DISTRIBUTED POLARIZING TRANSMISSIONS VIA BROADCAST SWITCHING ON FREQUENCY SELECTIVE FADING CHANNELS

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ABSTRACT

We address the scheme design issues by switching to polarizing frequency selective fading (FSF) channels while transmitting information symbols in a source-relay-destination wireless system. A simple polar-and-forward (PF) relay scheme, with source polar coding and relay polar coding, is proposed to provide an alternative solution for transmitting with high reliability. We analyze the bit error rate (BER) performance behaviors with the switching polar system equipped with four OFDM blocks, which is an idea approach to select OFDM symbols that tend to polarize in terms of the reliability under certain OFDM combining and splitting for the FSF channels.

***Index Terms*—** Polar codes, frequency selective fading channels, bit error rate, DFT, MIMO-OFDM

1. INTRODUCTION

The channel polarization shows an attractive construction of provably capacity-achieving coding sequences [1]. It has provided an attempt method to meet this elusive goal for multi-fold binary-input discrete memoryless channels, where channel combining and splitting operations were applied to improve its symmetric capacity.

Recently, MIMO relay communications, together with orthogonal frequency division multiplexing (OFDM) techniques, have proposed an effective way of increasing reliability as well as achievable rates in next generation wireless networks. In the MIMO-OFDM relay system, two or more nodes share and transmit jointly their information symbols in a multi-antenna array, which enables high data rate and diversity gain. A usual approach to share information is to tune in the transmitted signals and process the whole (or partial) received information in regenerative or non-regenerative way. The former employs a decode-and-forward (DF) relay scheme [2]. The latter scheme exploits an amplify-and-forward (AF) scheme without any attempt to decode the original information [3].

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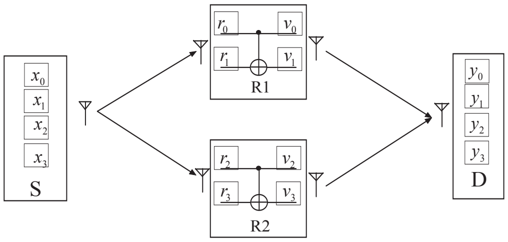
The problem with the previous relay system is the data rate loss as the number of relay nodes increases [4]. This leads to the use of polar sequences in the MIMO-OFDM system, where relay nodes are allowed to simultaneously transmit multiple OFDM symbols over the FSF channels. In the light of superiority of these relay strategies with the availability of channel state information (CSI), we consider the relay scheme design for the FSF channels using the polar-and-forward (PF) technique, in which each relay node polarizes and retransmits the partial signals with the fixed power constraint increasing the occurrence of capacity-achieving code sequences for the binary-input discret memoryless channels.

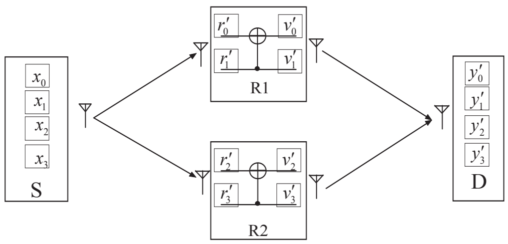
Some notations are defined throughout this paper. Z_N denotes an integer set $\{0, 1, \cdots, N-1\}$. Superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, and $(\cdot)^*$ represent the transpose, complex conjugate transpose, and complex conjugate of a matrix. $\mathrm{diag}(\mathbf{d}_0, \cdots, \mathbf{d}_{N-1})$ is a diagonal matrix with diagonal entries $\mathbf{d}_0, \cdots, \mathbf{d}_{N-1}$.

