussian and zero-mean, as well. For a specific set of locations, the elements of the corresponding covariance matrix $\mathbf{C_{xx}} = \mathbb{E}\left[\mathbf{x}\mathbf{x}^T\right]$ read $[\mathbf{C_{xx}}]_{j,j'} = k\left(\mathbf{s_j}, \mathbf{s_{j'}}\right)$ where $k(\cdot,\cdot)$ denotes the covariance function of the spatial random field. In order to enhance network lifetime, a pre-defined duty cycle of $p \times 100\%$ is set. Therefore, sensors are in active mode (i.e. transmitting information to the GW) or idle mode (i.e. saving power) with probability $p$ and $1-p$, respectively. In the sequel, we denote by $\mathcal{S}_A$ and $\mathcal{S}_A^c$ the subsets of active and idle sensors (with $\mathcal{S} = \mathcal{S}_A \cup \mathcal{S}_A^c$). Their corresponding cardinalities are $N_A = |\mathcal{S}_A|$, and $N_A^c = |\mathcal{S}_A^c|$.

At a given time instant, the subset of *active* sensors *simultaneously* transmit (i.e. beamform) their observations to GW. For the $i$-th transmission, the received signal reads[1]

$$r_i = \sqrt{\rho_i} \sum_{j=1}^{N} w_{i,j} h_j z_j X_j + n_i \quad i = 1, \ldots, N, \qquad (2)$$

where $z_1, \ldots, z_N$ are i.i.d. Bernoulli random variables accounting for sensor activity, namely, $z_j = 1$ with probability $p$ (and $z_j = 0$ with probability $1 - p$); $\{w_{i,j}\}_{i,j=1}^{N}$ denote the set of transmit weights[2] (to be designed); $h_1, h_2 \ldots, h_N$ stand for the sensor-to-GW channel coefficients; and $n_i$ is additive white Gaussian noise of variance $\sigma_n^2$, that is, $n_i \sim \mathcal{N}\left(0, \sigma_n^2\right)$. Further, we assume slow fading conditions and, hence, the channel coefficients remain unchanged for $N$ consecutive transmissions. From the $N \times 1$ received signal vector $\mathbf{r} = \left[r_1, \ldots, r_N\right]^T$, the GW attempts to estimate (reconstruct) the spatial field at the set of sampled locations, namely, $\hat{\mathbf{x}} = \left[\hat{x}_1, \ldots, \hat{x}_N\right]^T$. Due to channel impairments, the estimates are subject to some distortion which, in the sequel, will be characterized by the following metric:

$$D \triangleq \frac{1}{N} \sum_{j=1}^{N} \mathbb{E}\left[\left|\hat{x}_j - x_j\right|^2\right]. \qquad (3)$$

## III. COMPRESSED TRANSMISSION VIA THE KARHUNEN-LOÈVE (KL) TRANSFORM

Our goal here is to design a set of transmit weights $\{w_{i,j}\}_{i,j=1}^{N}$ which exploit the spatial correlation properties of the random field and, by doing so, allow for a more energy-efficient (i.e. compressed) transmission. To that aim, we resort to the well-known Karhunen-Loève (KL) Transform.

### III-A. Review of the Karhunen-Loève (KL) Transform

Let $\mathbf{x} = \left[x_1, \ldots, x_N\right]^T$ be a set of jointly Gaussian-distributed random variables. The eigendecomposition of its covariance matrix is given by

$$\mathbf{C_{xx}} = \mathbf{\Phi} \mathbf{\Lambda} \mathbf{\Phi}^T, \qquad (4)$$

where $\mathbf{\Phi} = \left[\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_N\right]$ and $\mathbf{\Lambda} = \mathsf{diag}\left[\lambda_1, \ldots, \lambda_N\right]$ are the unitary and diagonal matrices containing the corresponding eigenvectors and eigenvalues, respectively (with $\lambda_i \in \{\mathbb{R}^+, 0\}$ and $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N$). The KL transform of vector $\mathbf{x}$ can thus be defined as

$$\mathbf{y} = \mathbf{\Phi}^T \mathbf{x} = \begin{bmatrix} \boldsymbol{\phi}_\mathbf{1}^T \mathbf{x} \\ \vdots \\ \boldsymbol{\phi}_\mathbf{N}^T \mathbf{x} \end{bmatrix}. \qquad (5)$$

