

# Robustness of Xampling-based RF Receivers against Analog Mismatches

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**Abstract**—The analog imperfections in RF direct conversion receiver, of which I/Q imbalance is a major detriment, are examined. Existing literatures on I/Q imbalance compensation try to compensate for the imbalance by estimating the mismatches. In this work, xampling-based decoding algorithm is examined. This algorithm is shown to be very efficient in handling the impairments in the analog components. Simulation results are also presented to illustrate some of the benefits of the proposed approach.

## I. INTRODUCTION

The proliferation of different mobile communication standards catering to highly sophisticated modulation techniques calls for robust and flexible receiver architectures [1]. A wide plethora of receiver architectures such as heterodyne (typically single IF), dual-IF, homodyne (direct conversion receiver) have been investigated in literature that have relative advantages over each other in terms of selectivity, sensitivity, image suppression, DC offsets etc. [2]. The robustness of homodyne receivers to spurious images have rendered this architecture very attractive for present reception scenarios. But they are susceptible to DC-offsets, amplitude and phase mismatches between the real(I) and imaginary(Q) paths in complex signals during analog I/Q mixing process, which prove to be a potential detriment in the received signal SNR.

The problem of I/Q imbalance (see [3], [5], [6]) is typical of parallel-path analog signal processing, and is usually handled using calibration and/or estimation techniques ([4], [7], [8]), either blindly or by using pilot symbols.

Pushing the downconversion to the post-digitized signal (after the ADC) relaxes the analog matching requirements between the processed signal paths. One major issue with this approach, however, is that the front-end ADC would need to operate at an incredibly large sampling rate due to the high value of the carrier frequency, although the signal is concentrated in only a small frequency band.

In this work, we study a xampling-based demodulation algorithm (proposed in [9] as a sub-Nyquist sampling technique) that leverages this inherent sparsity of the RF signal to

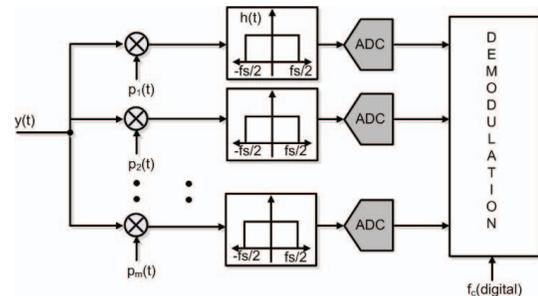


Fig. 1: Proposed receiver architecture

sample it at much lower rates. Xampling also belongs to the set of parallel-path analog signal processing solutions (albeit with a reduced front-end analog involvement). Hence, it is also prone to timing and gain mismatch errors. In this work, we analytically prove the implicit advantages that xampling furnishes in terms of mitigating the deleterious effects of analog mismatches. The technique enables a *calibration-free* (there is no need to estimate the degree of mismatch) solution to traditional problems in complex signal processing.

The remainder of the paper is organized as follows. In Section II, we describe our proposed demodulation strategy and provide mathematical analyses, while Section III gives detailed analyses of the possible effects of the analog imperfections in the proposed approach. In Section IV, we provide simulation results to illustrate the effectiveness of our algorithm in coping with the imperfections by comparing it with the standard direct conversion receiver. Section V concludes the paper.

## II. OUR APPROACH: XAMPLING BASED DEMODULATION

We now present an alternative demodulation scheme using xampling [9] that uses available devices and is more robust to the impairments in the analog components. The real RF signal at the receiver can be written as

$$y(t) = x(t)e^{+j2\pi f_c t} + x^*(t)e^{-j2\pi f_c t} + n(t),$$

where  $x(t)$  denotes the complex baseband equivalent of the frequency band of interest,  $f_c$  denotes its carrier frequency,  $n(t)$  is white Gaussian noise of finite variance and  $(\cdot)^*$  denotes (complex) conjugation. Considering QAM signaling,

