JOINT LINEAR PRECODER OPTIMIZATION AND BASE STATION SELECTION FOR AN UPLINK MIMO NETWORK: A GAME THEORETIC APPROACH

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ABSTRACT

We consider the problem of weighted sum rate optimization in a MIMO interfering multiple access channel (IMAC). We propose to jointly optimize the users' linear procoders as well as their base station (BS) associations. This approach enables the users to avoid congested BSs and can improve system performance as well as user fairness. We formulate the problem into a noncooperative game, and develop an algorithm that allows the players to distributedly reach the Nash Equilibrium (NE) of the game. We show that every NE of the game is a stationary solution of the weighted sum rate optimization problem, and propose an algorithm to compute the NE of the game. Simulation results show that the proposed algorithm performs well in the presence of BS congestion.

Index Terms— MIMO, Base Station Assignment, Linear Precoder Design, Game Theory

1. INTRODUCTION

We consider a general MIMO-IMAC in which a set of transmitters send data to their intended BSs at the same time. Both the transmitters and the BSs are equipped with multiple antennas. Such channel model is important as it can accurately describe many practical networks, e.g., the uplink of multicell heterogenous networks, in which the transmitters may represent either mobile users or the relays. The MIMO-IMAC is a generalization of the MIMO interference channel (IC), in which each BS has only one associated transmitter.

The problem of optimal transceiver design of MIMO-IC has been extensively studied recently. The authors of [1], among others, formulate the transmission covariance matrix optimization problem into a noncooperative game, in which the transmitters/users compete with each other for transmission. Simple distributed algorithms with convergence guarantees are derived, but the outcome of the game is inefficient in terms of system performance. This is due to the lack of coordination among the transmitters/users. Instead of the competitive design, one can optimize the system performance measured by some suitable system utility functions. However, these problems have been proven to be NP-complete in general [2]. As a result, many authors focus on developing high quality algorithms to compute sub-optimal solutions for these problems. Reference [3] proposes a local linear approximation algorithm based on first order Taylor expansion of the weighted sum rate objective. The algorithm allows the transmitters/users to update their transmission covariance matrices by solving a series of convex optimization problems. However no convergence result has been given. Reference [4] proposes a weighted Minimum Mean Square Error (WMMSE) algorithm in which the transmitters and receivers iteratively update their linear transmission and receiving strategies to optimize the system utility function. The authors show that as long as the system utility function satisfies some regularity conditions, their algorithm is guaranteed to converge to a stationary point of the problem.

All the above cited works aim at optimizing the linear transceiver structures *assuming that the transmitter-receiver association is known and fixed*. In our considered IMAC setting though, BS assignment becomes an important optimization variable. In the future heterogenous network, the cell sizes will become smaller, and the deployment of access points such as macro/pico/femto BSs will become denser. In this network configuration, the traditional strongest BS association approach is insufficient for congestion management and fairness provisioning [5]. Consequently, optimal BS assignment becomes a crucial aspect in the overall system performance optimization. The problem of joint cell site selection and power allocation in the traditional CDMA based network has been first considered in [6] and later in a game theoretical perspective in [7]. Recently [8] has considered the joint BS selection and power allocation in an OFDM network in which the BSs operate on non-overlapping spectrum. Our considered MIMO-IMAC is a generalization of all the network settings of the above referenced work and their approaches do not apply here.

In this work we formulate the joint linear precoder optimization and BS selection problem in a game theoretical framework. In our formulation both the transmitters and the BSs are the players of the game. Each transmitter aims at finding the best linear precoder as well as the least congested BS for transmission. Each BS computes a set of optimal prices to charge the transmitters for causing interference. These prices serve to coordinate the behavior of the transmitters so that they do not cause excessive interference. One advantage of our game theoretical formulation is that it naturally incorporates the joint optimization of BS association and linear precoder into individual transmitters' optimization problem. Moreover, our formulation allows us to find desirable operation points of the network such that: 1) the system is stable, in the sense that no single transmitter is willing to change its BS association; 2) the transmission strategies of the users are efficient, in the sense that they achieve a local optimal solution of the overall weighted sum rate objective.