Interestingly, the elements of the transform vector $\mathbf{y} = \left[y_1, \ldots, y_N\right]$ turn out to be independent random variables, that is, $\mathbf{y} = \left[y_1, \ldots, y_N\right] \sim \mathcal{N}\left(0, \mathbf{\Lambda}\right)$. Since $\mathbf{\Phi}$ is a unitary matrix, the inverse transform can be readily expressed as

$$\mathbf{x} = \mathbf{\Phi} \mathbf{y} = \sum_{i=1}^{N} \boldsymbol{\phi}_i y_i. \qquad (6)$$

Besides, the first $k$ components in $\mathbf{y}$ are known to provide the best $k$-length representation of vector $\mathbf{x}$, that is, the one that minimizes the quadratic error (MSE) in the estimate given by $\hat{\mathbf{x}}^{(k)} = \sum_{i=1}^{k} \boldsymbol{\phi}_i y_i$. For $k \leq N$, the vector $\left[y_1, \ldots, y_k\right]^T$ can be regarded as a *compressed* version of $\mathbf{x}$.

### III-B. Design of transmit weights

Inspired by [3], we let the transmit weights be

$$w_{i,j} = \sqrt{\rho_i} \frac{h_j *}{|h_j|^2} w'_{i,j}; \quad i, j = 1 \ldots N \qquad (7)$$

with $\rho_i; i = 1 \ldots N$ denoting a set of scalar factors to be determined later. The rationale for this approach is as follows: (i) the term $h_j^*$ allows sensors to coherently combine their transmissions at the GW ; (ii) the term $\frac{1}{|h_j|^2}$ carries out channel equalization (inversion); and (iii) the $w'_{i,j}$ coefficients are in charge of the compression task itself. Since (ii) makes the system oblivious to the channel gains, we simply let $w'_{i,j}$ be the coefficients of the (centralized) KL transform, that is, $w'_{i,j} = \phi_{i,j}$. In other words, in the $i$-th transmission we attempt to convey the observations over the $i$-th eigenmode of the covariance matrix $\mathbf{C_{xx}}$. This is strictly true when all sensors are active. However, in scenarios with duty cycle below $100\%$ (i.e. $p < 1$), only a *subset* of the sensors effectively transmit data; still, the same $w'_{i,j}$ coefficients are used[3]. This results into some degradation of the generated beampattern and, ultimately, an increased distortion in the reconstructed spatial field. This effect will be analyzed more in detail in subsequent sections.

The total transmit power at the $i$-th transmission reads

$$\begin{aligned} P_i &= \mathbb{E}\left[\sum_{j \in \mathcal{S}_A} \left|w_{i,j} x_j\right|^2\right] = \left(\sum_{j \in \mathcal{S}_A} \frac{1}{|h_j|^2} \phi_{i,j}^2 \sigma_x^2\right) \rho_i & (8) \\ &= \alpha_i \rho_i \quad ; \quad i = 1, \ldots, N, & (9) \end{aligned}$$

where we have defined $\alpha_i \triangleq \sum_{j \in \mathcal{S}_A}^{N} \frac{1}{|h_j|^2} \phi_{i,j}^2 \sigma_x^2$. From (2) and (7), and by letting $w'_{i,j} = \phi_{i,j}$, the received signal for the $i$-th transmission now reads:

$$r_i = \sqrt{\rho_i} \sum_{j=1}^{N} \phi_{i,j} z_j X_j + n_i \quad i = 1, \ldots, N. \qquad (10)$$

## IV. DISTORTION ANALYSIS

The Linear Minimum-Mean Square Error (LMMSE) estimator of $\mathbf{x}$ given $\mathbf{r}$ can be expressed as [6]

$$\hat{\mathbf{x}} = \mathbf{C_{xr}} \mathbf{C_{rr}}^{-1} \mathbf{r}, \qquad (11)$$

with $\mathbb{E}\left[\cdot\right]$ denoting the statistical expectation with respect to $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{n}$; and where $\mathbf{C_{xr}} = \mathbb{E}\left[\mathbf{x}\mathbf{r}^T\right]$ and $\mathbf{C_{rr}} = \mathbb{E}\left[\mathbf{r}\mathbf{r}^T\right]$. The average distortion in the estimate of $\mathbf{x}$ at the GW reads

$$D = \frac{1}{N} \mathrm{Tr}\left\{\mathbf{C_{xx}} - \mathbf{C_{xr}} \mathbf{C_{rr}}^{-1} \mathbf{C_{rx}}\right\}, \qquad (12)$$

[1]Perfect phase synchronization is assumed here.

[2]Notice that a different set of weights $w_{i,j}$ is used for each transmission.

