A DIFFERENTIAL FEEDBACK SCHEME FOR EQUAL GAIN TRANSMISSION IN TEMPORALLY CORRELATED CHANNELS

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ABSTRACT

The adoption of equal gain transmission (EGT) in MIMO systems can greatly reduce the computational complexity and the design effort of power amplifier. In a temporally correlated channel environment, the channel varies slowly and so does the corresponding beamforming vector. If the slowly varying channel condition can be taken into consideration while designing the feedback schemes of EGT, the computational complexity and feedback overhead of EGT are expected to be further diminished dramatically. In this paper, we propose a differential feedback scheme (DFS) with two algorithms to further improve the performance in EGT. Simulation results show the advantages of the proposed DFS in terms of the feedback overhead and the system performance.

Index Terms—MIMO, beamforming, scalar quantization (SQ), equal gain transmission (EGT), limited feedback.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques can improve transmission robustness against channel fading and thereby achieve high spectral efficiency. Among the MIMO techniques, the closed-loop MIMO can provide extra beamforming gain compared to the open-loop MIMO due to the provision of the channel state information (CSI) at the transmitter (CSIT). In time division duplex (TDD) wireless systems, full CSIT might be possible due to the channel reciprocity property. On the contrary, in frequency division duplex (FDD) wireless systems, owing to the separated and nearly independent channel characteristic between uplink and downlink channels, extra feedback information for CSIT is required. However, it may not be feasible to assume full CSIT is available. To overcome the dilemma of the CSI feedback in FDD systems, extensive researches have been done for closed-loop MIMO systems with limited feedback. The basic idea is to quantize the CSI before feedback. The quantization techniques can be categorized to vector quantization (VQ) and scalar quantization (SQ). VQ [1][2] in general performs better than SQ. However, SQ is done on a per-antenna basis. Hence, there is no need to search exhaustively to determine the closest codeword. Moreover, the storage requirement for the codebook in both transmitter and receiver can also be waived if SQ is used. In [3]-[5], the performance analysis of several quantization schemes can be found. In realistic communication systems, temporal correlation between channel realizations often exists especially in the static indoor environment, where the slow channel variations may result in a high correlation between two consecutive beamforming vectors. In environments with a high temporal correlation, the work in [7] showed that the feedback overhead can be reduced by compressing the CSI. In [8], a framework for the rotation-based differential feedback was proposed by constructing the rotation codebook to reflect the temporal correlation of channel. Moreover, the rotation-based differential feedback scheme has been proposed as a standard contribution for the IEEE 802.16m MIMO standard [9]. However, this rotation-based scheme requires computational effort to conduct the Householder operation and matrix multiplication. Meanwhile, due to the use of unequal gain beamforming vector, the design effort of the power amplifier (PA) is much higher than the equal gain beamforming schemes. According to [5], the maximum SNR loss between the optimal beamforming and EGT (equal gain transmission) is only 1.05 dB, whereas the computational complexity and PA design effort can be greatly reduced. If differential feedback can be used in EGT, we can reduce both the feedback overhead and the complexity. This motivates us to explore how to design differential feedback schemes for EGT.

In this paper, we study MIMO EGT beamforming with limited feedback in the temporally correlated channels, and proposed a differential feedback scheme (DFS) with two algorithms for the advanced performance improvement in EGT. From the simulation results, by ultilizing the proposed DFS in highly time-correlated channel environments, the EGT with little feedback information can achieve a performance very close to the EGT without quantization error. Moreover, the EGT with proposed DFS can outperform the conventional one-shot Grassmannian beamforming scheme [2].

Notation: Bold upper and lower case letters denote matrices and vectors, respectively. $[\mathbf{v}]_k$ is the kth entry of vector $\mathbf{v}$ and $[\mathbf{M}]_{i,j}$ is the entry of matrix $\mathbf{M}$ at the ith row and jth column. $(\cdot)^T$ is the transpose and $(\cdot)^H$ is the conjugate transpose. $\angle x$ is the angle of x. σ_x^2 denotes the variance of x.