2. TRANSMIT COVARIANCE OPTIMIZATION GAME FOR FIXED USER-BS ASSOCIATION

We consider a general MIMO IMAC with a set $\mathcal{N} = \{1, \cdots, N\}$ of users/transmitters that transmit to a set $\mathcal{Q} = \{1, \cdots, Q\}$ of BSs. For clarity of presentation, in this section we only consider fixed network topology in which each user/transmitter has its fixed intended BS. The general case with flexible association will be discussed in the next section. Define a $N \times 1$ vector $\mathbf{a}$ to represent the system

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association profile, $\mathbf{a}_n = q$ means user n connects to BS q. We will refer to a transmitter as a user in the sequel.

Suppose each user $n \in \mathcal{N}$ has T_n transmit antennas and each BS $q \in \mathcal{Q}$ has R_q receive antennas. Let $\mathbf{H}_{q,n} \in \mathbb{C}^{R_q \times T_n}$ be a $R_q \times T_n$ channel matrix which represents the channel gain from transmitter n to receiver q. Assuming $T_n \le R_q$ for all $n \in \mathcal{N}$ and all $q \in \mathcal{Q}$, or equivalently the channel matrices $\{\mathbf{H}_{q,n}\}$ are tall matrices. This assumption is natural as the number of antennas at the BS shoule be larger than that of the mobile stations. Let $\mathbf{x}_n \in \mathbb{C}^{T_n}$ be the transmitted signal of user n. Let $\mathbf{y}_q \in \mathbb{C}^{R_q}$ be the received signal of BS q. Then $\mathbf{y}_q$ can be expressed as

$$\mathbf{y}_q = \sum_{n \in \mathcal{N}} \mathbf{H}_{q,n}\mathbf{x}_n + \mathbf{z}_q \tag{1}$$

where $\mathbf{z}_q \sim \mathcal{CN}(0, \sigma_q^2 \mathbf{I}_{R_q})$ is an additive white Gaussian noise vector. Define the data symbol vector for user n to be $\mathbf{s}_n \in \mathbb{C}^{T_n}$, i.e., there are a maximum of T_n streams for user n. We assume that a linear precoder $\mathbf{P}_n \in \mathbb{C}^{T_n \times T_n}$ is used for the transmission, and the transmitted vector of user n is $\mathbf{x}_n = \mathbf{P}_n \mathbf{s}_n$. Assume that the data symbols are independent and have unit variance, i.e., $E[\mathbf{s}_n \mathbf{s}_n^H] = \mathbf{I}_{T_n}$. Then the transmit covariance matrix $\mathbf{S}_n$ for user n is given by $\mathbf{S}_n \triangleq E[\mathbf{x}_n \mathbf{x}_n^H] = \mathbf{P}_n \mathbf{P}_n^H \in \mathbb{S}_+^{T_n}$, where $\mathbb{S}_+^{T_N}$ denote the set of $T_N \times T_N$ hermitian semi-definite matrices. Once the covariance matrix $\mathbf{S}_n$ is obtained, the precoder can be calculated by Cholesky decomposition. Define the system joint transmission covariance and the joint covariance excluding user n as: $\mathbf{S} \triangleq \{\mathbf{S}_n\}_{n \in \mathcal{N}}$, $\mathbf{S}_{-n} \triangleq \{\mathbf{S}_m\}_{m \neq n, m \in \mathcal{N}}$. Then the interference covariance matrix for user n (at its intended BS $\mathbf{a}_n$), denoted by $\mathbf{C}_n(\mathbf{S}_{-n})$, can be expressed as

$$\mathbf{C}_n(\mathbf{S}_{-n}) \triangleq \sigma_{\mathbf{a}_n}^2 \mathbf{I}_{R_{\mathbf{a}_n}} + \sum_{m \neq n} \mathbf{H}_{\mathbf{a}_n,m}\mathbf{S}_m \mathbf{H}_{\mathbf{a}_n,m}^H.$$