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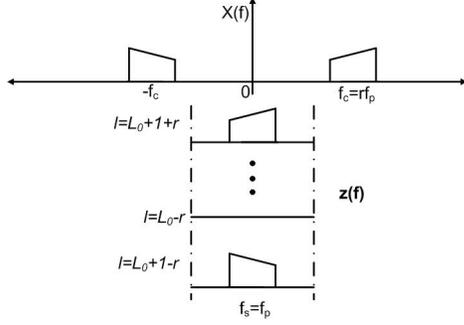


Fig. 2: Relation between  $Y(f)$  and the vector set  $\mathbf{z}(f)$

the complex baseband signal  $x(t)$  can be written as

$$x(t) = x_I(t) + jx_Q(t) = \sqrt{2E_{sym}B} \sum_n [I[n]s(t - nT_{sym}) + jQ[n]s(t - nT_{sym})],$$

where  $B$  is the bandwidth of the signal,  $E_{sym}, T_{sym} = 1/B$  are the energy and the duration of a symbol. The in-phase and quadrature bit streams are  $I[n]$  and  $Q[n]$ , respectively, while  $s(t)$  represents the pulse-shaping filter impulse response.

#### A. System Description

Our system (Fig. 1) exploits spread-spectrum techniques from communication theory [10]. More specifically, the received signal  $y(t)$  enters  $m$  channels simultaneously. In the  $i$ -th channel,  $y(t)$  is multiplied by a mixing function  $p_i(t)$ , which is  $T_p = 1/f_p$ -periodic. After mixing, the signal spectrum is truncated by a low-pass filter with cutoff  $1/(2T_s)$  and the filtered signal is sampled at rate  $f_s = 1/T_s$ .

#### B. Frequency Domain Analysis

For simplicity, let us carry out the analysis with zero noise. To this end, we introduce the definitions

$$f_j = 1/T_j, \mathcal{F}_j = [-f_j/2, f_j/2], j \in \{p, s\}.$$

The Fourier transform of the filter output in the  $i$ -th channel can be evaluated as

$$\begin{aligned} \tilde{Y}_i(f) &= \sum_{l=-L_0}^{l=L_0} c_{il} Y(f - lf_p), f \in \mathcal{F}_s, \\ &= \sum_{l=-L_0}^{l=L_0} c_{il} (X(f - f_c - lf_p) + X^*(-f - f_c - lf_p)), \end{aligned}$$

where  $c_{il}$ 's are the coefficients in the Fourier series expansion of the  $T_p$ -periodic mixing signal  $p_i(t)$  and  $L_0$  is as given in [9]. Collecting the outputs from different channels in a vector form, we can rewrite the equation as

$$\tilde{\mathbf{y}}(f) = \mathbf{A}\mathbf{z}(f), f \in \mathcal{F}_s,$$

where  $\tilde{\mathbf{y}}(f)$  is a vector of length  $m$ . The unknown vectors  $\mathbf{z}(f) = [z_1(f)z_2(f) \dots z_L(f)]$  are of length  $L = 2L_0 + 1$  with  $z_l(f) = Y(f + (l - L_0 - 1)f_p)$ ,  $f \in \mathcal{F}_s$ . The  $m \times L$  matrix  $\mathbf{A}$  contains the coefficients  $c_{il}$  with  $[\mathbf{A}]_{il} = c_{i, -l} = c_{il}^*$ . From

Fig. 2, it is obvious that in order to recover  $y(t)$ , it is sufficient to determine  $\mathbf{z}(f)$  in the interval  $\mathcal{F}_p$ .

