SPREADING CODE SELECTION AND POWER CONTROL STRATEGY FOR WARP CONVERGING MULTIUSER DETECTION IN ASYNCHRONOUS CDMA SYSTEMS

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ABSTRACT

In this work, we study the eigen-structure of the data covariance matrix in an *asynchronous* CDMA system. Our results reveal the joint effect of spreading codes and power control on warp converging Wiener filters. We further provide suggestions on spreading code selection and power control strategy in CDMA system design, where stable and predictable early convergence of the reduced-rank conjugate gradient Wiener filter (RRCG-WF) based multiuser detection (MUD) can be obtained.

1. INTRODUCTION

Wiener filter is widely used in multiuser detection as the optimum linear MMSE detector. For instance in a CDMA system, measurement $\mathbf{y}$ is a vector of chip-rate matched filtered and sampled data containing information from all active users; matrix $\mathbf{R_{yy}} \triangleq \mathbf{y}\mathbf{y}^T$ is the correlation matrix of such a data vector; and vector $\mathbf{s}$ represents the spreading code of a desired user. The Wiener filtering operation is then given by $\mathbf{s}^T\mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{y}$. However, the dimensionality in practical system often makes the direct evaluation of such filtering operation involving a large-size matrix inversion undesirable. Accordingly, reduced-rank schemes are introduced to alleviate the computational load.

In [1, 2], we have discovered and proved the conditions on the warp convergence of the RRCG-WF. It has been shown that in Gold code based synchronous CDMA systems with perfect power control, early convergence can be achieved in $L_r = 2 \sim 4$ steps, where L_r is independent of the user number K and the code spreading length N. We have also proved that the filter rank L_r is related to the number of distinct nonzero eigenvalues of the data covariance matrix $\mathbf{R_{yy}}$, as well as the code set Gram matrix $\mathbf{G_{SS}}$ under perfect power control. In a subsequent work [3], we have analytically derived the number of distinct eigenvalues for some commonly used spreading codes under synchronous scenario.

In this work, we extend our analysis of the code set Gram matrix to asynchronous case, a more practically encountered situation. Our study is focused on the so-called *preferred set* of *m*-sequences with *preferred* three-valued cross-correlation spectra. Instead of finding the exact number of distinct eigenvalues for the data covariance matrix, we are interested in determining the number of distinct *clusters* of eigenvalues. It is shown that advanced power control strategy can be used to bound the eigenvalues into clusters, hence enabling the warp convergence. We further conclude that for properly selected spreading codes and power control parameters, stable and predictable early convergence can be achieved for RRCG-WF based MUD even in asynchronous CDMA systems.

The conjugate gradient method is an iterative method for solving a linear system of equation $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ denotes an $N \times N$ symmetric and positive definite matrix. Specifically, our derivation is based on the following Theorems [4, 5, 6], connecting convergence of iterative solution to $\mathbf{Ax} = \mathbf{b}$ with matrix $\mathbf{A}$'s eigenvalues.

Theorem 1 (Convergence): If $\mathbf{A}$ has eigenvalues $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$, starting from $\mathbf{x}_0$ for the iterative solution $\mathbf{x}_{k+1}$, we have that

$$\| \mathbf{x}_{k+1} - \mathbf{x}^* \|_{\mathbf{A}}^2 \leq \left(\frac{\lambda_{N-k} - \lambda_1}{\lambda_{N-k} + \lambda_1}\right)^2 \| \mathbf{x}_0 - \mathbf{x}^* \|_{\mathbf{A}}^2 . \quad (1)$$

Here the weighted norm $\| \cdot \|_{\mathbf{A}}^2$ is with respect to $\mathbf{A}$ as defined by $\| \mathbf{z} \|_{\mathbf{A}}^2 = \mathbf{z}^T\mathbf{A}\mathbf{z}$.

Theorem 1 tells that for a small value of $(\lambda_{N-k} - \lambda_1)/(\lambda_{N-k} + \lambda_1)$, the CG iteration will provide a good approximate to the solution after only $k+1$ steps. An extension of this argument leads to a more interesting observation, upon which our results are based.