2. SYSTEM MODEL OF EGT IN CORRELATED CHANNELS

The MIMO beamforming systems considered in this work equip with N_t transmit and N_r receive antennas. At first, the modulated symbol $x_t \in \mathbb{C}$ is multiplied by the beamforming vector $\mathbf{w}_t \in \mathbb{C}^{N_t \times 1}$, where the subscription t is used to emphasize the channel environment is time-variant. For EGT, the beamforming vector can be expressed as [2]

$$\mathbf{w}_t = \frac{1}{\sqrt{N_t}} \left[1 \; e^{j\theta_{t,2}} \; ... \; e^{j\theta_{t,N_t}} \right]^T \qquad (1)$$

where $\frac{1}{\sqrt{N_t}}$ is used to normalize the transmit power so that the total power is kept the same for different N_t. After the beamforming

operation, the symbol vector $\mathbf{s}_t = [s_{t,1}\ s_{t,2}\ ...\ s_{t,N_t}]^T$, to be transmitted is given by $\mathbf{s}_t = \mathbf{w}_t x_t$. Then, $\mathbf{s}_t$ is transmitted to the MIMO channel. The received vector $\mathbf{r}_t = [r_{t,1}\ r_{t,2}\ ...\ r_{t,N_r}]^T$ from the channel can be expressed as $\mathbf{r}_t = \mathbf{H}_t \mathbf{s}_t + \mathbf{n}_t = \mathbf{H}_t \mathbf{w}_t x_t + \mathbf{n}_t$, where $\mathbf{H}_t \in \mathbb{C}^{N_r \times N_t}$ is the MIMO channel matrix at time instant t with its (i,j)th element being $[\mathbf{H}_t]_{i,j}$, and $\mathbf{n}_t \in \mathbb{C}^{N_r \times 1}$ is a white Gaussian noise vector with variance σ_n^2. In a temporally correlated channel environment, the channel variation may be modeled as a first-order Gauss-Markov process as follows [6]

$$\mathbf{H}_t = \sqrt{\alpha^2}\mathbf{H}_{t-1} + \sqrt{1-\alpha^2}\mathbf{G}_t \qquad (2)$$

where $\mathbf{G}_t \in \mathbb{C}^{N_r \times N_t}$ denotes the innovation process with i.i.d. elements according to $\mathcal{CN}(0,1)$ and α $(0 \leqslant \alpha \leqslant 1)$ denotes the timing correlation coefficient which reflects the correlation between $[\mathbf{H}_t]_{i,j}$ and $[\mathbf{H}_{t+1}]_{i,j}$. The value of α is used to judge the channel stationarity. For instance, for $\alpha = 0$, the channel is i.i.d. in time, whereas for $\alpha = 1$, the channel is time-invariant. We assume that the noise process is independent of $\mathbf{G}_t$ and the initial state of channel $\mathbf{H}_0$, and $\mathbf{H}_0$ is independent of $\mathbf{G}_{t+1}$ for all t. Also, we assume all the elements of $\mathbf{H}_t$ have the same time correlation coefficient α. The received signal $\mathbf{r}_t$ is multiplied by the combining vector $\mathbf{z}_t \in \mathbb{C}^{1 \times N_r}$ to form $\hat{x}_t$, i.e. $\hat{x}_t = \mathbf{z}_t \mathbf{r}_t = \mathbf{z}_t \mathbf{H}_t \mathbf{w}_t x_t + \mathbf{z}_t \mathbf{n}_t$. To achieve the best performance, the MRC technique [1] is applied at the receiver. Let the combiner be $\mathbf{z}_t = (\mathbf{H}_t \mathbf{w}_t)^H$. Thus $\hat{x}_t = \gamma x_t + \mathbf{z}_t \mathbf{n}_t$, where $\gamma = \mathbf{w}_t^H \mathbf{H}_t^H \mathbf{H}_t \mathbf{w}_t$ is the gain effect (including diversity gain and array gain) due to the spatial diversity and the beamforming. The channel is assumed to be quasi-static in time. $\mathbf{H}_t$ is assumed to be known perfectly at the receiver, but partially at the transmitter through a limited feedback channel. In addition, we assume that the feedback channel has no delay while conveying quantization information to the transmitter.

3. PROPOSED DIFFERENTIAL FEEDBACK SCHEME (DFS)

According to the channel model in (2), when α is large, channel varies slowly. In this case, the beamforming vector varies slowly as well, and there may be no need to feed back a new beamforming vector for each channel realization; it should be sufficient to send the differential information between the previous and the current beamforming vectors. This concept is called differential feedback. Differential feedback schemes for the optimal beamforming vector was proposed in [9]. Such schemes require Householder transformation for constructing the rotation matrix. Also, matrix multiplications are required to rotate the optimal beamforming vector to a certain region so that the differential codebook can cover. Hence, it may lead to high computational complexity and long feedback latency. Since the use of EGT can overcome the above drawbacks, this motivates us to explore how to design differential feedback schemes for EGT. In this section, we proposed a differential feedback scheme (DFS) with two algorithms for EGT as follows.