#### C. Reconstruction and Demodulation

The reconstruction of  $\mathbf{z}(f)$  is carried out in two steps. Consider that  $\mathbf{z}(f)$  is jointly sparse in  $f \in \mathcal{F}_s$ , e.g.,

$$|\text{support}(\mathbf{z}(\mathcal{F}_s))| = |\cup_{f \in \mathcal{F}_s} \text{support}(\mathbf{z}(f))| \leq K,$$

where  $\text{support}$  is the set containing the position of the non-zero elements in the vector  $\mathbf{z}(f)$  and  $|\cdot|$  calculates the cardinality of a set. Then using the theory of compressed sensing (see [9], [11] for details), first the support  $S$  of  $\mathbf{z}(f)$  is estimated. Once  $S$  is found,  $\mathbf{z}_S(f) = \mathbf{A}_S^\dagger \tilde{\mathbf{y}}(f)$ , where  $\mathbf{A}_S^\dagger = (\mathbf{A}_S^H \mathbf{A}_S)^{-1} \mathbf{A}_S^H$  is the pseudo-inverse of  $\mathbf{A}_S$ , where  $\mathbf{A}_S$  is obtained by choosing the columns of  $\mathbf{A}$  corresponding to the indices in the support set  $S$ .

Xampling was mainly proposed as a sub-Nyquist sampling technique (see [9]), but we observe (see Fig. 2) that with the prior knowledge of carrier frequency  $f_c$ , we can choose  $f_s = f_p = f_c/r \geq B$  (where  $r$  is an integer), which will enable us to integrate analog down-conversion with xampling (in the digital domain), as for this choice of  $f_p$ , the  $l = L_0 + 1 + r$ -th slice would be

$$\begin{aligned} z_l(f) &= Y(f + (l - L_0 - 1)f_p) \\ &= X(f - f_c + (l - L_0 - 1)f_p) \\ &= X(f - rf_p + (l - L_0 - 1)f_p) = X(f), f \in \mathcal{F}_p, \end{aligned}$$

and it is equal to the Fourier transform of the complex baseband signal  $x(t)$ . Hence by taking inverse discrete-time-Fourier-transform (DTFT) of the reconstructed  $z_{L_0+1+r}(f)$ , the real part will give us the in-phase component and the imaginary part will give us the quadrature-phase component.

### III. RF MISMATCHES IN XAMPLING BASED DEMODULATION

In this section, we show how the proposed approach is robust to typical analog imperfections.

#### A. Phase Imbalance Across Different Channels

Timing mismatches between the random sequences  $p_i(t)$  will introduce unwanted (and unknown) phase-shifts between the different channels in a xampling-based demodulation system. The sources of these mismatches are similar to those that give rise to I-Q imbalance in a conventional direct conversion receiver.

More specifically, in the  $i$ -th channel the received signal  $y(t)$  is multiplied by a mixing function  $p_i(t - \tau_i)$ , where  $\tau_i$  is uniformly distributed in  $[-T_p/(2M), T_p/(2M)]$ . The Fourier transform of this analog multiplication  $\tilde{y}_{mmi}(t) = y(t)p_i(t - \tau_i)$  can be written as

$$\tilde{Y}_{mmi}(f) = \sum_{l=-\infty}^{l=\infty} c_{il} e^{-j2\pi l f_p \tau_i} Y(f - lf_p).$$

Collecting all the outputs from different channels after they are passed through a low-pass filter of bandwidth  $f_s/2$ , we

can write  $\tilde{\mathbf{y}}_{mm}(f) = \mathbf{A}_{mm}\mathbf{z}(f)$ ,  $f \in \mathcal{F}_s$ , where  $\mathbf{z}(f)$  is the same as in the no mismatch case and the  $m \times L$  matrix  $\mathbf{A}_{mm}$  contains the coefficients  $[\mathbf{A}_{mm}]_{il} = [\mathbf{A}]_{il}e^{+j2\pi lf_p\tau_i}$ . Without the knowledge of the mismatches, the receiver tries to reconstruct  $\mathbf{z}(f)$  from  $\tilde{\mathbf{y}}_{mm}(f)$  using the matrix  $\mathbf{A}$  instead of the true mismatch matrix  $\mathbf{A}_{mm}$ .

