OPTIMAL WIRELESS MULTIUSER CHANNELS WITH IMPERFECT CHANNEL STATE INFORMATION

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ABSTRACT

This paper considers algorithms for optimal transmission over wireless multiuser channels where the transmitter has access to imperfect channel state information (CSI). We focus on downlink orthogonal frequency division multiple access and multiuser uplink random access. In both cases, frequency assignment, transmitted power, and coding mode are adapted to imperfect CSI in order to maximize expected transmission rate subject to average power constraints. Determination of optimal solutions is a non-convex stochastic optimization problem with infinitely many variables. Exploiting its property of null duality gap, we show that optimal solutions are determined by optimal dual variables. This affords considerable simplification because the dual optimization problem is convex and finite-dimensional. Iterative algorithms that find the optimal operating point based on imperfect CSI without having access to the channels' probability distributions are further developed.

1. INTRODUCTION

We develop algorithms to handle imperfect channel state information (CSI) in wireless multiuser channels. To exploit favorable channel conditions, transmitters adapt power and coding mode to the measured CSI. Due to the inaccuracy of CSI, channel outages occur when the rate selected turns out too aggressive for the actual channel realization [1]. From a practical perspective, it is recognized that to mitigate the negative effect of capacity outages caused by imperfect CSI, a rate backoff function is needed in addition to power control; see e.g. [2]. Ideally, power allocation and rate backoff should be jointly optimized but this results in a nonconvex problem that is difficult to solve. By imposing additional restrictions, the problem can be simplified to more tractable formulations. E.g., when power is fixed and only rate adaptation is considered the problem is reduced to the determination of the optimal backoff function; e.g. [3]. A second possibility is to fix a target outage probability and separate the optimization into the determination of a backoff function for target outage, followed by optimal power allocation [4]. A third possible restriction is to assume that the backoff function takes a certain parametric form and proceed to optimize the corresponding parameters, e.g. [2]. While yielding tractable formulations, the resultant transmission rates are not optimal. This paper focuses on downlink orthogonal frequency division multiple access (OFDMA) (Section 2) and uplink random access (RA) (Section 3) to develop algorithms that finds optimal operating points without imposing any additional simplifying assumptions. Numerical results are presented in Section 4.

2. ORTHOGONAL FREQUENCY DIVISION MULTIPLE ACCESS

Consider an orthogonal frequency division multiple access (OFDMA) channel where a common access point (AP) with an average power budget P_0 transmits to N terminals $\{T_n\}_{n=1}^N$ using a group of orthogonal frequencies $\mathcal{F}$. Time is slotted and indexed by t. The time-varying channel gain between the AP and T_n for all frequencies $f \in \mathcal{F}$ is modeled as block fading and denoted by $\gamma_n^f(t)$. In each time slot the AP observes imperfect channel gains denoted by $\hat{\boldsymbol{\gamma}}(t) := \{\hat{\gamma}_n^f(t) : n \in \mathcal{N}, f \in \mathcal{F}\}$. The accuracy of $\hat{\gamma}_n^f(t)$ is characterized through a known conditional probability distribution $m(\gamma_n^f|\hat{\gamma}_n^f)$ that determines the probability of the actual channel being γ_n^f when the observation is $\hat{\gamma}_n^f$. Although instantaneous imperfect CSI $\hat{\boldsymbol{\gamma}}(t)$ is observed, its distribution is assumed unknown.

Based on $\hat{\boldsymbol{\gamma}}(t)$, the AP decides on frequency assignment $a_n^f(t) := A_n^f(\hat{\boldsymbol{\gamma}}(t)) \in \{0,1\}$ and power allocation $p_n^f(t) := P_n^f(\hat{\boldsymbol{\gamma}}(t)) \in [0, P_{\mathsf{max}}]$. If $a_n^f(t) = 1$, it transmits to T_n using frequency f. Assume a given frequency cannot be used by more than one terminal in a time slot, we require $\sum_{n=1}^N a_n^f(t) \leq 1$. Defining the vector $\mathbf{A}^f(\hat{\boldsymbol{\gamma}}) := [A_1^f(\hat{\boldsymbol{\gamma}}), \cdots, A_n^f(\hat{\boldsymbol{\gamma}})]^T$ this frequency exclusion constraint is written as

$$\mathbf{A}^f(\hat{\boldsymbol{\gamma}}) \in \mathcal{A} := \left\{\mathbf{a} = [a_1, \cdots, a_N]^T : a_n \in \{0,1\}, \mathbf{a}^T\mathbf{1} \leq 1\right\}. \quad (1)$$