The proposed DFS sends back the quantized differential phase information for every channel realization after the initial full CSI is synchronized at both transmitter and receiver sides. Then, one dimensional scalar quantization (SQ) is used to represent the phase difference, and we call this *differential SQ*. For the differential SQ, the SQ is used to represent the phase difference between adjacent beamforming vectors. Let the phase difference between the adjacent EGT vectors without quantization be in the form of (1), and the previous quantized beamforming vectors be $\mathbf{d} = \angle\mathbf{w}_t - \angle\hat{\mathbf{w}}_{t-1} =$

$[0\ \varphi_2\ ...\ \varphi_{N_t}]^T$. For MISO channels, the optimal EGT vector without quantization can be derived by reversing the channel phase vector [1]. For MIMO channels, a benchmark solution can be obtained by the cyclic method proposed in [4]. Since SQ is used, it is reasonable to assume the number of bits allocated to each antenna is the same, denoted it by b_{diff}. The quantized vector of the phase difference is $\mathcal{Q}(\mathbf{d}) = (\hat{\varphi}_2\ ...\ \hat{\varphi}_{N_t})$, where $\hat{\varphi}_i = (2\pi n_i)/N_d$, $0 \leqslant n_i \leqslant N_d - 1$, $i = 2, ..., N_t$, with $N_d = 2^{b_{diff}}$ and n_i denoting the number of quantization levels and feedback index of φ_i, respectively. Then, the index set $(n_2, n_3, ..., n_{N_t})$ is sent back to the transmitter, totally requiring $B_{diff} = b_{diff}(N_t - 1)$ feedback bits. Please note that since the phase difference is not uniformly distributed, the differential SQ applied here may not have uniform step sizes, *i.e.* the distances between different quantization levels are different. Also, to reflect the channel variation due to varying α in (2), the proposed DFS allows the step sizes for different quantization levels vary adaptively. That is, the adaptive step size can help track the channel and further improve the performance. Since each antenna equips b_{diff} bits to quantize phase difference, first we consider how to determine the quantization levels as well as the variable step size for $b_{diff} > 1$; Then, we find that by letting $b_{diff} = 1$, there is a simple closed-form solution for quantization level and step size; this reduces the computational complexity while the corresponding performance degradation is still satisfactory.

3.1. Proposed DFS with $b_{diff} > 1$

The best quantization levels for one-dimensional scalar quantization can be determined by Lloyd-based algorithm summarized in Algorithm 1. This algorithm demands a on-line training of quantization levels; then after a long training period additional number b_{adapt} of bits is needed to indicate the quantization levels. Later simulation result shows that the performance with $b_{diff} = 2$ is very close to that without quantization effect when $\alpha = 0.999$.

Algorithm 1: Lloyd-based algorithm for adaptive quantization levels with $b_{diff} > 1$

Step 1) Start with an initial quantization level set. Set $m = 1$.

Step 2) Given the quantization level set, $Q^{(m)}$, perform the following Lloyd algorithm to generate the quantization level set, $Q^{(m+1)}$

    Step a) Given a quantization level set, $Q^{(m)} = \{y_i\}$, partition the phase differences into cluster sets R_i using the Nearest Neighbor Condition:
$R_i = \{\mathbf{x} : d(\mathbf{x}, \mathbf{y}_i) < d(\mathbf{x}, \mathbf{y}_j);\ \text{all}\ j \neq i\}$.

    Step b) Using the Centroid Condition, compute the centroids for the cluster sets just found to obtain the new quantization level set, $Q^{(m+1)} = \{cent(R_i)\}$.

Step 3) Compute the average distortion for $Q^{(m+1)}$. If the distortion is smaller than a pre-determined threshold, stop the program; Otherwise, $m \leftarrow m + 1$, goto Step 2.

