MINIMAX AND GENERALIZED LIKELIHOOD RATIO TEST FOR NONCOHERENT FH/BFSK IN BAND MULTITONE JAMMING

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ABSTRACT

We investigate non-coherent FH/BFSK detection in band multitone jamming when Bayesian detection is unfeasible due to lack of knowledge on *a priori* jamming state probabilities. We derive joint-suboptimal decision metrics of jamming states under Minimax hypothesis testing in band multitone jamming and AWGN. A sub-optimal generalized likelihood ratio test is also explored. Simulation results demonstrate the robustness of the proposed approach which combines sub-optimal generalized likelihood ratio test with generalized FH/BFSK modulation.

Index Terms— Anti-jamming, detection, hypothesis testing, frequency hopping, modulation.

1. INTRODUCTION

Frequency hopping (FH) is a widely used technique in military communications, due to its inherent security and capability to effectively combat various kinds of interference such as multipath fading, multiple access interference and interference from co-existence systems. In recent years, FH has also been adopted in Multiband OFDM Ultra Wideband (MB-OFDM UWB) systems [1]. From viewpoints of secure communications, band multitone jamming (BMJ) remains a harmful means that dominates the overall performance degradation [2, 3] in FH communications. In [4], a novel complexity-reduced scheme, Bayesian joint-suboptimum maximum likelihood (JSML) detector, was proposed to counteract the existence of BMJ for non-coherent frequency hopping binary shift keying (FH/BFSK) with no aid of jamming state indication and resultant energy of signal-plus-jammer tone. However, the Bayesian JSML detector [4] still requires knowledge on *a priori* probabilities of jamming states. Hostile band multitone jammer is able to keep randomly changing the amount of jamming tones and their spectrum locations accordingly. The Bayesisan approach thus is too ideal to be applied to practical situations.

In this paper, we remove assumptions that were made for Bayesian approach [4]. We present Minimax JSML: the JSML under Minimax hypothesis testing [5] with minimum probability of error cost for cases in which only the jamming state probabilities are unavailable. We further present a sub-optimal generalized likelihood ratio (GLR) test. In the scenario of BMJ and AWGN only, numerical results show that anti-jamming performances of both Minimax JSML and sub-optimum GLR test are pretty close to that of Bayesian JSML [4] when BMJ dominates the overall performance.

When both AWGN and BMJ dominate performances, however, it is found that Minimax JSML and sub-optimal GLR test no longer give satisfactory anti-jamming performance in high signal-to-jamming ratio (SJR) scenarios. On the other hand, in BMJ, AWGN and multipath fading environment, the delayed paths introduce ”non-orthogonality interference” which also degrades the performance. To overcome the above bottlenecks with no need to rely on *a priori* probabilities of jamming states, we propose an approach that combines sub-optimal GLR test and generalized modulation scheme of FH/BFSK [6]. This approach is shown by simulation to give robust anti-jamming and anti-multipath fading performances.

2. SYSTEM MODEL

In this paper, we consider the non-coherent FH/BFSK system model described in [3, 4]. We briefly restate this model as follows.

As shown in Fig. 1, the entire spread spectrum bandwidth W_{ss} is equally partitioned into $N_t = W_{ss}/R_c = 2N_b$ tones, where R_c is chip rate and N_b is total number of bands. These tones are equally spaced and further partitioned into N_b non-overlapping sub-bands. Fig. 1 depicts the structure of N_b FH/BFSK sub-bands and corresponding $N_t = 2N_b$ tones. We consider one-hop-per-symbol scenario in which the hop rate equals the chip rate, and so does the symbol rate $R_s(= 1/T_s)$. Let J be the total jamming power, S be the power of the transmitted signal for the desired user, E_s be the desired symbol energy, N_J be the power spectrum density of the jamming noise with $N_J = J/W_{ss}$, and Q be the total number of jamming tones, each of which has power $J/Q = S/\alpha$.