2. CHANNEL POLARIZATION

We consider the distributed wireless system based on OFDM modulation with N subcarriers. There is one source node S, one destination node D, and two relay nodes $R = \{R_1, R_2\}$, which are provided with one transmit antenna as shown in Fig.1. The design of the relay scheme that can mitigate relay synchronization errors is considered. The N_s independent OFDM symbols are transmitted simultaneously from source node S to destination node D in two stages. In the first stage the initial OFDM symbols are polarized and transmitted from source node S to each relay node R_k, $\forall\ k \in \{1, 2\}$. In the second stage each relay node R_k polarizes and forwards the (partial) symbols received from source node S to destination node D while source node S keeps silent. We further assume that each single-link between a pair of transmit and receive antenna is frequency selective Rayleigh fading with L independent propagation, which experiences quasi-static and remains unchanged in certain blocks. Denote the fading coefficients from source node S to relay node R_k as $\mathbf{h}_{SR_k} = \phi_k$ and coefficients from relay node R_k to destination node D as $\mathbf{h}_{R_kD} = \kappa_k$. Assume that ϕ_k and κ_k are all independent zero



(a) Down-polarizing OFDM Blocks



(b) Up-polarizing OFDM blocks

Fig. 1. The polar MIMO-OFDM relay system based on the OFDM polarizing for the FSF channels.

mean complex Gaussian random variables. Channel impulse responses $\phi_k(t)$ from source node S to destination node R are

$$\phi_k(t) = \sum_{l=0}^{L-1} \alpha_{sk}(l)\delta(t - \tau_{l,sk}),$$

where $\alpha_{sk}(l)$ represents the channel coefficient of the l^{th} path of the FSF channels, and $\tau_{l,sk}$ is the corresponding path delay. Each channel coefficient $\alpha_{sk}(l)$ is modelled as zero mean complex Gaussian random variables with variance $\sigma^2_{l,sk}$ such that $\sum_{l=0}^{L-1} \sigma^2_{l,sk} = 1$. Similarly, other channel impulse responses $\kappa_k(t)$ from R_k to D are

$$\kappa_k(t) = \sum_{l=0}^{L-1} \alpha_{rk}(l)\delta(t - \tau_{l,rk}),$$

where $\alpha_{rk}(l)$ represents the channel coefficient, and $\tau_{l,rk}$ is the corresponding path delay. In addition, we denote the average power of each relay R_k as p_r. The average transmit power at source node S is p_t. The constraint on the total network power is $p = p_t + 2p_r$ for $p_t = 2p_r = p/2$.

2.1. Down-polarizing MIMO-OFDM Relay System

At source node S the transmitted information are modulated into complex symbols x_{ij} and then each N modulated symbols as a block are poured into an OFDM modulator of N subcarriers. Denote four consecutive OFDM symbols by $x_i = (x_{i,0}, \cdots, x_{i,N-1})^{\mathrm{T}}, \forall\, i \in Z_4$. We define $x_i + x_j = (x_{i,0} + x_{j,0}, \cdots, x_{i,N-1} + x_{j,N-1})^{\mathrm{T}}$, $\forall\, i, j \in Z_4$, for polarization calculation.

Table 1. The PF scheme for the down-polarized system at relay nodes. OM_i denote the i^{th} OFDM block.

	Polar R_1	Polar R_2	Process R_1	Process R_2
OM_0	$\check{r}_{10}$	$\check{r}_{20}$	$\zeta(\check{r}_{10})$	0
OM_1	$\check{r}_{11}$	$\check{r}_{21}$	0	$\check{r}^*_{21}$
OM_2	$\check{r}_{10}+\check{r}_{12}$	$\check{r}_{22}$	$\zeta(\check{r}_{10}+\check{r}_{12})$	0
OM_3	$\check{r}_{13}$	$\check{r}_{23}+\check{r}_{21}$	0	$(\check{r}_{23}+\check{r}_{21})^*$

The four consecutive OFDM symbols are processed with the *down-polarizing* 4×4 matrix $\mathbf{Q}_4$ at source node S, i.e., $\mathbf{U} = \mathbf{X}\mathbf{Q}_4$, where $\mathbf{U} = (u_0, u_1, u_2, u_3)$ denotes the polarizing matrix of size $N \times 4$, $\mathbf{X} = (x_0, x_1, x_2, x_3)$ denotes the signal matrix of size $N \times 4$ corresponding to four OFDM blocks, the source polar matrix $\mathbf{Q}_4$ is given by $\mathbf{Q}_4 = \mathbf{I}_2 \otimes \mathbf{Q}_2$, where $\otimes$ denotes the Kronecker product and $\mathbf{Q}_2$ is the Arikan *down-polarizing* matrix [1], i.e., $\mathbf{Q}_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Namely, we have $u_{2k-2} = x_{2k-2}$ and $u_{2k-1} = x_{2k-2} + x_{2k-1}$, $\forall\, k \in \{1, 2\}$.