With transmission power $p_n^f(t)$ the maximum amount of information that can be delivered to T_n on frequency f is given by the capacity function $C(p_n^f(t), \gamma_n^f(t))$ which is a nonnegative and nondecreasing function of the signal to noise ratio (SNR) $p_n^f(t)\gamma_n^f(t)/N_0$, where N_0 is the channel noise. However, $\gamma_n^f(t)$ is the *actual* channel gain and is unknown to the AP. If the AP chooses a rate based on the imperfect CSI, i.e. $C(p_n^f(t), \hat{\gamma}_n^f(t))$, a channel outage will occur when this rate exceeds the maximum rate the channel can afford - i.e. when $C(p_n^f(t), \hat{\gamma}_n^f(t)) > C(p_n^f(t), \gamma_n^f(t))$ or simply $\hat{\gamma}_n^f(t) > \gamma_n^f(t)$. To reduce the negative effect of channel outage, the AP employs channel backoff functions $b_n^f(t) := B_n^f(\hat{\boldsymbol{\gamma}}(t))$ and the communication proceeds at rate $C(p_n^f(t), b_n^f(t))$. As a result, the information delivered to T_n at time t over all frequencies is

$$r_n(t) = \sum_{f\in\mathcal{F}} a_n^f(u) C\left(p_n^f(u), b_n^f(u)\right) \mathbb{I}\left\{b_n^f(u) \leq \gamma_n^f(u)\right\}, \quad (2)$$

where $\mathbb{I}\{\cdot\}$ is the indicator function. Upon defining $M_{\gamma_n^f|\hat{\gamma}_n^f}(\cdot)$ as the complementary cumulative distribution function (ccdf) of γ_n^f given $\hat{\gamma}_n^f$, we can express the ergodic rate from the AP to T_n as [5]

$$\begin{aligned} r_n &= \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\Bigg[\sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}) C\left(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}})\right) M_{\gamma_n^f|\hat{\gamma}_n^f}\left(B_n^f(\hat{\boldsymbol{\gamma}})\right)\Bigg] \\ &:= \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\Bigg[\sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}(t)) R_n^f(\hat{\boldsymbol{\gamma}}(t))\Bigg], \end{aligned} \quad (3)$$

where we defined $R_n^f(\hat{\boldsymbol{\gamma}}) := C\left(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}})\right) M_{\gamma_n^f|\hat{\gamma}_n^f}\left(B_n^f(\hat{\boldsymbol{\gamma}})\right)$ to denote the rate that terminal n expects to obtain in frequency f when the channel estimate is $\hat{\boldsymbol{\gamma}}$. By expected rate here we refer to the conditional expectation with respect to $\boldsymbol{\gamma}$ given $\hat{\boldsymbol{\gamma}}$. Contrast $R_n^f(\hat{\boldsymbol{\gamma}})$ with the rate r_n in (3) that also includes an expectation with respect to $\hat{\boldsymbol{\gamma}}$.

To evaluate performance of the system, assign utility functions $U_n(r_n)$ to ergodic rate r_n. The goal is then to design an algorithm that finds optimal frequency assignment, power allocation, and channel backoff functions such that the sum utility is maximized, i.e.,

$$\begin{aligned} \mathsf{P_b} &= \max \sum_{n=1}^N U_n(r_n) \quad (4) \\ \text{s.t. } r_n &\leq \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\Bigg[\sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}) R_n^f(\hat{\boldsymbol{\gamma}})\Bigg], \;\; P_0 \geq \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\Bigg[\sum_{n=1}^N \sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}) P_n^f(\hat{\boldsymbol{\gamma}})\Bigg], \\ &\mathbf{A}^f(\hat{\boldsymbol{\gamma}}) \in \mathcal{A}, \quad P_n^f(\hat{\boldsymbol{\gamma}}) \in [0, P_{\mathsf{max}}], \quad B_n^f(\hat{\boldsymbol{\gamma}}) \geq 0, \end{aligned}$$

where we relaxed the first equality constraint to inequality which can be done without loss of optimality. The second inequality reflects the AP's

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average power budget. Solving problem (4) presents several challenges: 1) infinite dimensionality due to the optimization variables being functions; 2) nonconvexity due to the presence of nonconvex channel capacity function; 3) channel pdfs are unknown; 4) presence of binary constraints on the variables variables $A_n^f(\hat{\boldsymbol{\gamma}})$. As we shall show in the next section, working in the dual domain resolves these issues.