Assume timing recovery is ideally done at the desired (anti-jamming intended) user's receiver. It is also assumed

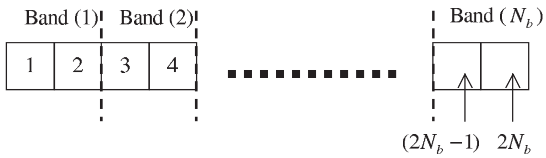

Fig. 1. Structure of N_b FH/BFSK sub-bands and tones

that one hopping interval of each jamming signal is synchronized to one symbol interval of the desired user. The received signal during the l-th symbol interval, $l = 0, 1, ...,$ is expressed by $R_l(t) = S_l(t) + J_l(t) + n(t)$, where $n(t)$ is zero mean AWGN with two-sided power spectrum density $N_0/2 \triangleq \sigma^2$. The transmitted signal corresponding to l-th symbol interval is $S_l(t) = \sqrt{2S}q(t - lT_s)\cos[2\pi(f_c(l) + f(l))t + \theta_s(l)]$. The jamming signal is denoted by $J_l(t)$, with expression $J_l(t) = \sum_{j=1}^{Q} \sqrt{2S/\alpha}\, q(t - lT_s)\cos[2\pi f_j(l)t + \theta_j(l)]$. We use $q(t)$ to denote the normalized rectangular pulse function with duration T_s. Assume $f_c(l) = 2h/T_s$, where h is a positive integer controlled by a hopping sequence. When $f(l)$ equals $f_0 \equiv -1/(2T_s)$, symbol 0 is sent. Likewise, when $f(l)$ equals $f_1 \equiv 1/(2T_s)$, symbol 1 is sent. $\{f_1(l), f_2(l), ..., f_Q(l)\}$ is a set of distinct jamming frequencies. That is, in each hopping interval, Q jamming signals randomly dwell at Q distinct tones. $f_j(l)$ equals a tone within W_{ss} that is used by the desired user.

In the absence of $n(t)$, E_0 and E_1 are respectively the output energies at f_0 and at f_1. These Q jamming tones are assumed distinct. Given that $b_m = i$ (i.e., the m-th symbol is transmitted at the symbol tone f_i), at most two jamming tones simultaneously dwell in a band (i.e., f_0 and f_1), which can be categorized to four jamming states. These four jamming states are denoted by $\mathbf{H}_{ik}$, where the subscript i stands for the symbol i, and $k \in \{1, 2, 3, 4\}$. For example, given that symbol 0 is sent, four hypotheses are described as follows.

$\mathbf{H}_{01}$ (Both tones are not jammed): $E_0(\mathbf{H}_{01}) = 1, E_1(\mathbf{H}_{01}) = 0$.

$\mathbf{H}_{02}$ (Symbol tone is not jammed and the other tone is jammed): $E_0(\mathbf{H}_{02}) = 1, E_1(\mathbf{H}_{02}) = 1/\alpha$.

$\mathbf{H}_{03}$ (Symbol tone is jammed and the other tone is not jammed): $E_0(\mathbf{H}_{03}) = 1 + 1/\alpha + 2\cos\phi/\sqrt{\alpha}, E_1(\mathbf{H}_{03}) = 0$.

$\mathbf{H}_{04}$ (Both tones are jammed): $E_0(\mathbf{H}_{04}) = 1 + 1/\alpha + 2\cos(\phi)/\sqrt{\alpha}, E_1(\mathbf{H}_{04}) = 1/\alpha$.

We let ϕ denote the phase difference between the symbol tone and the jamming tone. ϕ is assumed to be uniformly distributed in $[0, 2\pi)$.

3. MINIMAX HYPOTHESIS TEST AND GENERALIZED LIKELIHOOD RATIO TEST

Without loss of generality, we assume symbol 0 and symbol 1 are equiprobable. Nonetheless, jamming state *a priori* probabilities, $p(\mathbf{H}_{ik})$, are no longer known by the desired user. The Bayesian JSML [4] can not be applied to this case as it requires knowledge of $p(\mathbf{H}_{ik})$.