In the OFDM modulator, the four consecutive blocks are modulated by the N-point FFT. Then each block is precoded by a cyclic prefix (CP) with length l_{cp}. Thus each OFDM symbol consists of $L_s = N + l_{cp}$ samples, which are broadcasted to two relay nodes. Denote by τ_{sd2} the overall relative delay from source node S to relay node R_2, and then to destination node D, which is relative to relay node R_1. In order to combat against timing errors, we assume that $l_{cp} \geq \max_{l,k}\{\tau_{l,sk} + \tau_{l,rk} + \tau_{sd2}\}$. Denote four consecutive OFDM symbols by $\check{u}_i$, $\forall\, i \in Z_4$, where $\check{u}_i$ consists of $\mathrm{FFT}(u_i)$ and the CP.

At each relay R_k, the received noisy signals will be polarized, processed and forwarded to destination node D. We define two processed vectors $\check{\mathbf{u}}_1 = (\check{u}_0^{\mathrm{T}}, \check{u}_2^{\mathrm{T}})^{\mathrm{T}}$ and $\check{\mathbf{u}}_2 = (\check{u}_1^{\mathrm{T}}, \check{u}_3^{\mathrm{T}})^{\mathrm{T}}$, which are polarized at R_1 and R_2, respectively. Therefore, the received signals at each relay node R_k for four successive OFDM symbol durations can be given by

$$\begin{aligned}\check{r}_{k0} &= \sqrt{p_t}\check{u}_0 * \phi_k + \check{\check{n}}_{k0},\ \check{r}_{k1} = \sqrt{p_t}\check{u}_1 * \phi_k + \check{\check{n}}_{k1},\\ \check{r}_{k2} &= \sqrt{p_t}\check{u}_2 * \phi_k + \check{\check{n}}_{k2},\ \check{r}_{k3} = \sqrt{p_t}\check{u}_3 * \phi_k + \check{\check{n}}_{k3},\end{aligned} \quad (1)$$

where ϕ_k is an $L \times 1$ vector defined as $\phi_k = (\alpha_{sk}(0), \cdots, \alpha_{sk}(L-1))$, $*$ denotes the linear convolution, and $\check{\check{n}}_{ki}$, $\forall\, i \in Z_4$, denotes the additive white Gaussian noise (AWGN) at R_k with zero-mean and unit-variance.

Then each relay node R_k polarizes, processes and forwards the received noisy signals as shown in Table I, where $\zeta(\cdot)$ denotes the time-reversal of the signals [2], i.e., $\zeta(\check{r}_{ki}(\epsilon)) = \check{r}_{ki}(L_s - \epsilon)$, $\forall\, \epsilon \in Z_{L_s}$, $\forall\, k \in \{1, 2\}$ and $\forall\, i \in Z_4$. Denote by $\check{v}_0 = \zeta(\check{r}_{10})$, $\check{v}_1 = \zeta(\check{r}_{10} + \check{r}_{12})$, $\check{v}_2 = \check{r}^*_{21}$ and $\check{v}_3 = (\check{r}_{21} + \check{r}_{23})^*$. For the ϵ^{th} subcarrier of $\check{v}_i$ we also take the notations $\check{v}_{i,\epsilon} = \check{v}_i(\epsilon), \forall\, \epsilon \in Z_N$.

After the above-mentioned processing, each relay node R_k amplifies the yielded symbols with a scalar λ while remaining the average transmission power p_r. At destination node D, the CP is removed for each OFDM symbol. We note that relay node R_1 implements the time reversions of the noisy signals including both information symbols and CP. What we need is that after the CP removal, we obtain the time reversal version of only information symbols, i.e., $\zeta(\mathrm{FFT}(u_i)), \forall\, i \in Z_4$. Then by using some properties of $\mathrm{FFT}/\mathrm{IFFT}$, we achieve the feasible definition as follows.