Assuming Gaussian signaling and treating the interference as noises, the achievable rate for user n is given by [9]

$$R_n(\mathbf{S}_n, \mathbf{S}_{-n}) = \log\left|\mathbf{I}_{R_{\mathbf{a}_n}} + \mathbf{H}_{\mathbf{a}_n,n}\mathbf{S}_n\mathbf{H}_{\mathbf{a}_n,n}^H \mathbf{C}_n^{-1}(\mathbf{S}_{-n})\right|. \tag{2}$$

The weighted system sum rate with the set of weights $\{w_n\}_{n=1}^N$ can be expressed as $R(\mathbf{S}) \triangleq \sum_{n \in \mathcal{N}} w_n R_n(\mathbf{S}_n, \mathbf{S}_{-n})$. We aim at designing a noncooperative game with efficient equilibrium solution in terms of the sum rate of the system. Given a set of individual average power constraints of of the form $E[\mathrm{Tr}(\mathbf{x}_n\mathbf{x}_n^H)] = \mathrm{Tr}(\mathbf{S}_n) \le \bar{p}_n$, the weighted sum rate maximization problem (WSRM) can be stated as

$$\max_{\mathbf{S}} R(\mathbf{S}) \tag{WSRM}$$
$$s.t.\ \mathrm{Tr}(\mathbf{S}_n) \le \bar{p}_n,\ \mathbf{S}_n \in \mathbb{S}_+^{T_n},\ \forall\, n \in \mathcal{N}.$$

We let each user $n \in \mathcal{N}$ have the ability to change its own $\mathbf{S}_n$. Define its feasible set as

$$\mathcal{F}_n \triangleq \{\mathbf{S}_n | \mathrm{Tr}(\mathbf{S}_n) \le \bar{p}_n,\ \mathbf{S}_n \in \mathbb{S}_+^{T_n}\}. \tag{3}$$

Define the joint feasible set of all the users as: $\mathcal{F} \triangleq \prod_{n \in \mathcal{N}} \mathcal{F}_n$.

Due to the fact that all the users share the same spectrum, their individual transmissions cause interference at the non-intended BSs. In order to mitigate such interference, we allow each BS $q \in \mathcal{Q}$ to post a (matrix valued) price $\mathbf{T}_{q,n} \in \mathbb{S}_+^{R_q}$ to each user $n \in \mathcal{N}$. Define $\mathcal{H}_q = \prod_{n \in \mathcal{N}} \mathbb{S}_+^{R_q}$ as the feasible set of BS q's pricing strategies. Define $\mathcal{H} \triangleq \prod_{q \in \mathcal{Q}} \mathcal{H}_q$ as the joint feasible set of all the BSs. Let $\mathbf{T}_q = \{\mathbf{T}_{q,n}\}_{n \in \mathcal{N}}$, $\mathbf{T}_n = \{\mathbf{T}_{q,n}\}_{q \in \mathcal{Q}}$, and $\mathbf{T} = \{\mathbf{T}_q\}_{q \in \mathcal{Q}}$.

We model the users and the BSs as selfish agents, who are interested in choosing the optimal strategies (transmission covariances for the users, and price matrices for the BSs) to maximize some properly defined utility functions. Let $U_n(.)$ and $D_q(.)$ denote the user n and BS q's utility functions, respectively. We formulate a covariance optimization game $\mathcal{G}^C$ as follows

$$\mathcal{G}^C \triangleq \left\{\{\mathcal{N}, \mathcal{Q}\}, \{\mathcal{F}, \mathcal{H}\}, \{\{U_n(.)\}_{n \in \mathcal{N}}, \{D_q(.)\}_{q \in \mathcal{Q}}\}\right\}.$$

The aim of the *game design* is to properly specify the utility functions so that the game $\mathcal{G}^C$ induces efficient equilibrium in terms of the weighted system sum rate.