[3]By doing so, the $w'_{i,j}$ coefficients only need to be computed once, in the initialization phase , since they exclusively depend on matrix $\mathbf{C_{xx}}$.

where $\mathrm{Tr}(\cdot)$ denotes the trace operator. First of all, we need to compute matrix $\mathbf{C_{rr}}$. It is straightforward to show that $r_i$ is zero-mean and, hence, we can write $[\mathbf{C_{rr}}]_{i,i'} = \mathbb{E}[r_i r_{i'}]$. One can also prove that the diagonal elements (i.e. for $i = i'$) can be expressed as

$$\mathbb{E}\left[r_i^2\right] = \rho_i \lambda_i \mathbb{E}^2[z] + \rho_i \sigma_z^2 \sigma_x^2 + \sigma_n^2 \tag{13}$$

whereas, for the off-diagonal elements, we have $\mathbb{E}[r_i r_{i'}] = 0$ (the proof is omitted due to space limitation). In other words, the random variables $\{r_1, r_2, \ldots, r_N\}$ are uncorrelated and, hence, matrix $\mathbf{C_{rr}}$ turns out to be diagonal. Next, in order to compute matrix $\mathbf{C_{xr}}$, we recall from (6) that

$$x_j = \sum_{i=1}^{N} \phi_{i,j} y_i, \tag{14}$$

along with the expression for the received signal in (10). Bearing in mind that the random variables $x_j$ and $z_j$ are statistically independent, the $\mathbb{E}[x_j r_i]$ terms yield

$$\mathbb{E}[x_j r_i] = \mathbb{E}\left[\sum_{i=1}^{N} \phi_{i,j} y_i r_i\right] = \mathbb{E}[z]\sqrt{\rho_i}\phi_{i,j}\lambda_i \tag{15}$$

for $i, j = 1, \ldots, N$. Hence, matrix $\mathbf{C_{xr}}$ reads

$$\mathbf{C_{xr}} = \mathbb{E}[z]\mathbf{\Gamma}\mathbf{\Phi}\mathbf{\Lambda} \tag{16}$$

with $\mathbf{\Gamma} = \mathsf{diag}[\rho_1, \ldots, \rho_N]$. Finally, from (12), (13) and (16), the average distortion is given by

$$D = \sigma_x^2 - \frac{1}{N}\sum_{i=1}^{N} \frac{\rho_i p^2 \lambda_i^2}{\rho_i p^2 \lambda_i + \rho_i \sigma_x^2 p(1-p) + \sigma_n^2}, \tag{17}$$

where we have used that $\mathbb{E}[z] = p$ and $\sigma_z^2 = p(1-p)$. Clearly, for $p = 1$ (i.e. duty cycle of 100%) equation (17) reverts to that in our previous work [7].

### IV-A. Optimal Power Allocation

In order to compute the set of transmit weights $w_{i,j}$, we still need to determine the power allocation $\rho_i$ for each eigenmode (while explicitly taking into consideration the aforementioned duty cycle $p$). In particular, our interest lies in finding the power allocation which minimizes the overall distortion for a given sum-power constraint. From (17) and (9), the problem can be posed as:

$$\min_{\rho_1,\ldots,\rho_N} \quad \sigma_x^2 - \frac{1}{N}\sum_{i=1}^{N} \frac{\rho_i p^2 \lambda_i^2}{\rho_i p^2 \lambda_i + \rho_i \sigma_x^2 p(1-p) + \sigma_n^2} \tag{18}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \alpha_i \rho_i \leq P_t, \tag{19}$$

This is a convex problem and, similarly to [8], it can be solved in closed-form. This yields the following waterfilling-like solution:

$$\rho_i^* = \left[\frac{\sigma_n}{\sqrt{\alpha_i \mu} p + (1-p)\sigma_x^2} - \frac{\sigma_n^2}{\lambda_i p^2 + p(1-p)\sigma_x^2}\right]^+ \tag{20}$$

where $[x]^+ \triangleq \max\{x, 0\}$ and $\mu$ denotes the Lagrange multiplier associated to the power constraint, namely,

$$\mu = \left(\frac{P_t + \sum_{i=1}^{K} \frac{\alpha_i \sigma_n^2}{\lambda_i p^2 + p(1-p)\sigma_x^2}}{\sum_{i=1}^{K} \frac{\lambda_i \sigma_n \sqrt{\alpha_i}}{\lambda_i p + (1-p)\sigma_x^2}}\right)^{-2}. \tag{21}$$

In this last expression, $K$ stands for the largest number of transmissions such that (i) the optimal scaling factors verify $\rho_i^* \geq 0$ for $i = 1, \ldots, K$; and (ii) the sum-power constraint holds with equality, i.e. $\sum_{i=1}^{K} \rho_i^* \alpha_i = P_t$. Without loss of generality, we have also assumed that $\lambda_1/\alpha_1 > \lambda_2/\alpha_2 > \ldots > \lambda_N/\alpha_N$.