Although this algorithm achieves the best performance due to the use of Lloyd algorithm, there is no closed-form solution and it demands on-line training of multiple quantization levels. Thus this causes high computational effort and long feedback latency. To have a better trade-off, we found that there is a closed-form solution for $b_{diff} = 1$; hence the computational complexity can be greatly re-

duced and the resulting performance loss is still satisfactory. The closed-form solution is introduced below.

3.2. Proposed DFS with $b_{diff} = 1$

According to [5], when SQ is used for EGT, allocating three bits per antenna excluding the first antenna can generally lead to a performance very close to that of EGT without quantization. If the channel varies slowly, the phase difference of the current and previous beamforming vectors varies slowly as well. Thus, allocating one bit per antenna excluding the first antenna may be sufficient in this case, *i.e.* letting $b_{diff} = 1$. We derive a closed-form solution of the step size as follows.

If channel realizations are generated according to (2), the channel has the property of Markov process. Since the phase of correlated channel coefficient $[\mathbf{H}_t]_{i,j}$ has uniform distribution with zero mean, the phase of the jth beamforming element $\angle[\mathbf{w}_t]_j$ also has zero mean. Fig. 1 represents the statistics of phase difference for four
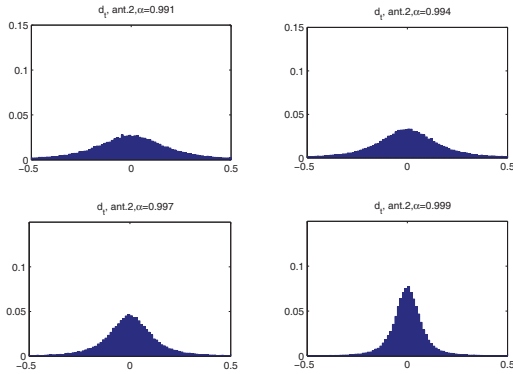

Fig. 1. The phase difference statistics for antenna 2 in 4 transmit antenna configuration.

transmit antennas with correlation coefficient α ranging from 0.991 to 0.999 with 60000 different channel realizations. We observe that the higher the α is, the more concentrated to zero the phase difference distribution is. For $b_{diff} = 1$, the two quantization levels are $\{\ell_{neg}, \ell_{pos}\}$ where $\ell_{neg} < 0$ and $\ell_{pos} > 0$. Actually ℓ_{neg} and ℓ_{pos} are two symmetric values centered by 0, *i.e.* $\ell_{neg} = -\ell_{pos}$; hence, only one level needs to be estimated. Without losing of generality, the least squares (LS) approach could be applied for the estimation of ℓ_{pos} to minimize the squared errors between ℓ_{pos} and the observed positive phase differences, which is given by

$$J(\ell_{pos}) = \sum_{t=1}^{N_1} ([\mathbf{d}]_j - \ell_{pos})^2, \quad [\mathbf{d}]_j \geqslant 0 \tag{3}$$

where the observation interval is assumed to be $t = 1, 2, ..., N_1$. By differentiating $J(\ell_{pos})$ with respect to ℓ_{pos}, *i.e.* $\frac{\partial J(\ell_{pos})}{\partial \ell_{pos}}$, and then setting it to zero, we obtain the LS estimator given by

$$\ell_{pos} = \frac{1}{N_1} \sum_{t=1}^{N_1} [\mathbf{d}]_j, \quad [\mathbf{d}]_j \geqslant 0. \tag{4}$$

The information of ℓ_{neg} and ℓ_{pos} is quantized before sending back. Due to the symmetric property of ℓ_{neg} and ℓ_{pos}, we let the number

of quantization bits for ℓ_{neg} and ℓ_{pos} be b_{adapt}. That is, $\hat{\ell}_{pos} = \hat{\ell}_{neg}$ and they have $2^{b_{adapt}}$ quantization levels.

The proposed DFS can be summarized in Algorithm 2.

Algorithm 2: Proposed Differential Feedback Scheme

Step 0)   Input: $\hat{\mathbf{w}}_{t-1}$: quantized beamforming vector at time $t - 1$; $\mathbf{H}_t$: channel realization at time t.