We first investigate the structure of the interference prices. Define $\mathcal{N}_q \triangleq \{n : \mathbf{a}_n = q\}$ as the set of users associated with BS q. At a given system covariance profile $\mathbf{S} \in \mathcal{F}$, user n's marginal influence to the rates the set of users $m \in \mathcal{N}_q \setminus n$ is given by

$$\begin{aligned}
&\sum_{m \in \mathcal{N}_q \setminus n} \nabla_{\mathbf{S}_n} w_m R_m(\mathbf{S}_m, \mathbf{S}_{-m}) \\
&= \sum_{m \in \mathcal{N}_q \setminus n} w_m \mathbf{H}_{q,n}^H \mathbf{C}_m^{-1}(\mathbf{S}_{-m}) \left(\mathbf{I}_{R_q} + \mathbf{H}_{q,m}\mathbf{S}_m\mathbf{H}_{q,m}^H\mathbf{C}_m^{-1}(\mathbf{S}_{-m})\right)^{-1} \\
&\times \mathbf{H}_{q,m}\mathbf{S}_m\mathbf{H}_{q,m}^H \mathbf{C}_m^{-1}(\mathbf{S}_{-m})\mathbf{H}_{q,n}
\end{aligned} \tag{4}$$

It is then natural to let BS q charge each user $n \in \mathcal{N}$ with its marginal influence. We define BS q's utility function as

$$D_q(\mathbf{T}_q, \mathbf{T}_{-q}, \mathbf{S}) \triangleq -\sum_{n \in \mathcal{N}} \left|\left|\mathbf{T}_{q,n} - \sum_{m \in \mathcal{N}_q \setminus n} \nabla_{\mathbf{S}_n} w_m R_m(\mathbf{S}_m, \mathbf{S}_{-m})\right|\right|. \tag{5}$$

Clearly, for a fixed set of $\mathbf{S} \in \mathcal{F}$, the set of $\mathbf{T}_q$ that maximizes BS q's utility is of the following form

$$\mathbf{T}_{q,n}(\mathbf{S}) = \sum_{m \in \mathcal{N}_q \setminus n} \nabla_{\mathbf{S}_n} w_m R_m(\mathbf{S}_m, \mathbf{S}_{-m}). \tag{6}$$

For a set of fixed $\mathbf{S} \in \mathcal{F}$, the interference price $\mathbf{T}_{q,n}(\mathbf{S})$ can be computed locally at BS q. We will occasionally make the dependency of $\mathbf{T}_{q,n}$ on all the users' covariance matrices explicit, as in the above expression. We note that this pricing idea is a generalization of [10], which consider a simpler IC with parallel channels.

We define user n's utility function $U_n(.)$ as the difference between its transmission rate and the total prices charged by the BSs

$$U_n(\mathbf{S}_n, \mathbf{S}_{-n}, \mathbf{T}_n) = w_n R_n(\mathbf{S}_n, \mathbf{S}_{-n}) - \sum_{q \in \mathcal{Q}} \mathrm{Tr}\left[\mathbf{T}_{q,n}\mathbf{S}_n\right]. \tag{7}$$

This utility function is a concave log det plus linear function of $\mathbf{S}_n$. Finding the optimal covariance $\mathbf{S}_n^*$ (assuming $\mathbf{S}_{-n}$ and $\mathbf{T}_n$ are known and fixed) falls in the category of *determinant maximization problem* (MAXDET), which is a convex optimization problem and can be solved efficiently [11].

In the following we show that the above design of the utility functions gives us a nice relationship between the utility of the users and the weighted system sum rate.