Definition 2.1 [2]: According to the processed four OFDM symbols at relay node R_1 we can obtain $\zeta(\phi_1') * \zeta(\mathrm{FFT}(u_i))$ at destination node D if we remove the CP as in a conventional OFDM system to get an N-point vector and shift the last $\tau_1' = l_{cp} - \tau_1 + 1$ samples of the N-point vector as the first τ_1' samples. Here ϕ_1' is an equivalent $N\times 1$ channel vector defined as $\phi_1' = (\alpha_{s1}(0), \cdots, \alpha_{s1}(L-1), 0, \cdots, 0)$, and τ_1 denotes the maximum path delay of channel ϕ_1 from source node S to relay node R_1, i.e., $\tau_1 = \max_l\{\tau_{l,s1}\}$. In a similar way, we define another equivalent $N \times 1$ channel vector $\kappa_1' = (\alpha_{r1}(0), \cdots, \alpha_{r1}(L-1), 0, \cdots, 0)$.

At destination node D, after the CP removal the received four successive OFDM symbols can be written as

$$\begin{aligned}
y_0 &= \lambda(\sqrt{p_t}\zeta(\mathrm{FFT}(u_0)) * \zeta(\phi_1') + \bar{n}_{10}) * \kappa_1' + n_0\\
y_1 &= \lambda(\sqrt{p_t}\zeta(\mathrm{FFT}(u_0+u_2)) * \zeta(\phi_1') + \bar{n}_{10} + \bar{n}_{12}) * \kappa_1' + n_1\\
y_2 &= \lambda(\sqrt{p_t}(\mathrm{FFT}(u_1))^* * t_{sd2} * t_1' * \phi_2' + \bar{n}_{21}^*) * \kappa_2' + n_2\\
y_3 &= \lambda(\sqrt{p_t}(\mathrm{FFT}(u_3+u_1))^* * t_{sd2} * t_1' * \phi_2' + \bar{n}_{21}^* + \bar{n}_{23}^*)\\
&\quad * \kappa_2' + n_3,
\end{aligned} \tag{2}$$

where t_{sd2} is an $N\times 1$ vector that represents the timing errors in the time domain denoted as $t_{sd2} = (\mathbf{0}_{\tau,sd2}, 1, 0, \cdots, 0)^{\mathrm{T}}$, and $\mathbf{0}_{\tau_{sd2}}$ is a $1 \times \tau_{sd2}$ vector of all zeros, and t_1' is the shift of τ_1' samples in the time domain defined as $t_1' = (\mathbf{0}_{\tau_1'}, 1, 0, \cdots, 0)^{\mathrm{T}}$. Since the signals transmitted from R_2 will arrive at the destination τ_{sd2} samples later and after the CP removal, the signals are further shifted by τ_1' samples. The total number of shifted samples is denoted by $\tau_2 = \tau_{sd2} + \tau_1'$. Here $\bar{n}_{ki}$ is the AWGN at relay node R_k and n_i denotes the AWGN at destination node D after the CP removal.

After that the received OFDM symbols are transformed by the N-point FFT. As mentioned before, because of timing errors, the OFDM symbols from relay node R_2 arrive at destination node D τ_{sd2} samples later than that of symbols from relay node R_1. Since l_{cp} is long enough, we can still maintain the orthogonality between subcarriers. The delay τ_{sd2} in the time domain corresponds to a phase change in the frequency domain, i.e., $f^{\tau_{sd2}} = (1, e^{-\iota 2\pi\tau_{sd2}/N}, \cdots, e^{-\iota 2\pi\tau_{sd2}(N-1)/N})^{\mathrm{T}}$, where $f = (1, e^{-\iota 2\pi/N}, \cdots, e^{-\iota 2\pi(N-1)/N})^{\mathrm{T}}$ and $\iota = \sqrt{-1}$. Similarly, the shift of τ_1' samples in the time domain also corresponds to a phase change $f^{\tau_1'}$, and hence the total phase change is f^{τ_2}.

Denote by $\check{y}_i = (\check{y}_{i0}, \check{y}_{i1}, \cdots, \check{y}_{i(N-1)}), \forall\, i \in Z_4$, the received four consecutive OFDM symbols at destination node D after the CP removal and the N-point FFT transformations. Therefore, we have

Table 2. The PF scheme for the up-polarized system at relay nodes.