2.1. Optimal solution

Despite being infinite dimensional and nonconvex, problem (4) has null duality gap [6]. Therefore, we can work on its dual problem which is finite dimensional and convex without loss of optimality. To do so, introduce multipliers λ_n and μ associated with the constraints in (4), define $\boldsymbol{\Lambda} := \{\lambda_n, \mu\}$, $\mathbf{P}(\hat{\boldsymbol{\gamma}}) := \left\{A_n^f(\hat{\boldsymbol{\gamma}}), P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}})\right\}$, $\mathbf{x} := \{r_n\}$, and write the Lagrangian as

$$\mathcal{L}(\mathbf{P}(\hat{\boldsymbol{\gamma}}), \mathbf{x}, \boldsymbol{\Lambda}) = \sum_{n=1}^{N} \sum_{f\in\mathcal{F}} \mathbb{E}_{\hat{\boldsymbol{\gamma}}} \left[A_n^f(\hat{\boldsymbol{\gamma}}) \left[\lambda_n R_n^f(\hat{\boldsymbol{\gamma}}) - \mu P_n^f(\hat{\boldsymbol{\gamma}})\right]\right] + \sum_{n=1}^{N} [U_n(r_n) - \lambda_n r_n] + \mu P_0, \quad (5)$$

where we reordered and grouped terms by primal variables in the second equality. The dual function and the dual problem are then given by

$$\mathsf{D_b} = \min_{\lambda_n \geq 0, \mu\geq 0} g(\lambda_n, \mu) = \min_{\lambda_n \geq 0, \mu\geq 0} \max_{\mathbf{P}(\hat{\boldsymbol{\gamma}}), \mathbf{x}} \mathcal{L}(\mathbf{P}(\hat{\boldsymbol{\gamma}}), \mathbf{x}, \boldsymbol{\Lambda}). \quad (6)$$

By leveraging the property of null duality gap, i.e., $\mathsf{P_b} = \mathsf{D_b}$, we can characterize the optimal solution of the primal problem using the optimal solution of the dual problem, as shown in the following theorem:

Theorem 1 The optimal frequency assignment function $A_n^{f*}(\hat{\boldsymbol{\gamma}})$, channel backoff function $B_n^{f*}(\hat{\boldsymbol{\gamma}})$ and power allocation function $P_n^{f*}(\hat{\boldsymbol{\gamma}})$ for solving problem (4) are determined by the optimal variables λ_n^* and μ^* of the dual problem (6). In particular, for a given frequency $f \in \mathcal{F}$ we define $R_n^{f*}(\hat{\boldsymbol{\gamma}}) = C\left(P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}})\right) M_{\gamma_n^f|\hat{\gamma}_n^f}\left(B_n^{f*}(\hat{\boldsymbol{\gamma}})\right)$ and compute

$$\left\{P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}})\right\} \in \operatorname*{argmax}_{p\in[0,P_{\mathsf{max}}], b\geq 0} \lambda_n^* C(p, b)\, M_{\gamma_n^f|\hat{\gamma}_n^f}(b) - \mu^* p, \quad (7)$$

$$n^f = \operatorname*{argmax}_{n} \lambda_n^* R_n^{f*}(\hat{\boldsymbol{\gamma}}) - \mu^* P_n^{f*}(\hat{\boldsymbol{\gamma}}), \quad (8)$$

and set $A_n^{f*}(\hat{\boldsymbol{\gamma}}) = 0$ for all $n \neq n^f$. For $n = n^f$, we set $A_n^{f*}(\hat{\boldsymbol{\gamma}}) = 1$ if $\lambda_n^* R_n^{f*}(\hat{\boldsymbol{\gamma}}) - \mu^* P_n^{f*}(\hat{\boldsymbol{\gamma}}) > 0$.

Proof: See [5]. □

The maximization in (7) is a one dimensional problem for a given terminal n, on a given frequency f, and for given channel estimate $\hat{\boldsymbol{\gamma}}$. This problem is easy to solve even if the maximand is not concave. Comparing this simplicity to the primal problem (4) which contains $3 \times N \times |\mathcal{F}|$ function variables. Further note that Theorem 1 indicates that the optimal solution is opportunistic because frequency f is used only when at least one terminal's channel on this frequency is above a threshold.

2.2. Online learning algorithm

For cases when channel pdf of $\hat{\boldsymbol{\gamma}}$ is unknown, we develop an online learning algorithm that performs subgradient descent in the dual domain using instantaneous channel observations $\hat{\boldsymbol{\gamma}}(t)$ only. For given multipliers $\lambda_n(t)$ and $\mu(t)$, the algorithm first computes primal variables:

$$r_n(t) = \operatorname*{argmax}_{r_n\in[0,C_{\mathsf{max}}]} U_n(r_n) - \lambda_n(t) r_n, \quad (9)$$

$$\{a_n^f(t), p_n^f(t), b_n^f(t)\} = \operatorname*{argmax}_{\mathbf{a}\in\mathcal{A}, p\in[0,P_{\mathsf{max}}], b>0} a_n \left[\lambda_n(t) C(p, b)\, M_{\gamma_n^f|\hat{\gamma}_n^f(t)}(b) - \mu(t) p\right], \quad (10)$$