Proposition 1 *For any $\mathbf{S}_n, \widehat{\mathbf{S}}_n \in \mathcal{F}_n$ and any $\widehat{\mathbf{S}}_{-n} \in \mathcal{F}_{-n}$, we have the following implication*

$$\begin{aligned}
&U_n(\mathbf{S}_n, \widehat{\mathbf{S}}_{-n}, \mathbf{T}_n(\widehat{\mathbf{S}})) - U_n(\widehat{\mathbf{S}}_n, \widehat{\mathbf{S}}_{-n}, \mathbf{T}_n(\widehat{\mathbf{S}})) > 0 \\
&\Longrightarrow R(\mathbf{S}_n, \widehat{\mathbf{S}}_{-n}) - R(\widehat{\mathbf{S}}_n, \widehat{\mathbf{S}}_{-n}) > 0.
\end{aligned} \tag{8}$$

The following lemma is needed to prove Proposition 1.

Lemma 1 *User m's achievable rate $R_m(\mathbf{S}_m, \mathbf{S}_{-m})$ is a convex function of $\mathbf{S}_n$ for each $n \neq m$.*

Let $\mathbb{S}^{T_n}$ denote the set of all $T_n \times T_n$ Hermitian matrices. Lemma 1 can be shown by checking that for all $\mathbf{D} \in \mathbb{S}^{T_n}$ and $\mathbf{S}_n + t\mathbf{D} \in \mathbb{S}_+^{T_n}$, the following function is convex in the constant t

$$R_m(t) \triangleq \log \left|\mathbf{I}_{R_q} + \mathbf{H}_{q,m}\mathbf{S}_m\mathbf{H}_{q,m}^H \left(\mathbf{C}_m(\mathbf{S}_{-m}) + t\mathbf{H}_{q,n}\mathbf{D}\mathbf{H}_{q,n}^H\right)^{-1}\right|.$$

Proposition 1 can be proved by the following main steps. Fixed any $\mathbf{S} \in \mathcal{F}$ and $\widehat{\mathbf{S}} \in \mathcal{F}$. Let $R_{-n}(\mathbf{S}_n, \mathbf{S}_{-n}) \triangleq \sum_{m \neq n} w_m R_m(\mathbf{S}_m, \mathbf{S}_{-m})$. Using Lemma 1, we can lower bound $R_{-n}(\mathbf{S}_n, \widehat{\mathbf{S}}_{-n})$ at $\widehat{\mathbf{S}}$ as

$$R_{-n}(\mathbf{S}_n, \widehat{\mathbf{S}}_{-n}) \geq \mathbf{L}_n(\widehat{\mathbf{S}}) - \sum_{q \in \mathcal{Q}} \text{Tr}\left[\mathbf{T}_{q,n}(\widehat{\mathbf{S}})\mathbf{S}_n\right] \quad (9)$$

where $\mathbf{L}_n(\widehat{\mathbf{S}})$ is a constant that is only related to $\widehat{\mathbf{S}}$. The equality in (9) is achieved when $\mathbf{S}_n = \widehat{\mathbf{S}}_n$. Utilizing (9), we can show

$$\begin{aligned} &U_n(\mathbf{S}_n, \widehat{\mathbf{S}}_{-n}, \mathbf{T}_n(\widehat{\mathbf{S}})) - U_n(\widehat{\mathbf{S}}_n, \widehat{\mathbf{S}}_{-n}, \mathbf{T}_n(\widehat{\mathbf{S}})) \\ &\leq R(\mathbf{S}_n, \widehat{\mathbf{S}}_{-n}) - R(\widehat{\mathbf{S}}_n, \widehat{\mathbf{S}}_{-n}). \end{aligned} \quad (10)$$

Then (8) immediately follows.

We mention that the property (8) is reminiscent of the *generalized potential property* [1] for a wide range of games referred to as *Potential Games* [12], with the subtle difference that in (8) the implication is dependent upon a "state variable" $\mathbf{T}_n(\widehat{\mathbf{S}})$.