	Polar R_1	Polar R_2	Process R_1	Process R_2
OM_0	$\check{r}_{10}'+\check{r}_{12}'$	$\check{r}_{20}'$	$\zeta(\check{r}_{10}'+\check{r}_{12}')$	0
OM_1	$\check{r}_{11}'$	$\check{r}_{21}'+\check{r}_{23}'$	0	$(\check{r}_{21}'+\check{r}_{23}')^*$
OM_2	$\check{r}_{12}'$	$\check{r}_{22}'$	$\zeta(\check{r}_{12}')$	0
OM_3	$\check{r}_{13}'$	$\check{r}_{23}'$	0	$(\check{r}_{23}')^*$

$$\begin{aligned}
\check{y}_0 &= \lambda[\sqrt{p_t}\mathrm{FFT}(\zeta(\mathrm{FFT}(u_0))) \circ \check{\phi}_1 \circ \check{\kappa}_1 + \check{\bar{n}}_{10} \circ \check{\kappa}_1] + \check{n}_0\\
\check{y}_1 &= \lambda[\sqrt{p_t}\mathrm{FFT}(\zeta(\mathrm{FFT}(u_0+u_2))) \circ \check{\phi}_1 \circ \check{\kappa}_1\\
&\quad + (\check{\bar{n}}_{10} + \check{\bar{n}}_{20}) \circ \check{\kappa}_1] + \check{n}_1\\
\check{y}_2 &= \lambda[\sqrt{p_t}\mathrm{FFT}((\mathrm{FFT}(u_1))^*) \circ f^{\tau_2} \circ \check{\phi}_2 \circ \check{\kappa}_2 + \check{\bar{n}}_{21} \circ \check{\kappa}_2] + \check{n}_2\\
\check{y}_3 &= \lambda[\sqrt{p_t}\mathrm{FFT}((\mathrm{FFT}(u_3+u_1))^*) \circ f^{\tau_2} \circ \check{\phi}_2 \circ \check{\kappa}_2\\
&\quad + (\check{\bar{n}}_{21}^* + \check{\bar{n}}_{23}^*) \circ \check{\kappa}_2] + \check{n}_3,
\end{aligned} \tag{3}$$

where $\circ$ denotes the Hadamard product, $\check{\phi}_1 = \mathrm{FFT}(\zeta(\phi_1'))$, $\check{\kappa}_1 = \mathrm{FFT}(\kappa_1')$, $\check{\phi}_2 = \mathrm{FFT}((\phi_1')^*)$, $\check{\kappa}_2 = \mathrm{FFT}(\kappa_2')$, $\check{\bar{n}}_{ki} = \mathrm{FFT}(\bar{n}_{ki})$, and $\check{\bar{n}}_i = \mathrm{FFT}(\bar{n}_i)$, $\forall\, k \in \{1,2\}$ and $\forall\, i \in Z_4$.

According to the properties of the well-known N-point FFT transforms for an $N\times 1$ vector x, we have $(\mathrm{FFT}(x))^* = \mathrm{IFFT}(x^*)$ and $\mathrm{FFT}(\zeta(\mathrm{FFT}(x))) = \mathrm{IFFT}(\mathrm{FFT}(x)) = x$ [2]. Therefore, the formulas in (3) can be written in the polar form for each subcarrier ϵ, $\forall\, \epsilon \in Z_N$, as follows