where C_{max} in (9) is a constant representing the hard limit for channel capacity. Follow the logic used for deriving (7)-(8) and define $R_n^f(t) = C\left(p_n^f(t), b_n^f(t)\right) M_{\gamma_n^f|\hat{\gamma}_n^f(t)}\left(b_n^f(t)\right)$, the maximization in (10) can be further simplified to computing

$$\{p_n^f(t), b_n^f(t)\} = \operatorname*{argmax}_{p\in[0,P_{\mathsf{max}}], b\geq 0} \lambda_n(t) C(p, b)\, M_{\gamma_n^f|\hat{\gamma}_n^f(t)}(b) - \mu(t) p, \quad (11)$$

$$n^f(t) = \operatorname*{argmax}_{n} \lambda_n(t) R_n^f(t) - \mu(t) p_n^f(t), \quad (12)$$

and setting $a_n^f(t) = 0$ for $n \neq n^f(t)$ and $a_n^f(t) = 1$ for $n = n^f(t)$ if $\lambda_n(t)R_n^f(t) - \mu(t)p_n^f(t) > 0$. By evaluating the instantaneous constraint violations, the algorithm completes with dual updates

$$\lambda_n(t+1) = \left[\lambda_n(t) - \epsilon(t)\left[\sum_{f\in\mathcal{F}} a_n^f(t) R_n^f(t) - r_n(t)\right]\right]^+, \quad (13)$$

$$\mu(t+1) = \left[\mu(t) - \epsilon(t)\left[P_0 - \sum_{n=1}^{N}\sum_{f\in\mathcal{F}} a_n^f(t) p_n^f(t)\right]\right]^+, \quad (14)$$

where step size $\epsilon(t)$ can be either diminishing or constant. With diminishing step size, $\lambda_n(t+1)$ and $\mu(t)$ converge to the optimal dual variables almost surely. With constant step size, convergence is established in an ergodic sense as we describe in the following property [7].

Property 1 If constant step size $\epsilon(t) = \epsilon > 0$ is used in (13) and (14), then the average power constraint is almost surely satisfied

$$\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} \left[\sum_{n=1}^{N}\sum_{f\in\mathcal{F}} a_n^f(u) p_n^f(u)\right] \leq P_0 \quad \text{a.s.}, \quad (15)$$

and the ergodic limit of the transmission rates almost surely converges to a value within $\kappa\epsilon$ of optimal,

$$\mathsf{P_b} - \sum_{n=1}^{N} U_n\left(\lim_{t\to\infty} \frac{1}{t}\sum_{u=1}^{t} r_n(t)\right) \leq \kappa\epsilon, \quad (16)$$

where κ is a constant [7].

3. RANDOM ACCESS

Consider a multiple access channel in which N terminals contend for communication to a common AP using random access. A set of frequencies $\mathcal{F}$ is used for communication and the channel between terminals and the AP for all tones is modeled as block fading and denoted as $\gamma_n^f(t)$. Assume each terminal only observes an imperfect version of its local channel $\hat{\boldsymbol{\gamma}}_n := \left\{\hat{\gamma}_n^f(t) : f \in \mathcal{F}\right\}$. Channels for different users are assumed independent. Based on its local channel, terminals decide frequency assignment $a_n^f(t) := A_n^f(\hat{\boldsymbol{\gamma}}_n) \in \{0, 1\}$, power allocations $p_n^f(t) := P_n^f(\hat{\boldsymbol{\gamma}}_n) \in [0, P_{\mathsf{max}}]$ and channel backoffs $b_n^f(t) := B_n^f(\hat{\boldsymbol{\gamma}}_n) \geq 0$. We remark that $A_n^f(\hat{\boldsymbol{\gamma}}_n)$, $P_n^f(\hat{\boldsymbol{\gamma}}_n)$ and $B_n^f(\hat{\boldsymbol{\gamma}}_n)$ are functions of local channel only. Since terminals contend for channel access, transmission of T_n in a time slot t on frequency f is successful if and only if $a_n^f(t) = 1$ and $a_m^f(t) = 0$ for all $m \neq n$. If the transmission of T_n is successful, its transmission rate is determined by $C(p_n^f(t), b_n^f(t))$. Therefore, the instantaneous transmission rate for T_n in time slot t on frequency f is

$$r_n^f(t) = C\left(p_n^f(t), b_n^f(t)\right) \mathbb{I}\left\{b_n^f(t) \leq \hat{\gamma}_n^f(t)\right\} \cdot a_n^f(t) \prod_{m=1, m\neq n}^{N} \left[1 - a_m^f(t)\right], \quad (17)$$

and the ergodic rate is given by

$$r_n^f = \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\left[R_n^f(\hat{\boldsymbol{\gamma}}_n) A_n^f(\hat{\gamma}_n) \prod_{m=1, m\neq n}^{N} \left[1 - A_m^f(\hat{\boldsymbol{\gamma}}_m)\right]\right], \quad (18)$$