## V. COMPRESSED TRANSMISSION VIA THE PARTIAL KL TRANSFORM

In the previous section, we simply let $w'_{i,j}$ be the coefficients of the KL transform associated to the covariance matrix of the entire set of sensors $\mathcal{S}$ (namely, $w'_{i,j} = \phi_{i,j}$). Yet computationally efficient, this approach results into some mismatch between the designed and effective beampatterns since only a subset of sensors will actually transmit data. Hence, we ask ourselves whether it pays off to specifically design the $w'_{i,j}$ weights (and the corresponding LMMSE estimator in equation (11)) for each realization of the subset of active sensors. To answer that, we resort to the *partial* KL transform (pKLT) of [4] which we particularize for the problem at hand.

Let $\mathbf{x}_A$ and $\mathbf{x}_A^c$ denote the $N_A \times 1$ and $N_A^c \times 1$ vectors of observations collected by the subsets of active and idle sensors, respectively. From [4], the best representation of the $N \times 1$ vector $\mathbf{y}$ from $\mathbf{x}_A$ is given by the pKLT, namely,

$$\mathbf{y} = \mathbf{\Psi}^T \begin{pmatrix} \mathbf{I}_{N_A} \\ \mathbf{C}_{\mathbf{x}_{A^c}\mathbf{x}_A}\mathbf{C}_{\mathbf{x}_A\mathbf{x}_A}^{-1} \end{pmatrix} \mathbf{x}_A, \tag{22}$$

with $\mathbf{I}_{N_A}$ standing for the $N_A \times N_A$ identity matrix, $\mathbf{C}_{\mathbf{x}_{A^c}\mathbf{x}_A} = \mathbb{E}\left[\mathbf{x}_{A^c}\mathbf{x}_A^T\right]$ and $\mathbf{C}_{\mathbf{x}_A\mathbf{x}_A} = \mathbb{E}\left[\mathbf{x}_A\mathbf{x}_A^T\right]$. Besides, $\mathbf{\Psi}$ is the $N \times N$ unitary matrix of the eigenvectors of the covariance matrix $\mathbf{C}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$, namely,

$$\mathbf{C}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} = \begin{pmatrix} \mathbf{C}_{\mathbf{x}_A\mathbf{x}_A} & \mathbf{C}_{\mathbf{x}_A\mathbf{x}_{A^c}} \\ \mathbf{C}_{\mathbf{x}_{A^c}\mathbf{x}_A} & \mathbf{C}_{\mathbf{x}_{A^c}\mathbf{x}_A}\mathbf{C}_{\mathbf{x}_A\mathbf{x}_A}^{-1}\mathbf{C}_{\mathbf{x}_A\mathbf{x}_{A^c}} \end{pmatrix} \tag{23}$$

$$= \mathbf{\Psi}\mathbf{\Delta}\mathbf{\Psi}^T \tag{24}$$

and $\mathbf{\Delta} = \mathsf{diag}[\delta_1, \ldots, \delta_N]$ is a (diagonal) matrix with the corresponding eigenvalues. This follows from the fact that the product of the last two terms on the left handside of (22) can be regarded as a new vector $\tilde{\mathbf{x}}$ defined as

$$\tilde{\mathbf{x}} = \begin{bmatrix} \mathbf{x}_A \\ \tilde{\mathbf{x}}_A^c \end{bmatrix} = \begin{pmatrix} \mathbf{I}_{N_A} \\ \mathbf{C}_{\mathbf{x}_{A^c}\mathbf{x}_A}\mathbf{C}_{\mathbf{x}_A\mathbf{x}_A}^{-1} \end{pmatrix} \mathbf{x}_A, \tag{25}$$

In order to implement the partial KL transform in a distributed fashion, it suffices to define matrix $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_N]$ as

$$\mathbf{B}^T = \mathbf{\Psi}^T \begin{pmatrix} \mathbf{I}_{N_A} \\ \mathbf{C}_{\mathbf{x}_{A^c},\mathbf{x}_A}\mathbf{C}_{\mathbf{x}_A,\mathbf{x}_A}^{-1} \end{pmatrix} \tag{26}$$