$$\begin{aligned}
\begin{pmatrix} y_{0\epsilon} \\ y_{1\epsilon} \\ y_{2\epsilon}^* \\ y_{3\epsilon}^* \end{pmatrix} &= \lambda\sqrt{p_t}\left(\begin{array}{cc|cc} \check{\phi}_{1\epsilon}\check{\kappa}_{1\epsilon} & 0 & 0 & 0 \\ \check{\phi}_{1\epsilon}\check{\kappa}_{1\epsilon} & 0 & \check{\phi}_{1\epsilon}\check{\kappa}_{1\epsilon} & 0 \\ \Phi_{2\epsilon}^* & \Phi_{2\epsilon}^* & 0 & 0 \\ \Phi_{2\epsilon}^* & \Phi_{2\epsilon}^* & \Phi_{2\epsilon}^* & \Phi_{2\epsilon}^* \end{array}\right)\begin{pmatrix} x_{0\epsilon} \\ x_{1\epsilon} \\ x_{2\epsilon} \\ x_{3\epsilon} \end{pmatrix} + \mathbf{e}_\epsilon\\
&= \mathcal{H}_I \mathbf{x}_{I\epsilon} + \mathcal{H}_F \mathbf{x}_{F\epsilon} + \mathbf{e}_\epsilon,
\end{aligned} \tag{4}$$

where $\Phi_{2\epsilon} = f_\epsilon^{\tau_2}\check{\phi}_{2\epsilon}\check{\kappa}_{2\epsilon}$, $\Phi_{2\epsilon}^* = (f_\epsilon^{\tau_2}\check{\phi}_{2\epsilon}\check{\kappa}_{2\epsilon})^*$, $f_\epsilon^{\tau_2} = \exp(-\iota 2\pi\epsilon\tau/N)$, $\mathbf{x}_{I\epsilon} = (x_{0\epsilon}, x_{1\epsilon})^{\mathrm{T}}$, $\mathbf{x}_{F\epsilon} = (x_{2\epsilon}, x_{3\epsilon})^{\mathrm{T}}$, $x_{i\epsilon}$ is the ϵ^{th} element of x_i, $\check{\kappa}_{k\epsilon}$ and $\check{\phi}_{k\epsilon}$ denote the ϵ^{th} element of $\check{\kappa}_k$ and $\check{\phi}_k$, $\forall\, k \in \{1,2\}$ and $\forall\, i \in Z_4$. Two vectors $\mathbf{e}_{0\epsilon}$ and $\mathbf{e}_\epsilon$ denote the corresponding polarized noises.

2.2. Up-polarizing MIMO-OFDM Relay System

In this system, the four consecutive OFDM symbols are processed at source node S with the *up-polarizing* 4×4 matrix $\mathbf{Q}_4'$ at S, i.e., $\mathbf{U}' = \mathbf{X}\mathbf{Q}_4'$, where $\mathbf{U}' = (u_0', u_1', u_2', u_3')$, $\mathbf{X} = (x_0, x_1, x_2, x_3)$, $\mathbf{Q}_4' = \mathbf{I}_2 \otimes \mathbf{Q}_2'$, and $\mathbf{Q}_2' = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. Namely, we have $u_{2k-2} = x_{2k-2} + x_{2k-1}$ and $u_{2k-1} = x_{2k-1}$.

At destination node D, the received noisy OFDM symbols for each subcarrier ϵ can be written in the *up-polarizing* structure as follows,

$$(y_{0\epsilon}, y_{1\epsilon}, y^*_{2\epsilon}, y^*_{3\epsilon})^T = \mathcal{H}'_F \mathbf{x}'_{F\epsilon} + \mathcal{H}'_I \mathbf{x}'_{I\epsilon} + \mathbf{e}'_\epsilon, \tag{5}$$

where $\mathbf{x}'_{F\epsilon} = (x_{0\epsilon}, x_{1\epsilon})^{\mathrm{T}}$ and $\mathbf{x}'_{I\epsilon} = (x_{2\epsilon}, x_{3\epsilon})^{\mathrm{T}}$.

Due to the benefits of the polarizing OFDM symbols for the large number $N_s = 2^n$ it also shares the good BER performance behaviors of the polarizing FSF channels in terms of its capacity-achieving properties as the non-negative integer n goes to infinity.