Let $\mathbf{r}_i \triangleq [r_{ic}, r_{is}]$, where r_{ic} and r_{is} are in-phase and quadrature phase components of normalized received signal at f_i, respectively. In other words,

$$r_{ic} = \int_{mT_s}^{(m+1)T_s} \sqrt{\tfrac{2R_m(t)^2}{E_sT_s}} \cos[2\pi(f_c(m) + f_i)t]dt,$$
$$r_{is} = \int_{mT_s}^{(m+1)T_s} \sqrt{\tfrac{2R_m(t)^2}{E_sT_s}} \sin[2\pi(f_c(m) + f_i)t]dt.$$

We define $R(\mathbf{p}, \delta(\mathbf{r}_0, \mathbf{r}_1))$ as the expected cost for a given decision rule δ when jamming states' *a priori* probability distribution denoted by $\mathbf{p} \triangleq [p(\mathbf{H}_{01}), p(\mathbf{H}_{02}), p(\mathbf{H}_{03}), p(\mathbf{H}_{04})]$, is true. Let $\delta^*_{\mathbf{p}}(\mathbf{r}_0, \mathbf{r}_1)$ be the Bayesian-optimal decision rule under *a priori* probability distribution $\mathbf{p}$. Clearly, we have

$$R(\mathbf{p}, \delta^*_{\mathbf{p}}(\mathbf{r}_0, \mathbf{r}_1)) \leq R(\mathbf{p}, \delta(\mathbf{r}_0, \mathbf{r}_1)), \forall \delta \in \Delta,$$

where Δ represents the set of all admissible decision rules. In Minimax hypothesis testing, we find out $\mathbf{p}^*$ that has the maximum expected Bayesian cost among all legitimate $\mathbf{p}$'s. That is,

$$R(\mathbf{p}^*, \delta^*_{\mathbf{p}^*}(\mathbf{r}_0, \mathbf{r}_1)) \geq R(\mathbf{p}, \delta^*_{\mathbf{p}}(\mathbf{r}_0, \mathbf{r}_1)), \forall \mathbf{p} \neq \mathbf{p}^*, \quad (1)$$

where $\delta^*_{\mathbf{p}^*}(\mathbf{r}_0, \mathbf{r}_1)$ denotes the Minimax decision rule.

We omit notations $\mathbf{r}_0$ and $\mathbf{r}_1$ for simplicity. In this paper, we consider the the minimum probability of error (MPE) cost function. Since symbol 0 and symbol 1 are equiprobable and their sub-hypotheses (i.e., $\mathbf{H}_{ik}$'s) are symmetric, we have $p(\delta^*_{\mathbf{p}} = \mathbf{H}_{jl}|\mathbf{H}_{ik}) = p(\delta^*_{\mathbf{p}} = \mathbf{H}_{il}|\mathbf{H}_{jk})$. Then it can be shown that

$$R(\mathbf{p}, \delta^*_{\mathbf{p}}) = \sum_{k=1}^{4}\sum_{l=1}^{4} p(\delta^*_{\mathbf{p}} = \mathbf{H}_{1l}|\mathbf{H}_{0k})p(\mathbf{H}_{0k}). \quad (2)$$

Equation (2) is the probability of error under the assumption that $\mathbf{p}$ is the true *a priori* probability distribution of jamming states. Hence, $\delta^*_{\mathbf{p}^*}$ is the Minimax decision rule with MPE cost, where $\mathbf{p}^* \triangleq [p^*(\mathbf{H}_{01}), p^*(\mathbf{H}_{02}), p^*(\mathbf{H}_{03}), p^*(\mathbf{H}_{04})]$ satisfies

$$\mathbf{p}^* = \arg\max_{\mathbf{p}} R(\mathbf{p}, \delta^*_{\mathbf{p}}). \quad (3)$$