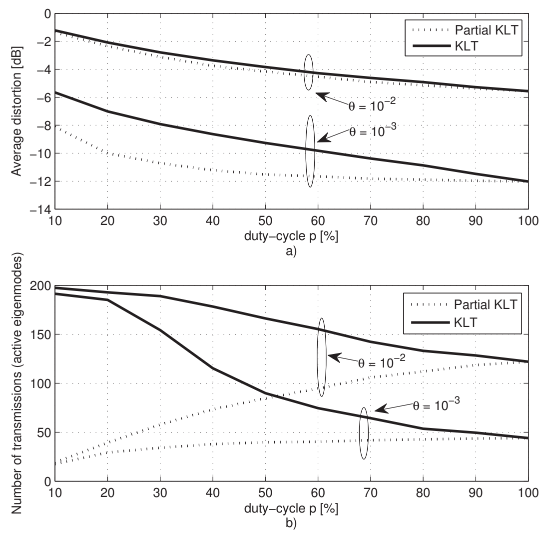


**Fig. 1**. a) Average distortion vs duty-cycle p [%] and b) Number of transmissions vs duty-cycle p [%] ($N = 200$, $\sigma_n^2 = 1$, $\sigma_x^2 = 1$)

and then let $w'_{i,j} = b_{i,j}$ for $i, j = 1, \ldots, N$, where $b_{i,j}$ stands for the $j$-th element of vector $\mathbf{b}_i$. Following a similar derivation to that in Section IV, the distortion for the optimal MMSE estimator can be readily expressed as:

$$D = \sigma_x^2 - \frac{1}{N}\sum_{i=1}^{N} \frac{\rho_i \delta_i^2}{\rho_i \delta_i + \sigma_n^2}. \tag{27}$$

From this, it is straightforward to find a closed-form expression of the optimal power allocation (see Section IV-A).

## VI. SIMULATION RESULTS AND CONCLUSIONS

The simulation scenario consists of $N = 200$ sensors deployed at random locations within a $1000 \times 1000$ rectangular area (uniform distribution). As in [9], the spatial field is modeled as a Gaussian Markov Ornstein-Uhlenbeck process with correlation (covariance) function given by $k(\mathbf{s}_i, \mathbf{s}_j) = \sigma_x^2 \exp(-\theta \|\mathbf{s}_i - \mathbf{s}_j\|_2)$. Clearly, the higher the parameter $\theta$, the lower the correlation among observations. Due to space limitations, we show results for AWGN channels only (i.e., $h_j = 1$ for $j = 1, \ldots, N$).

In Fig. 1a, we depict the average distortion in the estimated random field as a function of the sensor duty-cycle $p$. We illustrate the performance of both the KLT- and pKLT-based cooperative beamforming schemes. Unsurprisingly, distortion is a monotonically decreasing function in $p$. By increasing $p$ the active number of sensors increases on average and, hence, the beamforming gain is larger. Besides, the pKLT-based scheme outperforms the KLT-based one for all $p$ (for $p = 1$, both schemes are identical). This stems from the fact that pKLT exploits the information on the subset of active sensors in each realization not only for power allocation purposes but also for the computation of the *compression* term ($w'_{ij}$) in the beamforming weights (and, consequently, the design of the corresponding estimator in the GW). Despite of the computational burden that pKLT entails, the gain can be regarded as moderate for highly correlated fields (2 $dB$ for 30% duty cycle when $\theta = 10^{-3}$) or negligible in fields with lower correlation ($\theta = 10^{-2}$).

Fig. 1b provides further insights on the average number of active eigenmodes (or, equivalently, consecutive transmissions needed, or transmission latency). The impact of duty cycle in the KLT- and pKLT-based approaches is radically different. In the KLT case, the power allocation scheme activates more eigenmodes for small $p$. In this way, it attempts to (partly) compensate for the increasing distortion in the beampatterns by sending more (in principle redundant) information. On the contrary, the pKLT scheme tends to activate less eigenmodes. This is attributed to the fact that, when $p$ diminishes, the attempt to infer the observations of idle sensors by conditioning on those of the active ones (the $\tilde{\mathbf{x}}_A^c$ term in equation (25)) is less accurate. Likewise, matrix $\mathbf{C}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ becomes substantially different from $\mathbf{C}_{\mathbf{xx}}$ this having a direct impact on the designed (eigen)beamformers. Their *combined* effects result into a reduction of the number of active modes (i.e. focusing on the more reliable ones). Interestingly, the gap is larger for scenarios with low correlation (large $\theta$) since the difficulty to infer unknown data is higher.

In conclusion, diminishing the duty cycle has a negative impact in the performance of the proposed pre-coding schemes. This loss can be partly compensated by resorting to the partial KL transform, at the expense of a substantial increase in computational burden and associated signalling.

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