3. DEPOLARIZING MIMO-OFDM RELAY SYSTEM

So far we have established the polar system based on the OFDM polarizing for the FSF channels. Next, we analyze the reliability of the the FSF channels with transmission probabilities $W_4^{(i)}$ for the i^{th} OFDM symbol based on the Bhattacharyya parameter vector $\mathbf{z}_4 = (z_{4,0}, z_{4,1}, z_{4,2}, z_{4,3})$, which can be calculated from the recursion formula [1], i.e.,

$$z_{2k,j} = \begin{cases} z^2_{k,j}, & \text{for } 0 \le j \le k-1; \\ 2z_{k,j-k} - z^2_{k,j-k}, & \text{for } k \le j \le 2k-1, \end{cases} \tag{6}$$

for $\forall\ k \in \{1, 2\}$ starting with $z_{1,0} = 1/2$. From scratch, we form a permutation $\pi_4 = (i_0, i_1, i_2, i_3)$ of $(0, 1, 2, 3)$ corresponding to entries of $\mathbf{x} = (x_0, x_1, x_2, x_3)^{\mathrm{T}}$ so that the inequality $z_{4,i_j} \le z_{4,i_k}, \forall\ 0 \le j < k \le 3$, is true. Thus we have the reliability of OFDM splitting for the FSF channels given by $z_4 = (1/16, 7/16, 9/16, 15/16)$, which creates a permutation $\pi_4 = (0, 1, 2, 3)$. It implies that for each subcarrier of the source OFDM symbols $\mathbf{x}_\epsilon$, the first two signals $\{x_{0,\epsilon}, x_{1,\epsilon}\}$ can be transmitted with higher reliability than that of the last two signals $\{x_{2,\epsilon}, x_{3,\epsilon}\}$. Therefore, for the reliable transmission of signals, we let $\{x_{0,\epsilon}, x_{1,\epsilon}\}$ to be the *information* bits that are required to be transmitted from relay nodes, and $\{x_{2,\epsilon}, x_{3,\epsilon}\}$ to be *frozen* bits that provide assistance for transmissions. In practice, the *frozen* bits $\{x_{2,\epsilon}, x_{3,\epsilon}\}$ are always be set zeros for simplicity, i.e., $\{x_{2,\epsilon} = 0, x_{3,\epsilon} = 0\}$. This property can be utilized for the flexible transmission of signals on the FSF channels with high reliability [1].

In Fig. 2, we present the BER curves of the stacked Alamouti code for four OFDM symbols transmitted at source node S. For the present polar system, it shows that the slope of the BER performance curve of the proposed PF scheme with the stacked Alamouti code for the polar system via the OFDM depolarizing algorithm approaches the direct transmitting system when power p_t increases. It implies that the PF scheme can achieve full diversity with the depolarizing algorithm. Furthermore, the BER performance behavior of the present polar system outperforms that of the direct transmission approach which verifies our analysis of the transmission reliability of the FSF channels. Simulations demonstrate that the proposed PF scheme has a similar performance as that of the Alamouti scheme with the ML decoding when the depolarizing is applied at the receiver.

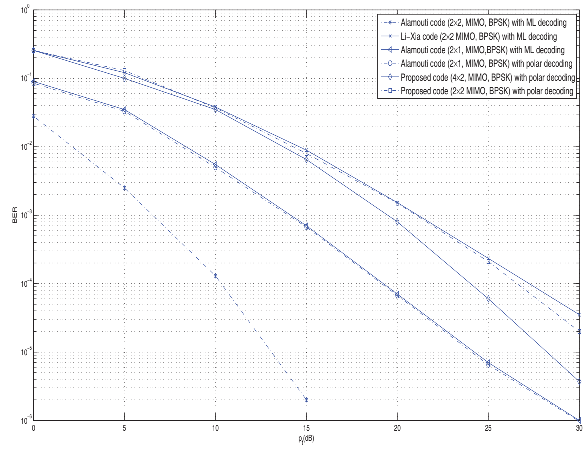


Fig. 2. BER performance behaviors with the depolarizing receiver.

4. CONCLUSION

In this paper, we have presented a simple design of the PF scheme based on the switching polar systems over the FSF channels, i.e., the *down-polarizing* system and the *up-polarizing* system using two polarizing operations $\mathbf{Q}_2$ and $\mathbf{Q}'_2$ first suggested by E. Arikan. The present polar wireless system has a salient recursiveness feature and can be decoded with the SIC decoder, which renders the PF relay scheme analytically tractable and provides a low-complexity coding algorithm while multiple OFDM symbols are equipped and broadcasted from source node S.

5. REFERENCES

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