where we used $R_n^f(\hat{\boldsymbol{\gamma}}_n) = C\big(P_n^f(\hat{\boldsymbol{\gamma}}_n), B_n^f(\hat{\boldsymbol{\gamma}}_n)\big) M_{\gamma_n^f|\hat{\gamma}_n^f}\left(B_n^f(\hat{\boldsymbol{\gamma}}_n)\right)$ as in (3), which in this case represents the average transmission rate for terminal n on frequency f if there is no collision. An important observation here is that since terminals are required to make channel access and power control decisions independently of each other, varaibles $A_n^f(\hat{\boldsymbol{\gamma}}_n)$, $P_n^f(\hat{\boldsymbol{\gamma}}_n)$, and $B_n^f(\hat{\boldsymbol{\gamma}}_n)$ are independent of $A_m^f(\hat{\boldsymbol{\gamma}}_m)$, $P_m^f(\hat{\boldsymbol{\gamma}}_m)$, and $B_m^f(\hat{\boldsymbol{\gamma}}_m)$ for all $n \neq m$. This allows us to rewrite r_n^f as

$$r_n^f = \mathbb{E}_{\hat{\gamma}_n}\left[R_n^f(\hat{\boldsymbol{\gamma}}_n)A_n^f(\hat{\boldsymbol{\gamma}}_n)\right]\prod_{m=1,m\neq n}^{N}\left[1-\mathbb{E}_{\hat{\boldsymbol{\gamma}}_m}\left[A_m^f(\hat{\boldsymbol{\gamma}}_m)\right]\right]. \quad (19)$$

The objective is to maximize a weighted proportional fair utility of r_n^f,

$$U(\mathbf{r}) = \sum_{n=1}^{N}\sum_{f\in\mathcal{F}} w_n^f \log(r_n^f), \quad (20)$$

where $\mathbf{r} := \left\{r_n^f : n \in \{1,\cdots,N\}, f\in\mathcal{F}\right\}$ and w_n^f is the positive weight coefficient for T_n using frequency f. Maximizing $U(\mathbf{r})$ yields solutions that are fair since it prevents users from having very low transmission rates. With constraints and objective defined, the optimal random access with imperfect CSI is formulated as the following

$$\begin{aligned}
\mathsf{P_r} \;=\; & \max\; U(\mathbf{r}) \\
\text{s.t. }\; & r_n^f = \mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[R_n^f(\hat{\boldsymbol{\gamma}}_n)A_n^f(\hat{\boldsymbol{\gamma}}_n)\right]\prod_{m=1,m\neq n}^{N}\left[1-\mathbb{E}_{\hat{\boldsymbol{\gamma}}_m}\left[A_m^f(\hat{\boldsymbol{\gamma}}_m)\right]\right], \\
& \mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[\sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}_n)P_n^f(\hat{\boldsymbol{\gamma}}_n)\right] \leq P_{0n}, \\
& A_n^f(\hat{\boldsymbol{\gamma}}_n)\in\{0,1\}, P_n^f(\hat{\boldsymbol{\gamma}}_n)\in[0,P_{\max}], B_n^f(\hat{\boldsymbol{\gamma}}_n)\geq 0, \quad (21)
\end{aligned}$$

where the second inequality indicates each terminal has an average power budget of P_{0n}. The formulation in (21) is not amenable for distributed implementations because the rate constraint involves actions of all terminals. By substituting r_n^f into $U(\mathbf{r})$, we express the global utility as the sum of local utilities [5], i.e. $U(\mathbf{r}) = \sum_{n=1}^{N}\sum_{f\in\mathcal{F}} U_n^f$ with

$$U_n^f := w_n^f \log \mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[R_n^f(\hat{\boldsymbol{\gamma}}_n)A_n^f(\hat{\boldsymbol{\gamma}}_n)\right] + \tilde{w}_n^f \log\left[1-\mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[A_n^f(\hat{\boldsymbol{\gamma}}_n)\right]\right]$$

where $\tilde{w}_n^f = \sum_{m\neq n} w_m^f$. To maximize $U(\mathbf{r})$ for the whole system it suffices to separately maximize $\sum_{f\in\mathcal{F}} U_n^f$ for each terminal n. Introduce variables $x_n^f = \mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[A_n^f(\hat{\boldsymbol{\gamma}}_n)R_n^f(\hat{\boldsymbol{\gamma}}_n)\right]$ and $y_n^f = \mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[A_n^f(\hat{\boldsymbol{\gamma}}_n)\right]$, we have the following per terminal subproblems