- It is generally true that if the eigenvalues occur in k distinct clusters, the CG iterates will *approximately* solve the problem after k steps.

Theorem 2 (Gersgorim): Let $\mathbf{A} = [a_{ij}]$ denotes an arbitrary $N \times N$ matrix, and let $R_i'(\mathbf{A}) \equiv \sum_{j=1, j\neq i}^{N} |a_{ij}|$, (for $1 \leq i \leq N$, denote the *deleted absolute row sums* of $\mathbf{A}$. Then all the eigenvalues of $\mathbf{A}$ are located in the union of N discs:

$$\bigcup_{i=1}^{N} \left\{ \lambda \in \mathbf{C} : |\lambda - a_{ii}| \leq R_i'(\mathbf{A}) \right\} \equiv G(\mathbf{A}). \quad (2)$$

Furthermore, if a union of k of these N discs forms a connected region that is disjoint from all the remaining $N-k$ discs, then there are precisely k eigenvalues of A in this region.

For Asynchronous CDMA systems, the direct calculation of the eigenvalues for the code set Gram matrix is almost impossible. But, it is possible to get an approximate range of the eigenvalues based on Theorem 2. Theorem 1 further links this approximate eigenvalue bound to an approximate step number that CG iteration converges.

2. EIGEN-STRUCTURE ANALYSIS OF THE DATA COVARIANCE MATRIX

When a set of K spreading codes of length N (assuming $K \leq N$) is chosen, we can construct a $N \times K$ code set matrix $\mathbf{S} = [\mathbf{s}_1\, \mathbf{s}_2 \ldots \mathbf{s}_K]$, and further form the $K \times K$ code set Gram matrix $\mathbf{G_{SS}} = \mathbf{S}^T\mathbf{S}$. Notice that for asynchronous CDMA systems, $\{\mathbf{s}_k\}_{k=1}^K$ represent shifted versions of the spreading codes based on different time delays of multiple users. Now we study in detail the spectrum property of this matrix, and the number of steps required for the RRCG-WF to converge, or approximately converge, to the full-rank Wiener filter.

2.1. Cross-correlation Spectrum of Connected Sets of m-sequences

If linear *m*-sequences $\mathbf{u}$ and $\mathbf{v}$ are generated from different primitive polynomials, their cross-correlation function (CCF) $\theta_{\mathbf{u},\mathbf{v}}$ produces *at least* three values. While there are sets of *m*-sequences available that produce only three values for the CCF. The property of *m*-sequences given below exhibits specific decimation which produces *exactly* three-valued CCF except when n is a power of 2. This result is a composite one, as various parts of it were proved by Gold, Kasami, Solomon, and Welch [8].

- *Cross-Spectrum Property:* Let $\mathbf{u}$ and $\mathbf{v}$ denote *m*-sequences of period $N = 2^n - 1$. If $\mathbf{v} = \mathbf{u}[q]$, where either $q = 2^k - 1$ or $q = 2^{2k} - 2^k + 1$, and if $e = \gcd(n,k)$ is such that n/e is odd, then the spectrum of $\theta_{\mathbf{u},\mathbf{v}}$ is three valued and,

$$\begin{array}{llll} -1+2^{(n+e)/2} & \text{occurs} & 2^{n-e-1}+2^{(n-e-2)/2} & \text{times} \\ -1 & \text{occurs} & 2^n - 2^{n-e} - 1 & \text{times} \\ -1-2^{(n+e)/2} & \text{occurs} & 2^{n-e-1}-2^{(n-e-2)/2} & \text{times} \end{array}$$

(Notice that in most instances, small values of e are desirable. If we wish to have $e = 1$ then clearly n must be odd in order that n/e be odd.)

CCFs taking on these three values only are the so-called *preferred* three-valued CCFs, and the corresponding pair of *m*-sequences is called a *preferred* pair of *m*-sequences. A *connected set* of *m*-sequences is a collection of *m*-sequences which has the property that each pair in the collection is a preferred pair. Our following derivation is restricted to connected sets of *m*-sequences. While our results can be generalized to larger sets of periodic sequences that have good periodic correlation properties (with small cross-correlations). Such sequences can be constructed from the *m*-sequence, including the Gold sequences, the Gold-like and Dual-BCH sequences, and the Kasami sequences.