Step 1)   Quantization: calculate the EGT vector at time t, *i.e.* $\mathbf{w}_t$, and the phase difference $\mathbf{d} = \angle \mathbf{w}_t - \angle \hat{\mathbf{w}}_{t-1}$. Then, quantize the phase difference $\mathbf{d}$ using either one of the following quantization levels (let the quantized phase difference be $\hat{\mathbf{d}}$):

1) fixed quantization levels (e.g. $\{\pm \frac{\pi}{10}\}$)

2) adaptive quantization levels

2.1) with $b_{diff} > 1$ (see the procedure in Sec. 3.1)

2.2) with $b_{diff} = 1$ (see the procedure in Sec. 3.2)

Step 2)   Output: obtain the quantized beamforming vector $\tilde{\mathbf{w}}_t$ via $\tilde{\mathbf{w}}_t = \frac{1}{\sqrt{N_t}} e^{j(\angle \hat{\mathbf{w}}_{t-1} + \hat{\mathbf{d}})}$

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4. SIMULATION RESULTS

In the following simulations, the time-correlated channel model in (2) is used. We generate α according to the suggestions in [6] and [8]. That is, in indoor environment for mobile speed 1-5 km/hr and carrier frequency ranging from 800 MHz to 5 GHz, the correlation coefficient is from $\alpha_{min} = 0.999$ to $\alpha_{max} \approx 1$. Also, in urban environment for mobile speed 50 km/h and carrier frequency of 5 GHz, $\alpha_{min} = 0.991$ and $\alpha_{max} = 0.999$ have been assumed. We use 60000 different channel realizations. For the parameters not particularly specified in the example, the default modulation is BPSK and the default correlation coefficient is $\alpha = 0.999$. The initial quantization levels are set to be $\{\pm \pi/4\}$ for $b_{diff} = 1$; if adaptive quantization levels are applied, we use $b_{adapt} = 3$ bits to represent the quantization levels. The notation mTnR or $m \times n$ denotes $N_t = m$ and $N_r = n$.

Example 1.   The proposed DFS with and without adaptive quantization levels. Fig. 2 shows the performance comparison among the EGT without quantization, Grassmannian beamforming (GS) in [2], and the proposed DFS scheme for EGT with and without adaptive quantization levels. The channel environment is set to be 4T1R. We have the following observations: *1)* In a highly time-correlated channel with $\alpha = 0.999$, the proposed DFS outperforms the one-shot Grassmannian beamforming. *2)* The use of adaptive quantization levels introduced in Sec. 3 can better catch the specific temporal correlation property of channel in α, which not only considerably outperforms that without adaptive quantization levels but also achieves a performance very close to the EGT without quantization. For the adaptive quantization level scheme, we use two different numbers of differential feedback bits; one is $b_{diff} = 2$ and the other is $b_{diff} = 1$. We use Algorithm 1 for $b_{diff} = 2$, and the closed-form solution in (4) for $b_{diff} = 1$. Note that using $b_{diff} = 1$ can greatly reduce the computational complexity. Observe that for $b_{diff} = 2$, the performance is very close to EGT without quantization, and thus letting $B_{diff} = b_{diff}(N_t - 1) = 6$ for $N_t = 4$ is a satisfactory setting. For $b_{diff} = 1$, there is a performance degradation compared to

$b_{diff} = 2$ with Algorithm 1; however, it still outperforms the case without the adaptive quantization levels by around 0.35 dB.
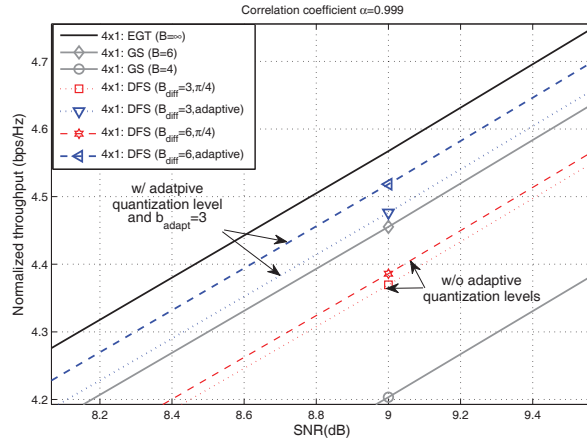

Fig. 2. Capacity performance for the proposed DFS with and without adaptive quantization levels in the temporally correlated channels with correlation coefficient $\alpha = 0.999$.