$$\begin{aligned}
\mathsf{P_{r,n}} \quad = \quad & \max \sum_{f\in\mathcal{F}} w_n^f \log x_n^f + \tilde{w}_n^f \log(1-y_n) \quad (22) \\
& \text{s.t. }\; x_n^f \leq \mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[A_n^f(\hat{\boldsymbol{\gamma}}_n)R_n^f(\hat{\boldsymbol{\gamma}}_n)\right], y_n^f \geq \mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[A_n^f(\hat{\boldsymbol{\gamma}}_n)\right], \\
& P_{0n} \geq \mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[\sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}_n)P_n^f(\hat{\boldsymbol{\gamma}}_n)\right], \\
& A_n^f(\hat{\boldsymbol{\gamma}}_n)\in\{0,1\}, P_n^f(\hat{\boldsymbol{\gamma}}_n)\in[0,P_{\max}], B_n^f(\hat{\boldsymbol{\gamma}}_n)\geq 0.
\end{aligned}$$

In particular, we have $\mathsf{P_r} = \sum_{n=1}^{N}\mathsf{P_{r,n}}$. Therefore, to solve problem (21) we only need to solve problem (22) for all terminals in a distributed manner. Next, we characterize the optimal solution for (22) by exploiting its property of null duality gap and devise adaptive algorithm by using stochastic subgradient descent as we did in the case of OFDMA.

3.1. Optimal solution

Associate multipliers α_n^f, β_n^f and ν_n with the first three constraints in problem (22), define $\boldsymbol{\Lambda}_n := \{\alpha_n^f, \beta_n^f, \nu_n : f\in\mathcal{F}\}$, $\mathbf{P}_n(\hat{\boldsymbol{\gamma}}_n) := \{A_n^f(\hat{\boldsymbol{\gamma}}_n), P_n^f(\hat{\boldsymbol{\gamma}}_n), B_n^f(\hat{\boldsymbol{\gamma}}_n) : f\in\mathcal{F}\}$, $\mathbf{x}_n = \{x_n^f, y_n^f : f\in\mathcal{F}\}$, and write the Lagrangian as

$$\begin{aligned}
&\mathcal{L}_n(\mathbf{P}_n(\hat{\boldsymbol{\gamma}}_n), \mathbf{x}_n, \boldsymbol{\Lambda}_n) \quad (23) \\
&= \sum_{f\in\mathcal{F}}\Bigg[\left[w_n^f\log x_n^f - \alpha_n^f x_n^f\right] + \left[\tilde{w}_n^f\log(1-y_n^f) + \beta_n^f y_n^f\right] \\
&+ \mathbb{E}_{\hat{\boldsymbol{\gamma}}_n}\left[A_n^f(\hat{\boldsymbol{\gamma}}_n)\left[\alpha_n^f R_n^f(\hat{\boldsymbol{\gamma}}_n) - \beta_n^f - \nu_n P_n^f(\hat{\boldsymbol{\gamma}}_n)\right]\right] + \nu_n P_{0n}\Bigg],
\end{aligned}$$

where we reordered and grouped terms by primal variables. As in the case of OFDMA, we define the dual function and dual problem as

$$\begin{aligned}
\mathsf{D_{r,n}} &= \min_{\alpha_n^f\geq 0, \beta_n^f\geq 0, \nu_n\geq 0} g_n(\alpha_n^f, \beta_n^f, \nu_n) \\
&= \min_{\alpha_n^f\geq 0, \beta_n^f\geq 0, \nu_n\geq 0}\; \max_{\mathbf{P}_n(\hat{\boldsymbol{\gamma}}_n), \mathbf{x}_n} \mathcal{L}_n(\mathbf{P}_n(\hat{\boldsymbol{\gamma}}_n), \mathbf{x}_n, \boldsymbol{\Lambda}_n). \quad (24)
\end{aligned}$$

By leveraging the property of null duality gap, i.e., $\mathsf{P_{r,n}} = \mathsf{D_{r,n}}$, we can characterize the optimal solution of the primal problem using the optimal solution of the dual problem, as shown in the following theorem.

Theorem 2 The optimal frequency assignment function $A_n^{f*}(\hat{\boldsymbol{\gamma}})$, channel backoff function $B_n^{f*}(\hat{\boldsymbol{\gamma}})$ and power allocation function $P_n^{f*}(\hat{\boldsymbol{\gamma}})$ for solving problem (22) are determined by the optimal variables $\alpha_n^{f*}, \beta_n^{f*}$ and ν_n^* of the dual problem (24). Define $R_n^{f*}(\hat{\boldsymbol{\gamma}}_n) = C\left(P_n^{f*}(\hat{\boldsymbol{\gamma}}_n), B_n^{f*}(\hat{\boldsymbol{\gamma}}_n)\right) M_{\gamma_n^f|\hat{\gamma}_n^f}\left(B_n^{f*}(\hat{\boldsymbol{\gamma}}_n)\right)$, then for a given terminal n and frequency $f\in\mathcal{F}$ we have