*Lemma 1:* If the relative delays  $\tau_i$  across  $m$  channels are uniformly distributed with 0 mean, then the support of  $\mathbf{z}(f)$  reconstructed from  $\tilde{\mathbf{y}}_{mm}(f)$  using the matrix  $\mathbf{A}$  is the same as the support of  $\mathbf{z}(f)$ , if reconstructed using  $\mathbf{A}_{mm}$  instead.

The lemma can be proved by showing that  $l_2$ -norm of error vector  $(\mathbf{A}_{mm} - \mathbf{A})\mathbf{z}(f)$  approaches 0 for large values of  $m$  and is omitted here for brevity. Once the support is recovered,  $\mathbf{z}_{mmS}(f)$  is reconstructed using the following equation

$$\mathbf{z}_{mmS}(f) = \mathbf{A}_S^\dagger \tilde{\mathbf{y}}_{mm}(f) = (\mathbf{A}_S^H \mathbf{A}_S)^{-1} \mathbf{A}_S^H \mathbf{A}_{mmS} \mathbf{z}_S(f)$$

Under no phase imbalance,  $\mathbf{A}_{mmS} = \mathbf{A}_S$  and hence  $\mathbf{z}_{mmS}(f) = \mathbf{z}_S(f)$ . So to study the effect of phase imbalance, we need to examine the matrix  $(\mathbf{A}_S^H \mathbf{A}_S)^{-1} \mathbf{A}_S^H \mathbf{A}_{mmS}$ . The next theorem summarizes the effect of phase imbalance.

*Theorem 2:* When the number of channels  $m$  is large and the delays  $\tau_i$  in different channels are independent and uniformly distributed with zero mean, then

$$\mathbf{W} = (\mathbf{A}_S^H \mathbf{A}_S)^{-1} \mathbf{A}_S^H \mathbf{A}_{mmS} \xrightarrow{a.s.} c\mathbf{I},$$

where *a.s.* stands for almost sure convergence,  $\mathbf{I}$  is the identity matrix of appropriate dimensions and  $c$  is a constant which is a function of  $m$ ,  $M$  and indices of support  $S$ .

*Proof:* Due to space constraints, we provide a brief sketch of the proof. Let the reconstructed support be  $S = \{L_0 + l + 2, L_0 + l + 1, L_0 - l + 1, L_0 - l\}$  (note that  $|S| = 2N = 4$  in our case, see [9] for details) and  $(\mathbf{A}_S^H \mathbf{A}_S)^{-1} \mathbf{A}_S^H = \mathbf{B} = [B_{L_0+l+2} \ B_{L_0+l+1} \ B_{L_0-l+1} \ B_{L_0-l}]^T$  is a  $2N \times m$  matrix with  $B_i$  is a row-vector of length  $m$ . Now let's look at one of the off-diagonal term in  $\mathbf{W} = \mathbf{B}\mathbf{A}_{mmS}$  (the analysis would be similar for all the off-diagonal terms)

$$[\mathbf{W}]_{12} = \sum_{n=1}^m [\mathbf{B}]_{L_0+l+2,n} [\mathbf{A}_S]_{n,L_0+l+1} e^{+j2\pi(L_0+l+1)\tau_n}.$$

In the no mismatch case, these off-diagonal terms would be 0, e.g.,  $\mathbf{B}\mathbf{A}_S = \mathbf{I}$ , or in other words,

$$\sum_{n=1}^m [\mathbf{B}]_{L_0+l+2,n} [\mathbf{A}_S]_{n,L_0+l+1} = 0.$$

Now both the real and imaginary parts of  $[\mathbf{W}]_{12}$  is the sum of  $m$  independent but not identically distributed (different variance and higher order moments) random variables. Then according to the Kolmogorov's strong law of large numbers (see [12]),

$$[\mathbf{W}]_{12} = \frac{1}{m} \sum_{n=1}^m [\mathbf{B}]_{L_0+l+2,n} [\mathbf{A}_S]_{n,L_0+l+1} e^{+j2\pi(L_0+l+1)\tau_n} \xrightarrow{a.s.} \frac{1}{m} E[e^{+j2\pi(L_0+l+1)\tau}] \sum_{n=1}^m [\mathbf{B}]_{L_0+l+2,n} [\mathbf{A}_S]_{n,L_0+l+1} = 0,$$