Example 2. Comparisons of proposed and various beamforming schemes. We compare the bit error probability (BEP) performance for the following beamforming schemes: antenna selection (AS), Grassmannian beamforming (GS) in [2], the differential feedback scheme in [9] named DFS-Pri, the proposed DFS without adaptive quantization levels for EGT, the EGT without quantization and the optimal MRT beamforming. Regarding the number of bits for each feedback, we let $B = 3$ for the proposed DFS, and let $B = 3$ for the DFS-Pri. Fig. 3 shows the performance comparisons in 4T1R and 4T2R channel environments. We have the following observations: *1)* Compared to the conventional one-shot AS and GS beamforming schemes, the proposed DFS not only improves the performance in time-correlated channels but also significantly reduces the computational complexity, since there is no need to use exhaustive search to select the beamforming vector; especially in a MISO channel, the EGT vector without quantization can be directly obtained from the channel (see [4][5]). *2)* Although the DFS-Pri in [9] outperforms the proposed DFS, it is worthwhile to emphasize that the DFS employs equal transmit power and thus the design effort for power amplifier can be greatly reduced. In addition, the computational complexity can also be reduced significantly.

5. CONCLUSION

In this paper, we have presented the differential feedback scheme with two algorithms for the adaptive quantization levels in MIMO EGT systems. The proposed DFS not only lowers the specification of power amplifier but also exploits the temporal channel correlation property for the performance improvement and the feedback overhead reduction. Moreover, by utilizing the adaptive quantization levels with the Lloyd-based algorithm or the closed-form solution, it further makes a good trade-off between the performance and the complexity. With the assumption of the robust feedback channel, the simulation results have shown the significant performance improvement can be achieved compared to the conventional one-shot
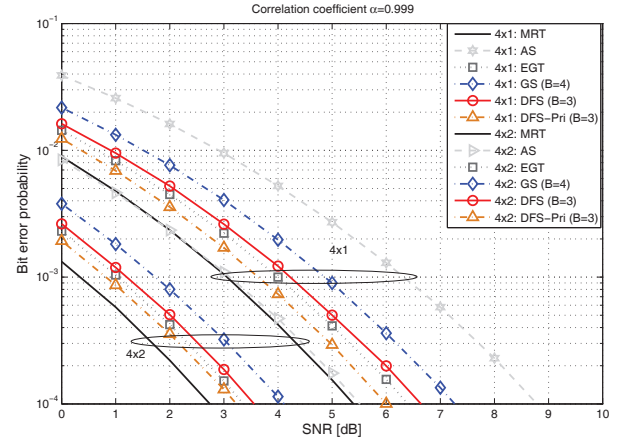

Fig. 3. BEP performance for different beamforming schemes in 4T1R and 4T2R channel environments in the temporally correlated channels with correlation coefficient $\alpha = 0.999$.

beamforming schemes even using less feedback information in the temporally correlated channels.

6. REFERENCES

[1] D. J. Love and R. W. Heath Jr., "Equal gain transmission in multiple-input multiple-output wireless systems," *IEEE Trans. Commun.*, vol. 51, pp. 1102-1110, July 2003.

[2] D. J. Love, R. W. Heath, Jr. and T. Strohmer, "Grassmannian Beamforming for Multiple-Input Multiple-Output Wireless Systems," *IEEE Trans. Info. Theory*, vol. 49, Oct. 2003, pp. 2735-45.

[3] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple antenna systems," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2562-2579, Oct. 2003.

[4] X. Zheng, Y. Xie, J. Li and P. Stoica, "MIMO transmit beamforming under uniform elemental power constraint," *IEEE Trans. Signal Processing*, vol. 55, pp.5395-5406, Nov. 2007.

[5] S.-H. Tsai, "Transmit equal gain precoding in rayleigh fading channels," *IEEE Trans. Signal Processing*, vol. 57, pp. 3717-3721, Sep 2009.

[6] R. H. Etkin and D. N. C. Tse, "Degree of freedom in some underspread MIMO fading channel," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, Apr. 2006.

[7] K. Huang, B. Mondal, R. W. Heath, and J. G. Andrews, "Multi-antenna limited feedback for temporally correlated channels: feedback compression," *Proceedings of IEEE Globecom 2006*, Nov.-Dec 2006.

[8] T. Kim, D. J. Love, B. Clerckx, and S. Kim, "Differential rotation feedback MIMO system for temporally correlated channels," *in Proc. of IEEE Global Commun. Conf.*, New Orleans, LA, Nov. 30-Dec. 4, 2008.

[9] IEEE C802.16m-09/0058r4, Intel, "*Differential feedback for IEEE 802.16m MIMO Schemes.*"