$$\begin{aligned}
&\left\{P_n^{f*}(\hat{\boldsymbol{\gamma}}_n), B_n^{f*}(\hat{\boldsymbol{\gamma}}_n)\right\} \in \\
&\qquad \operatorname*{argmax}_{p\in[0,P_{\max}], b\geq 0} \alpha_n^{f*} C(p,b)\, M_{\gamma_n^f|\hat{\gamma}_n^f}(b) - \beta_n^{f*} - \nu_n^* p, \quad (25) \\
&A_n^{f*}(\hat{\boldsymbol{\gamma}}_n) = H\left(\alpha_n^{f*}R_n^{f*}(\hat{\boldsymbol{\gamma}}_n) - \beta_n^{f*} - \nu_n^* P_n^{f*}(\hat{\boldsymbol{\gamma}}_n))\right), \quad (26)
\end{aligned}$$

where $H(a)$ denotes the Heaviside's step function, i.e. $H(a) = 1$ for $a > 0$ and $H(a) = 0$ otherwise.

Proof: See [5]. □

Theorem 2 shows that the determination of power, frequency and channel backoff can be done distributedly for each terminal and is only based on terminal's local channel state. Comparing Theorem 2 with Theorem 1, we notice two differences: 1) for a given frequency $f\in\mathcal{F}$, determination of $P_n^{f*}(\cdot)$ and $B_n^{f*}(\cdot)$ for OFDMA [cf. Thoerem 1] is done for all n jointly while the ones for RA [cf. Thoerem 2] is done separately for each n; 2) the frequency assignment variable $A_n^{f*}(\hat{\boldsymbol{\gamma}})$ in OFDMA must satisfy the constraint $A_n^{f*}(\hat{\boldsymbol{\gamma}})\in\mathcal{A}$ while the one in RA only needs to be binary. This is because in the case of RA all terminals act independently of each other while in the case of broadcast channels the AP plays the role of a central decision maker.

3.2. Online learning algorithm

To solve problem (22) without accessing to channel pdf, we implement stochastic subgradient descent algorithm in the dual domain. The algorithm begins with primal iterations which compute primal variables according to the following

$$x_n^f(t) = \operatorname*{argmax}_{x\geq 0} w_n^f\log x - \alpha_n^f(t)x = \frac{w_n^f}{\alpha_n^f(t)}, \quad (27)$$

$$y_n^f(t) = \operatorname*{argmax}_{0\leq y\leq 1} \tilde{w}_n^f\log(1-y) + \beta_n^f(t)y = \left[1-\frac{\tilde{w}_n^f}{\beta_n^f(t)}\right]^+, \quad (28)$$

$$\begin{aligned}
&\{a_n^f(t), p_n^f(t), b_n^f(t)\} = \quad (29) \\
&\operatorname*{argmax}_{a\in\{0,1\}, p\in[0,P_{\max}], b\geq 0} a\left[\alpha_n^f(t)C(p,b)\, M_{\gamma_n^f|\hat{\gamma}_n^f}(b) - \beta_n^f(t) - \nu_n(t)p\right].
\end{aligned}$$

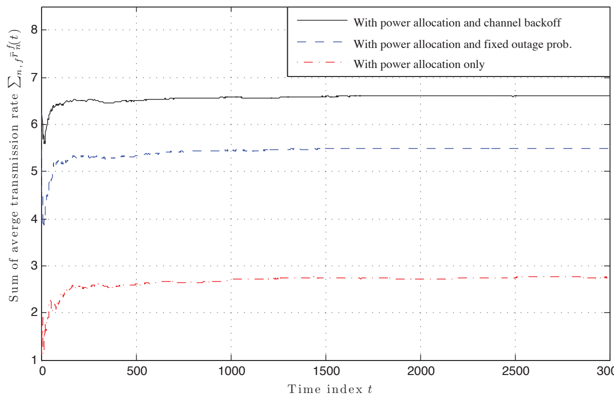

Fig. 1. Comparison of performances of different algorithms for multiuser downlink OFDMA channel.

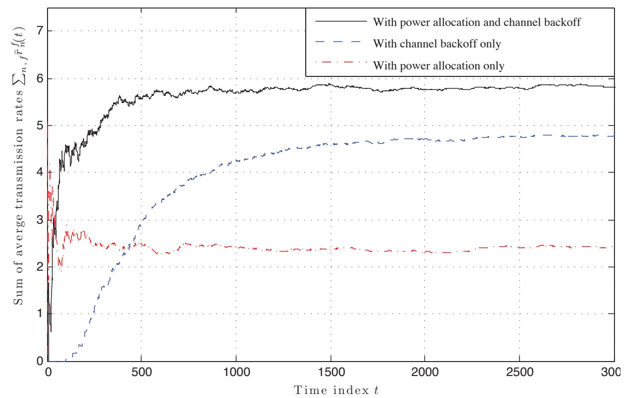

Fig. 2. Comparison of performances of different algorithms for multiuser uplink RA channel.