Parameter	Value
Carrier frequency ( $f_c$ )	2.4GHz
Tsymbol ( $T_{sym}$ )	100ns
Number of Bands ( $N$ )	2
Sampling frequency ( $f_s$ )	120MHz
Signal bandwidth ( $B$ )	10MHz
Frequency of mixing signal ( $f_p$ )	120MHz
Channels ( $m$ )	50

TABLE I: Design parameters for the receiver

provided each of the random variable has finite 2nd order moment and

$$\sum_{n=1}^{\infty} \frac{1}{n^2} |[\mathbf{B}]_{L_0+l+2,n}^2 [\mathbf{A}_S]_{n,L_0+l+1}^2| < \infty,$$

where  $E[\cdot]$  is over the distribution of the delay and  $|\cdot|$  is the square of the  $l_2$ -norm. This condition can be shown for the particular choice of  $p_i(t)$  given in [9]. Similar analysis can be done for each of the diagonal elements of  $\mathbf{W}$  to show that they converge almost surely to the constant  $c$  in Theorem 2 and that concludes our proof. This proves that timing mismatches or phase imbalance does not degrade the demodulation process. ■

### B. Gain Mismatch Across Different Channels

Mathematically the filtered output with gain mismatch is given by

$$\tilde{\mathbf{y}}_{gm}(f) = \mathbf{A}_{gm}\mathbf{z}(f), \quad f \in [-f_s/2, f_s/2],$$

where  $[\mathbf{A}_{gm}]_{il} = [\mathbf{A}]_{il}(1 + g_i)$ , where  $g_i$  follows normal distribution with mean 0. As in the timing mismatch case, we can show similar results for gain imbalance.

*Theorem 3:* When the number of channels  $m$  is large and the gains  $g_i$  in different channels are independent and identically distributed with zero mean, then

$$(\mathbf{A}_S^H \mathbf{A}_S)^{-1} \mathbf{A}_S^H \mathbf{A}_{gmS} \xrightarrow{a.s.} \mathbf{I},$$

And thus gain mismatch also does not affect the demodulation process at all.

## IV. SIMULATION RESULTS

Random data-bits mapped to a 16-QAM constellation is selected as the test vector for the receiver. Table I presents the design parameters for both the conventional direct conversion receiver and the proposed receiver.

The systems are simulated for a SNR of 10dB, under varying imperfections. Fig. 3(a),(b) shows the received signal constellation with uniformly distributed timing or phase-error with  $\sigma = T_p/M$  between 2 channels for I/Q and between  $m$  channels for the proposed approach. Fig. 3(d),(e) extends this analysis for a 10% gain mismatch between the paths for both the cases. We define the demodulated SNR as  $SNR = 20 \log \|\mathbf{I} + j\mathbf{Q}\|_2 / \|(\mathbf{I} - \mathbf{I}_{rec}) + j(\mathbf{Q} - \mathbf{Q}_{rec})\|_2$ , where  $\mathbf{I}, \mathbf{Q}$  are the original bit streams and  $\mathbf{I}_{rec}, \mathbf{Q}_{rec}$  are their reconstructed counterparts in presence of additive noise. This SNR degradation with varying degrees of timing mismatch