Now that we have a complete characterization of the utility functions, we are ready to investigate the properties of the pure NE of the game $\mathcal{G}^C$. A pure strategy NE of the game $\mathcal{G}^C$ is a tuple of strategies $\{\mathbf{S}^*, \mathbf{T}^*\}$ such that the following set of inequalities are satisfied

$$\begin{aligned} &U_n(\mathbf{S}_n^*, \mathbf{S}_{-n}^*, \mathbf{T}_n^*) \geq U_n(\mathbf{S}_n, \mathbf{S}_{-n}^*, \mathbf{T}_n^*), \ \forall\, \mathbf{S}_n \in \mathcal{F}_n, \ \forall\, n \in \mathcal{N} \\ &D_n(\mathbf{T}_q^*, \mathbf{T}_{-q}^*, \mathbf{S}^*) \geq D_n(\mathbf{T}_q, \mathbf{T}_{-q}^*, \mathbf{S}^*), \ \forall\, \mathbf{T}_q \in \mathcal{H}_q, \ \forall\, q \in \mathcal{Q}. \end{aligned}$$

By utilizing Proposition 1, we have the following characterization of the NEs of game $\mathcal{G}^C$.

Theorem 1 *$(\mathbf{S}^*, \mathbf{T}^*)$ is a NE of the game $\mathcal{G}^C$ if and only if $\mathbf{S}^*$ is a KKT point of the problem (WSRM).*

3. JOINT BS SELECTION AND TRANSMIT COVARIANCE OPTIMIZATION GAME

When allowing each user $n \in \mathcal{N}$ to have the ability to optimize both its transmission covariance and BS association, we define its joint strategy as $\mathbf{J}_n \triangleq (\mathbf{S}_n, \mathbf{a}_n)$, and define its feasible space as $\mathcal{J}_n = \mathcal{F}_n \times \mathcal{Q}$. Let $\mathbf{J}_{-n} \triangleq (\mathbf{S}_{-n}, \mathbf{a}_{-n})$, and $\mathbf{J} \triangleq \{\mathbf{J}_n\}_{n \in \mathcal{N}}$. In this case, each user's rate is still defined as in (2), but we have to make the dependency of association profile explicit. We use $R_n(\mathbf{J}_n, \mathbf{J}_{-n})$ to denote user n's rate. Let $\mathcal{N}_q(\mathbf{a})$ denote the set of users associated with BS q under association profile $\mathbf{a}$. Moreover, the sum rate maximization problem is also dependent on the underlying user-BS association. We use WSRM($\mathbf{a}$) to indicate such dependency.

Let $\bar{U}_n(.)$ and $\bar{D}_q(.)$ denote user n and BS q's utility functions, respectively. The joint BS selection and precoder optimization game $\mathcal{G}^J$ is defined as

$$\mathcal{G}^J \triangleq \left\{\{\mathcal{N}, \mathcal{Q}\}, \{\mathcal{J}, \mathcal{H}\}, \{\{\bar{U}_n(\mathbf{J}, \mathbf{T}_n)\}_{n \in \mathcal{N}}, \{\bar{D}_q(\mathbf{T}, \mathbf{J})\}_{q \in \mathcal{Q}}\right\}.$$

[1]The generalized potential property is referred to the following relationship between the users' utility function and a "potential function" $P(.)$: let $\mathbf{x}_n$ be player n's action profile; for any two $\widehat{\mathbf{x}}_n, \mathbf{x}_n \in \chi_n$, for all $\mathbf{x}_{-n} \in \chi_{-n}$, and for all player n, $U_n(\widehat{\mathbf{x}}_n, \mathbf{x}_{-n}) - U_n(\mathbf{x}_n, \mathbf{x}_{-n}) > 0$ implies $P(\widehat{\mathbf{x}}_n, \mathbf{x}_{-n}) - P(\mathbf{x}_n, \mathbf{x}_{-n}) > 0$.