Optimizations for $x_n^f(t)$ and $y_n^f(t)$ are relatively easy because their objectives are convex functions with single variable [cf. (27) and (28)]. Determinations for $a_n^f(t), b_n^f(t)$ and $p_n^f(t)$ in (29) can be simplified since $a_n^f(t)$ can only take value 0 or 1. Thus, we can rewrite (29) as

$$\{b_n^f(t), p_n^f(t)\} = \operatorname*{argmax}_{p\in[0,P_{\max}],b\geq 0} \alpha_n^f(t)C(p,b)\, M_{\gamma_n^f|\hat{\gamma}_n^f}(b) - \beta_n^f(t) - \nu_n(t)p, \quad (30)$$

$$a_n^f(t) = H\left(\alpha_n^f(t)R_n^f(t) - \beta_n^f(t) - \nu_n(t)p_n^f(t)\right), \quad (31)$$

where we defined $R_n^f(t) = C\left(p_n^f(t), b_n^f(t)\right) M_{\gamma_n^f|\hat{\gamma}_n^f}\left(b_n^f(t)\right)$ in (31). The next step of the algorithm is the dual iterations which evaluate instantaneous constraints violations and update dual variables by

$$\alpha_n^f(t+1) = \left[\alpha_n^f(t) - \epsilon(t)\left[a_n^f(t)R_n^f(t) - x_n^f(t)\right]\right]^+, \quad (32)$$

$$\beta_n^f(t+1) = \left[\beta_n^f(t) - \epsilon(t)\left[y_n^f(t) - a_n^f(t)\right]\right]^+, \quad (33)$$

$$\nu_n^f(t+1) = \left[\nu_n^f(t) - \epsilon(t)\left[P_n - \sum_{f\in\mathcal{F}} a_n^f(t)p_n^f(t)\right]\right]^+. \quad (34)$$

Similar to the case of OFDMA, there are two possible step size rules: 1) Diminishing step size: in this case algorithm converges almost surely; 2) Constant step size: in this case algorithm is almost surely feasible and optimal in an ergodic sense.

4. NUMERICAL RESULTS

In the first set of simulations, we compare algorithms for downlink OFDMA channel. Assume the number of user is $N = 8$ and there are $|\mathcal{F}| = 4$ frequencies available. Model the real channel coefficient h as random variables with complex Gaussian distribution $\mathcal{CN}(0,2)$ and the channel estimation error as complex Gaussian distribution $\mathcal{CN}(0,\sigma_e^2)$ with $\sigma_e^2 = 0.1$. The total average power budget is $P_0 = 4$ and the channel capacity function takes the form of $\log(1 + p_n^f(t)\gamma_n^f(t))$. Sum utility is used, i.e, $U_n(r_n) = r_n$. We run the proposed algorithm with constant step size $\epsilon(t) = 0.01$ and compare its performance with two suboptimal solutions: 1) We do channel backoff first such that a fixed outage probability is achieved, followed by optimal power allocation. In particular, we set the maximum allowed outage probability to 0.01, i.e., we compute $b_n^f(t)$ such that $M_{\gamma(t)|\hat{\gamma}(t)}(b_n^f(t)) = 1 - 0.01 = 0.99$ followed by power allocation. 2) We do not perform channel backoff, i.e. $b_n^f(t) = \hat{\gamma}_n^f(t)$. Fig. 1 compares the total average transmission rate achieved by all three algorithms over 3000 time slots. We conclude that: 1) Channel backoff is very important when dealing with imperfect CSI (6.6 vs. 2.8); 2) Jointly optimizing power allocation and channel backoff results in considerate performance improvement (6.6 vs. 5.5).

For the simulation of algorithms for uplink RA channel, we assume similar parameters as in the case of OFDMA except for the the power constraint which we set $P_{0,n} = 4$ for all n. All coefficients for the weighted proportional fair utility are set to 1. We run the proposed algorithm with constant step size $\epsilon(t) = 0.01$ and compare its performance with two suboptimal solutions: 1) Without power control, i.e. $p_n^f(t)$ is always constant such that average power constraint is satisfied. 2) Without channel backoff, i.e. $b_n^f(t) = \hat{\gamma}_n^f(t)$. Fig. 2 compares the total average transmission rate achieved by all three algorithms over 3000 time slots. Again, by jointly optimizing the frequency access, power allocation and channel backoff the proposed algorithm achieves the highest total transmission rate.

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