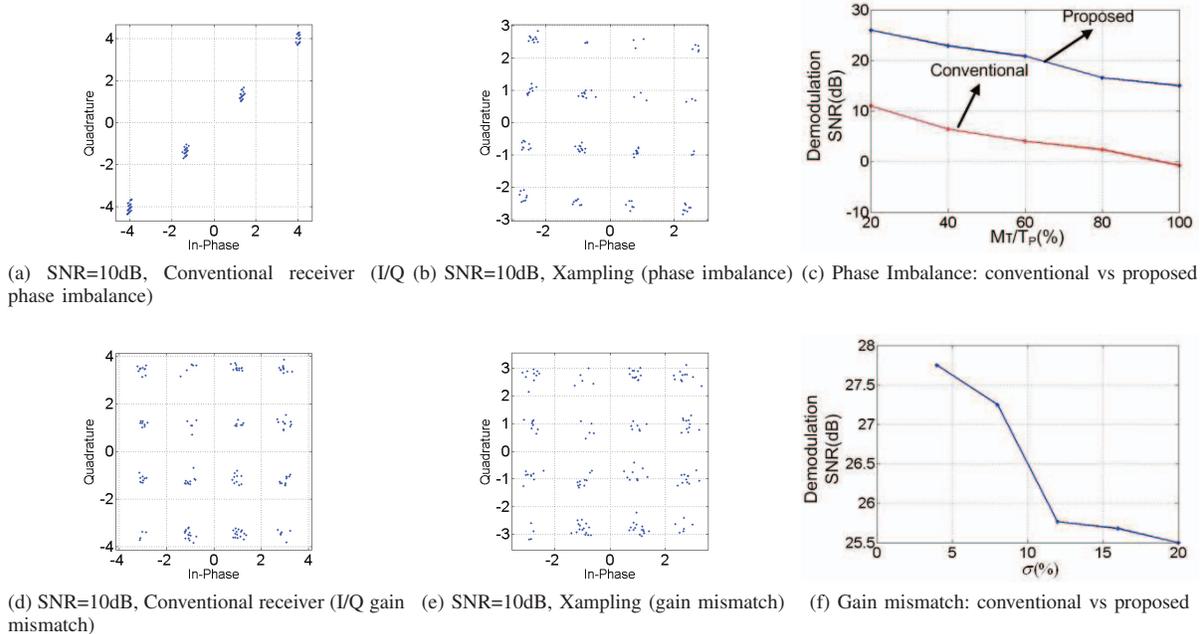


Fig. 3: Plots comparing conventional and proposed approach for analog imperfections

error for both the receivers is presented in Fig. 3(c) ( $\tau$  is the mismatch standard deviation in each of the channels). As can be clearly seen from Fig. 3(a), (b) and (c), the proposed solution is superior to the conventional analog mixing for similar all-white Gaussian noise (AWGN) added in both cases. Both the techniques show, however, a lot less sensitivity to the gain mismatch(es) between the parallel paths. Fig. 3(f) captures the demodulated SNR degradation for both (they have almost similar performances). Fig. 3(d) and (e) show the robustness of both the schemes to similar amounts of gain mismatches.

As can be appreciated from the figures, traditional reception techniques exhibit severe deterioration of performance under these errors. It should be understood that the basic sources of errors in both the approaches are the same (mismatch between parallel paths due to systematic routing skews, random mismatches etc.). On the other hand, the proposed approach is immune to these errors, attributed mostly to the averaging they undergo over multiple channels. In fact, as has been shown in the previous section, for a large number of channels (within practical limits), such errors tend almost to zero!

## V. CONCLUSION

A novel technique to tackle conventional analog mismatch errors in direct conversion receivers is presented. The robustness of the technique under similar error environments is mathematically derived. Simulation results proving the effectiveness of the technique are also presented. Besides the inaccuracies discussed here, the dynamic range requirements for the channel ADCs operating at a frequency of  $f_s$  (noise lineup between multiplier, filter and ADC) will also be important

considerations for the practical implementation of the scheme. Furthermore, the robustness of our approach comes at a price of additional hardware complexity. We would like to study this robustness vs complexity trade-off in the future, which will provide the optimal choice for the parameters ( $f_s$ ,  $f_p$ ,  $m$ ,  $p_i(t)$ ) to implement a truly power-efficient, calibration-free reception module.

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