2.2. Eigen-structure Analysis of the Code Set Gramian

Based on Theorem 1 (G-disc), all the eigenvalues of matrix $\mathbf{G_{SS}}$ are located in the union of K discs, centered at g_{11} through g_{KK}, with g_{ij} $(1 \leq i, j \leq K)$ denoting the element on the i-th row and j-th column of matrix $\mathbf{G_{SS}}$. Since the diagonal elements are the same for matrix $\mathbf{G_{SS}}$ (all 1's if normalized codes are in use), all the eigenvalues then fall into a single disc:

$$\left\{\lambda \in \mathbf{C} : |\lambda - 1| \leq R(\mathbf{G_{SS}})\right\} \equiv G(\mathbf{G_{SS}}),$$

where $R(\mathbf{G_{SS}})$ represents the radius of the G-disc. Here we have $R(\mathbf{G_{SS}}) = \max_{i=1}^K \left[R_i(\mathbf{G_{SS}})\right]$, with $R_i(\mathbf{G_{SS}}) \equiv \sum_{j=1, j\neq i}^K |g_{ij}|$ denoting the i-th deleted absolute row sums of $\mathbf{G_{SS}}$ based on Theorem 1. According to the three-value property of the *m*-sequences, we are able to calculate an asymptotic radius of the G-disc, based on the assumption that the CCFs in the code set Gramian $\mathbf{G_{SS}}$ are evenly distributed according to the CCF spectra given by the cross-spectrum property.

Specifically, we obtain the distribution of $|g_{ij}|$ shown below (for a given number of i with $1 \leq j \leq K$ and $j \neq i$).

$$\begin{array}{llll} \dfrac{2^{(n+e)/2}-1}{2^n-1} & \sim \text{occurs} & \dfrac{2^{n-e-1}+2^{(n-e-2)/2}}{2^n-1}(K-1) & \text{times} \\ \dfrac{1}{2^n-1} & \sim \text{occurs} & \dfrac{2^n-2^{n-e}-1}{2^n-1}(K-1) & \text{times} \\ \dfrac{2^{(n+e)/2}+1}{2^n-1} & \sim \text{occurs} & \dfrac{2^{n-e-1}-2^{(n-e-2)/2}}{2^n-1}(K-1) & \text{times} \end{array}$$

Here $\sim$ denotes "approximately". Consequently, we have,

$$\begin{aligned} R(\mathbf{G_{SS}}) &\approx \text{ mean}_{i=1}^K\left[R_i(\mathbf{G_{SS}})\right] \\ &\approx \frac{2^{(3n-e)/2}+2^n-2^{n-e}-2^{(n-e)/2}-1}{(2^n-1)^2} \times (K-1) \\ &\approx \frac{K-1}{2^{(n+e)/2}}. \end{aligned} \tag{3}$$

Unfortunately the results at this step dose not provide much useful information as all the eigenvalues fall into the same G-disc. However, it dose provide a lead on how to approximately bound the eigenvalues based on the spectrum of the CCF. It will be discussed in the next sub-section that the eigenvalues can be further bounded into several clusters with the help of advanced power control strategy.

2.3. Eigen-Structure Analysis of the Power controlled Code Set Gramian

Referring to [7], an implicit assumption for the conventional power control strategy is that the system under study is for one class of service. This assumption can be extended to allow multiple classes of services to be accommodated simultaneously, which may offer different bit rates, or different bit error rates.

Let us assume that the system is designed to provide J classes of services, which results in J levels of the received power p_j. Specifically, we assume that for $j = 1, 2, \dots, J$, there are K_j users within class j with received power σ_j^2 $(K_1 + K_2 + \cdots + K_J = K)$. Matrix $\mathbf{\Sigma} = \mathrm{diag}\{\sigma_1, \sigma_2, \dots, \sigma_K\}$ contains the users' amplitudes. Based on the fact that matrix $\mathbf{S}\mathbf{\Sigma}^2\mathbf{S}^T$ and matrix $\mathbf{\Sigma}^2\mathbf{S}^T\mathbf{S}$ have the same non-zero eigenvalues, we then try to determine the eigen-structure of the power scaled Gram matrix $\mathbf{\Sigma}^2\mathbf{G}_{\mathbf{SS}}$.