We use L^*_{ik} to denote the Minimax JSML decision metric of jamming state $\mathbf{H}_{ik}$. Through similar procedure in [4], it can be shown that

$$\begin{aligned}
&L^*_{01} \equiv r_0 + A + C^*_{01}, && L^*_{11} \equiv r_1 + A + C^*_{11},\\
&L^*_{02} \equiv r_0 + \sqrt{\tfrac{r_1^2}{\alpha}} + A + B + C^*_{02}, && L^*_{12} \equiv r_1 + \sqrt{\tfrac{r_0^2}{\alpha}} + A + B + C^*_{12},\\
&L^*_{03} \equiv r_0^2/2 + C^*_{03}, && L^*_{13} \equiv r_1^2/2 + C^*_{13},\\
&L^*_{04} \equiv r_0^2/2 + \sqrt{\tfrac{r_1^2}{\alpha}} + B + C^*_{04}, && L^*_{14} \equiv r_1^2/2 + \sqrt{\tfrac{r_0^2}{\alpha}} + B + C^*_{14},
\end{aligned} \quad (4)$$

where $A = -1/2$; $B = -1/(2\alpha) - \sigma^2\ln(2\pi/(\sigma^2\sqrt{\alpha}))/2$; and $C^*_{ik} = \sigma^2\ln(p^*(\mathbf{H}_{ik}))$.

A test called GLR test can be applied to a case in which there does not exist an Uniformly most powerful test (UMP) test. The corresponding decision rule δ_{GLR} first takes the maximum among likelihood functions under sub-hypothesis $\Theta_i \in \{\mathbf{H}_{i1}, .., \mathbf{H}_{i4}\}$. Then it decides $\mathbf{H}_1$ if $\frac{\max_{\Theta_1} p(\mathbf{r}_0,\mathbf{r}_1|\Theta_1)}{\max_{\Theta_0} p(\mathbf{r}_0,\mathbf{r}_1|\Theta_0)} \geq \gamma$, and $\mathbf{H}_1$ otherwise. Here we take $\gamma = 1$, it can be shown that the corresponding decision metric $\tilde{L}_{ik}$ the same as L^*_{ik} without C^*_{ik} in (4).

4. NUMERICAL RESULTS

In our simulation, we consider a slow frequency hopping (SFH) system for single-diversity and multiple-diversity scenarios. We follow the same parameter setting as in [4], where N_b is set by 100 in the worst case (WC) BMJ scenario. The optimal parameter setting for Q and α in the WC BMJ is described in Chapter 2 of [3]. For $n = 1$ BMJ, α is set equal to 0.95 to make the jamming tone's energy slightly greater than the desired symbol tone's energy. We consider moderate SNR (13.35 *dB*) conditions. We compare Minimax JSML detector and sub-optimal GLR detector with the Bayesian JSML detector [4], and other commonly used diversity combining approaches such as hard-decision majority-vote (HDMV) receiver, product-combining (PC) receiver, and self-normalized (SN) receiver [7]. Note that the Bayesian JSML detector [4] is not optimal and it requires knowledge on *a priori* jamming state probabilities. Fig. 2 shows single diversity ($U = 1$) and triple ($U = 3$) diversity anti-jamming performances in $n = 1$ BMJ; whereas Fig. 3 shows single diversity ($U = 1$) and double ($U = 2$) diversity anti-jamming performances in $n = 2$ BMJ. We observe that the HDMV, PC and SN receivers are more vulnerable to BMJ, compared to the proposed detectors. In addition, the ML-I receiver [7] with incomplete jammer state information is unable to deliver satisfactory performance. One would expect that Bayesian JSML detector outperforms Minimax JSML and GLR detectors according to the fact that Bayesian JSML detector utilizes the knowledge on $p(\mathbf{H}_{ik})$ while the other two do not. Nonetheless, Fig. 2 reveals negligible differences among Bayesian JSML, Minimax JSML and sub-optimal GLR detectors. On the other hand, in $n = 2$ BMJ and moderate SNR (13.35*dB*), AWGN cannot be ignored. In this scenario, the Minimax test no longer gives diminishing error probabilities in high SJR conditions, as shown in Fig. 3. We can further see that when SJR becomes higher, the GLR test can no longer give satisfactory anti-jamming performances accordingly. However, combining the sub-optimal GLR test with a generalized FH/BFSK signaling scheme makes it possible to lower the error probability curve in high SJRs. Intuitively, generalized modulation scheme of FH/BFSK destroys jammer's side information on the original FH/BFSK band structure and thus improves anti-jamming performances. In Fig. 3 we show curves of GLR test combined with Random FH/BFSK scheme in which the two tones defined in a FH/BFSK modulation band are randomly chosen from all possible tones within the whole spread spectrum W_{ss}. These two curves ($U = 1$ and $U = 2$) perform better than that of the conventional FH/BFSK with Bayesian JSML when SJR is lower than approximately 17*dB*, and their anti-jamming performances remain competitive when SJR is 17*dB* or even higher.