We refer to the game $\mathcal{G}^J$ as a *hybrid game*, because the strategies of a subset of the players consist of a continuous matrix and a discrete index. We define the utility functions $\bar{U}_n(.)$ and $\bar{D}_q(.)$ similarly as in (7) and (5)

$$\bar{U}_n(\mathbf{J}_n, \mathbf{J}_{-n}, \mathbf{T}_n) \triangleq R_n(\mathbf{J}_n, \mathbf{J}_{-n}) - \sum_{q \in \mathcal{Q}} \text{Tr}\left[\mathbf{T}_{q,n}\mathbf{S}_n\right].$$

$$\bar{D}_q(\mathbf{T}_q, \mathbf{T}_{-q}, \mathbf{J}) \triangleq -\sum_{n \in \mathcal{N}} \left|\left|\mathbf{T}_{q,n} - \sum_{m \in \mathcal{N}_q(\mathbf{a}) \setminus n} \nabla_{\mathbf{S}_n} w_m R_m(\mathbf{J}_m, \mathbf{J}_{-m})\right|\right|.$$

Note that both utility functions defined above are *dependent* on the user-BS association vector $\mathbf{a}$. On the one hand user n's transmission rate is different when it associates to different BSs. On the other hand BS q charges a user n for the interference caused to all the users *that are currently associated to it*. Similarly to the previous case, in order to emphasize the relationship between the optimal solution of BS q and the users' strategies $\mathbf{J}$, we occasionally use $\mathbf{T}_{q,n}(\mathbf{J})$ or $\mathbf{T}_{q,n}(\mathbf{S}, \mathbf{a})$ to denote the optimal prices charged by BS q to user n via maximizing its utility.

The pure NE of the game $\mathcal{G}^J$ is the tuple $(\mathbf{J}^*, \mathbf{T}^*)$ that satisfies

$$\begin{aligned} &\bar{U}_n\left(\mathbf{J}_n^*, \mathbf{J}_{-n}^* \mathbf{T}_n^*\right) \geq \bar{U}_n\left(\mathbf{J}_n, \mathbf{J}_{-n}^*, \mathbf{T}_n^*\right), \ \forall\, \mathbf{J}_n \in \mathcal{J}_n, \ \forall\, n \in \mathcal{N} \\ &\bar{D}_q(\mathbf{T}_q^*, \mathbf{T}_{-q}^*, \mathbf{J}^*) \geq \bar{D}_q(\mathbf{T}_q, \mathbf{T}_{-q}^*, \mathbf{J}^*), \ \forall\, \mathbf{T}_q \in \mathcal{H}_q, \ \forall\, q \in \mathcal{Q}. \end{aligned}$$

The following proposition is instrumental in characterizing the pure NE of game $\mathcal{G}^J$.

Proposition 2 *For any $\mathbf{J}_n, \widehat{\mathbf{J}}_n \in \mathcal{J}_n$ and $\widehat{\mathbf{J}}_{-n} \in \mathcal{J}_{-n}$, we have that*

$$\bar{U}_n\left(\mathbf{J}_n, \widehat{\mathbf{J}}_{-n}\mathbf{T}_n(\widehat{\mathbf{J}})\right) - \bar{U}_n\left(\widehat{\mathbf{J}}_n, \widehat{\mathbf{J}}_{-n}\mathbf{T}_n(\widehat{\mathbf{J}})\right) > 0 \quad (11)$$

$$\Longrightarrow R\left(\mathbf{J}_n, \widehat{\mathbf{J}}_{-n}\right) - R\left(\widehat{\mathbf{J}}_n, \widehat{\mathbf{J}}_{-n}\right) > 0. \quad (12)$$

Proof (sketch): The following identities can be proved

$$R_m((\widehat{\mathbf{S}}_n, \mathbf{a}_n), \widehat{\mathbf{J}}_{-n}) = R_m((\widehat{\mathbf{S}}_n, \widehat{\mathbf{a}}_n), \widehat{\mathbf{J}}_{-n}), \ \forall\, m \neq n \quad (13)$$

$$\mathbf{T}_n(\widehat{\mathbf{S}}, \widehat{\mathbf{a}}) = \mathbf{T}_n(\widehat{\mathbf{S}}, [\mathbf{a}_n, \widehat{\mathbf{a}}_{-n}]). \quad (14)$$