For J classes of services, we have J distinct power levels corresponding to the diagonal elements of matrix $\mathbf{\Sigma}^2\mathbf{G}_{\mathbf{SS}}$. Referring to the Gersgorim theorem, the eigenvalues of this matrix are located in the union of J discs centered at σ_j^2, for $j = 1, 2, \dots, J$. The corresponding radius for the j-th G-circle can be approximately calculated as follows:

$$
\begin{aligned}
& R_j(\mathbf{\Sigma}^2\mathbf{G}_{\mathbf{SS}}) \\
\approx\ & \frac{2^{(3n-e)/2} + 2^n - 2^{(n-e)/2} + 2^{n-e} - 1}{(2^n - 1)(2^n + 2^{n-e+1} - 1)} \times K_j \times \sigma_j^2 \\
\approx\ & \frac{K_j}{2^{(n+e)/2} + 2^{(n-e+2)/2}} \times \sigma_j^2. \qquad (4)
\end{aligned}
$$

It can be observed that the key factors for the bound of eigenvalues are the number of users K_j's, and the power levels σ_j^2's. For small K_j's and distinctly separated σ_j^2's, the eigenvalues of matrix $\mathbf{\Sigma}^2\mathbf{G}_{\mathbf{SS}}$ can be nicely bounded into J clusters, which results in an approximate early convergence of J steps for the RRCG-WF of the CDMA system.

Consequently, the proposed power control strategy enabling fast converging RRCG-WF solutions in asynchronous CDMA systems is given by the following procedures:

- Determine the number of classes J and number of users K_j (for $j = 1, 2, \dots, J$) within each class; set a base power σ^2 and assume that $\sigma_j^2 = \rho_j\sigma^2$ with $\rho_1 = 1$; and calculate the constant $t = 2^{(n+e)/2} + 2^{(n-e+2)/2}$ based on the spreading codes used in the system.

- Sequentially calculate the minimum required value of ρ_j, for $j = 2, \dots, J$, according to Eq. (5), which is the constraint that the G-discs from different classes do not overlap with each other, and choose a relatively small and convenient value for ρ_j.

$$\rho_j > \frac{t + K_{j-1}}{t - K_j}\rho_{j-1} \qquad (5)$$

The following Fig. 1 shows a three-cluster distribution of eigenvalues for the data covariance matrix under a typical three-level power control setting. We have a connected set of length $N = 63$ *m*-code with $n = 6$ and $e = 2$. A $J = 3$ group power control strategy is applied. There are 8 users within group-1 with power σ^2; 4 users within group-2 with power $2\sigma^2$; and 2 users within group-3 with power $4\sigma^2$, respectively. And the radius of the G-discs turns out to be $\sigma^2/3$ for all three groups. As can be seen in Fig. 1, the blue circles represent the calculated bound, and the red starts represent the eigenvalues of a specific trial.

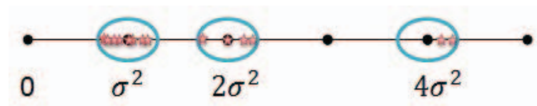

Fig. 1. Three clusters of eigenvalues bounded by the power control strategy.

3. NUMERICAL EXAMPLES

Computer generated simulations are provided to demonstrate the warp convergence of the RRCG-WF in asynchronous CDMA applications with either *connected set* of *m*-sequences, or Gold sequences.