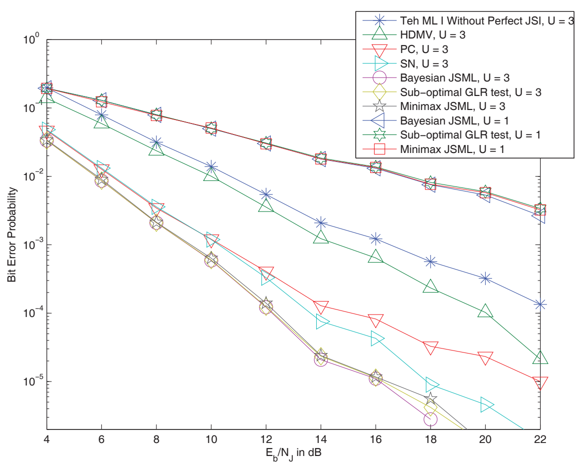

Fig. 2. BERs in the WC $n = 1$ BMJ, SNR = 13.35 *dB*

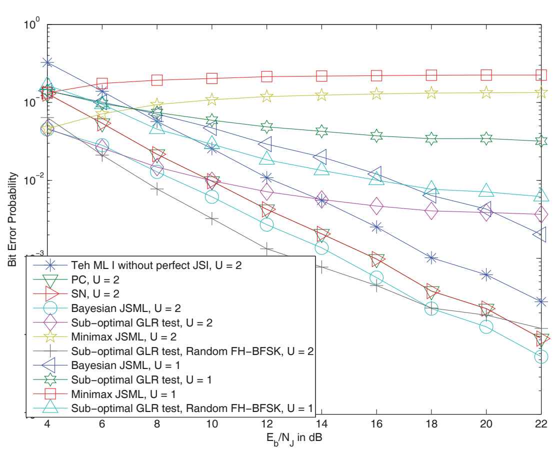

Fig. 3. BERs in the WC $n = 2$ BMJ, SNR = 13.35 *dB*

In realistic wireless environment, multipath fading is a major cause of performance degradation. It has been pointed out by [8, 9, 10] that a two-path channel with independent Rayleigh fading is a better model of a real wireless channel. In this simulation setting we consider two-path Rayleigh fading. The impulse response of two-path Rayleigh fading channel is $h(t) = \sum_{n=1}^{2} \beta_n \delta(t - \tau_n) \exp(j\theta_n)$, where $\tau_1 = 0$, and τ_2 is uniformly distributed in $[0, T_s)$. The path amplitude β_n is Rayleigh distributed with parameter $\sqrt{\bar{\beta}_n^2/2}$. We denote the average power of the n-th path by $\bar{\beta}_n^2$, which stands for the mean square value of the n-th path amplitude and is expressed by $\bar{\beta}_n^2 = \bar{\beta}_1^2 \exp(-\tau_n/T_s)$. θ_n is the corresponding random phase of the n-th path and is statistically independent of β_n. Fig. 4 and Fig. 5 illustrate simulation results when $\beta_1^2 = 1$. In $n = 1$ BMJ, by comparing to Fig. 2, we can see apparent performance degradation in Fig. 4, due to the existence of two-path fading. Although the Bayesian JSML, Minimax JSML and Sub-optimal GLR receivers give better performances than HDMV, PC and SN, they all perform fairly close to one another. In addition, since $n = 1$ BMJ dominates the performance as well, these receivers remain with similar performance relationships to those in Fig. 2. For $n = 2$ BMJ, by comparing to Fig. 3, we observe one major difference shown in Fig. 5: the Bayesian JSML is not comparable to Sub-optimal GLR test (with Random FH/BFSK). The Bayesian JSML proposed in [4] is no longer joint suboptimal under the