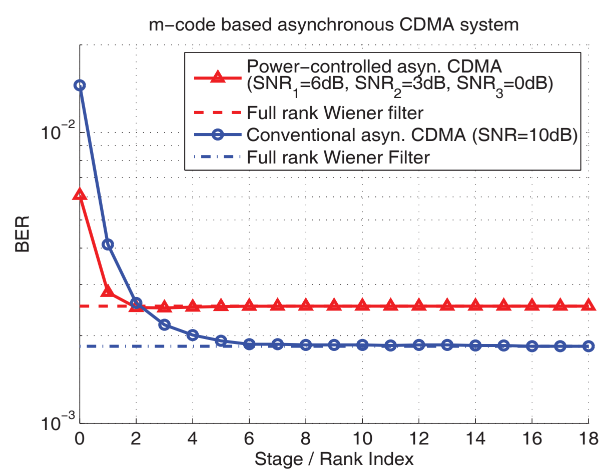

Fig. 2. BER Performance of the RRCG-WF in *m*-sequence (of a *connected* set) based asynchronous CDMA applications.

For the first simulation example, we assume to have $K = 30$ active users, each using a distinct length $N = 63$ *m*-code chosen from a *connected* set of *m*-sequences (with $n = 6$ and $e = 2$). Case 1 (red lines) is for a $J = 3$ group power control strategy, which results in users 1 through 10 with $SNR_1 =$ 0dB, users 11 through 20 with $SNR_2 =$ 4dB, and users 21 through 30 with $SNR_3 =$ 4dB. On the other hand, case 2 (blue lines) is for a conventional asynchronous CDMA system with a common $SNR =$ 10dB for all 30 users. As can be seen in Fig. 2, a warp convergence is achieved in just $2 \sim 3$ steps for the group-wise power-controlled scenario, while 6 steps are needed for the conventional system.

Our second simulation example is for a set of $K = 15$ users using length $N = 31$ Gold codes. Here, case 1 (red lines) follows a $J = 3$ group-wise power control strategy, where users 1 through 5 are with $SNR_1 = 2$dB, users 6 through 10 are with $SNR_2 = 6$dB, and users 11 through 15 are with $SNR_3 = 9$dB. Case 2 (blue lines) is for a conventional power-controlled asynchronous CDMA system with a common $SNR = 6$dB. The simulation results shown in Fig. 3 is similar to the *m*-codes results shown in Fig. 2, where warp convergence in 3 steps is obtained for the power controlled asynchronous CDMA system, and 5 steps for the conventional asynchronous CDMA system.

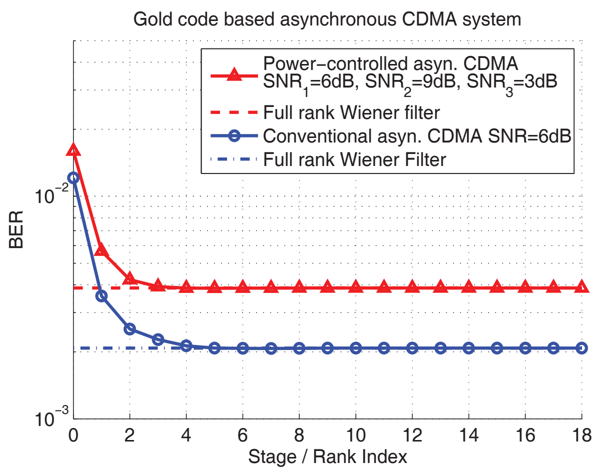

Fig. 3. BER Performance of the RRCG-WF in Gold code based asynchronous CDMA applications.

It should be pointed out that the different performance on convergency is related to the choice of spreading codes along with group power control, which results in different periodic correlation properties. The simulation results also show that even for the conventional single-power-level asynchronous CDMA system, early convergence in just a few more steps can be obtained. It also reminds us that the Convergence theorem together with the Gersgorim theorem only provide an approximate upper bound for the convergence performance of the RRCG-WF.

4. CONCLUSION

We have shown in this paper that for a general asynchronous CDMA system, spreading codes and power control strategies have joint effect on the eigen-structure of the data covariance matrix. And such eigen-structure directly control the warp convergence of the RRCG-WF for the system. For properly chosen spreading codes with small cross-correlations, and power control strategies with distinct power levels, stable and predictable early convergence can be obtained for the RRCG-WF based MUD of the CDMA system. This property may not be well recognized in existing literature, while here we show its important implications in designing fast converging solutions to CDMA system, as well as other signal processing and communication problems.

5. REFERENCES

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