condition of multipath Rayleigh fading and it even performs worse than SN and PC receivers. In $n = 2$ BMJ, the multipath fading becomes dominant. The delayed second path removes the orthogonality relation between signals which correspond to two BFSK symbols in conventional FH systems, where both symbol tones are located adjacently in a common modulation band. Hence, the "non-orthogonality interference" degrades the performance. Sub-optimal GLR test with Random FH/BFSK does not require *a priori* jammer state probabilities and it possesses a property that the probability of both symbol tones being located contiguously to each other is low enough given the total number of tones allowed to use is assumed sufficiently large (in other words, both symbol tones are usually far away from each other), which alleviates the "non-orthogonality interference" from the delayed second path. The GLR test under this kind of randomized modulation scheme is comparable to PC and SN receivers. The reader can see differences between performance curves of the GLR test and PC/SN receivers, as SJR goes higher.

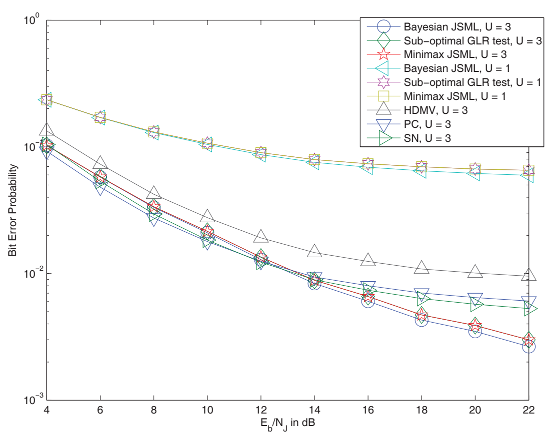

Fig. 4. BERs in the WC $n = 1$ BMJ, 2-path Rayleigh fading

5. CONCLUSIONS

For anti-jamming objectives, we present Minimax JSML with MPE cost and a sub-optimal GLR test without the need to know *a priori* jamming state probabilities. Though both Minimax JSML and sub-optimal GLR test can no longer be robust as SJR goes high, it is found that an alternative approach which combines the sub-optimal GLR test and generalized modulation scheme of FH/BFSK, is able to have robust anti-jamming performances in high SJRs and in multipath fading environment.

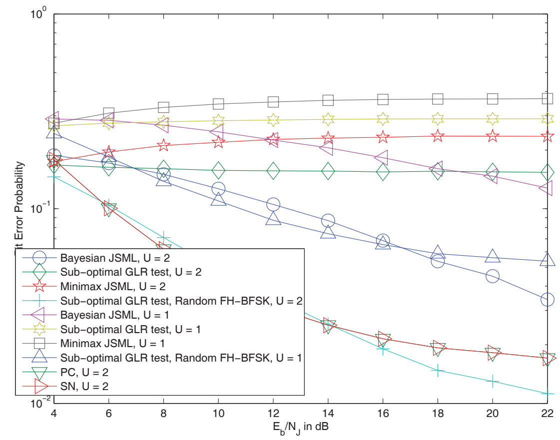

Fig. 5. BERs in the WC $n = 2$ BMJ, 2-path Rayleigh fading

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