This set of equations says that if user n unilaterally switches from BS $\widehat{\mathbf{a}}_n$ to $\mathbf{a}_n$ but keeps its covariance matrix unchanged, then all other users' transmission rates as well as the price charged for user n do not change. Using (14) and (10), we can show that

$$\begin{aligned} &\bar{U}_n\left((\mathbf{S}_n, \mathbf{a}_n), \widehat{\mathbf{J}}_{-n}, \mathbf{T}_n(\widehat{\mathbf{S}}, \widehat{\mathbf{a}})\right) - \bar{U}_n\left((\widehat{\mathbf{S}}_n, \mathbf{a}_n), \widehat{\mathbf{J}}_{-n}, \mathbf{T}_n(\widehat{\mathbf{S}}, [\mathbf{a}_n, \widehat{\mathbf{a}}_{-n}])\right) \\ &\leq R((\mathbf{S}_n, \mathbf{a}_n), \widehat{\mathbf{J}}_{-n}) - R((\widehat{\mathbf{S}}_n, \mathbf{a}_n), \widehat{\mathbf{J}}_{-n}) \end{aligned} \quad (15)$$

Utilizing (13) and (14), we can show that

$$\begin{aligned} &\bar{U}_n\left((\widehat{\mathbf{S}}_n, \mathbf{a}_n), \widehat{\mathbf{J}}_{-n}, \mathbf{T}_n(\widehat{\mathbf{S}}, [\mathbf{a}_n, \widehat{\mathbf{a}}_{-n}])\right) - \bar{U}_n\left((\widehat{\mathbf{S}}_n, \widehat{\mathbf{a}}_n), \widehat{\mathbf{J}}_{-n}, \mathbf{T}_n(\widehat{\mathbf{S}}, \widehat{\mathbf{a}})\right) \\ &= R_n((\widehat{\mathbf{S}}_n, \mathbf{a}_n), \widehat{\mathbf{J}}_{-n}) - R_n((\widehat{\mathbf{S}}_n, \widehat{\mathbf{a}}_n), \widehat{\mathbf{J}}_{-n}) \\ &= R((\widehat{\mathbf{S}}_n, \mathbf{a}_n), \widehat{\mathbf{J}}_{-n}) - R((\widehat{\mathbf{S}}_n, \widehat{\mathbf{a}}_n), \widehat{\mathbf{J}}_{-n}). \end{aligned} \quad (16)$$

Combining (15) and (16), we obtain the desired inequality. ■

Due to the hybrid structure of the strategy space of the users, conventional existence results of the NE for a N-person concave game do not apply here. Proposition 2 is used to derive the following existence result of the NE of game $\mathcal{G}^J$.

Theorem 2 *The game $\mathcal{G}^J$ must admit at least a pure NE. Moreover, if $(\mathbf{S}^*, \mathbf{a}^*, \mathbf{T}^*)$ is a NE of the game $\mathcal{G}^J$, then $\mathbf{S}^*$ must be a KKT solution of the problem $WSRM(\mathbf{a}^*)$.*

S1) **Initialization**: Let $t = 0$, each user $n \in \mathcal{N}$ randomly choose $\mathbf{J}_n^0 \in \mathcal{J}_n$; each BS $q \in \mathcal{N}$ set $\mathbf{T}_q^0 = \mathbf{0}$.

S2) **Message Passing**: Randomly choose a user $n \in \mathcal{N}$ to act at time t. Each BS $q \in \mathcal{Q}$, $q \neq \mathbf{a}_n^t$ sends the price term $\mathbf{T}_{q,n}^t$ for user n to BS $\mathbf{a}_n^t$. The BS $\mathbf{a}_n^t$ feeds user n the sum of all prices $\sum_{q\in\mathcal{Q}} \mathbf{T}_{q,n}^t$.

S3) **User Utility Maximization**: User n computes its best-reply strategy $\mathbf{J}_n^{t+1}$ by solving:
$(\mathbf{S}_n^{t+1}, \mathbf{a}_n^{t+1}) = \arg\max_{q\in\mathcal{Q}} \max_{\mathbf{S}_n\in\mathcal{F}_n} \bar{U}_n((\mathbf{S}_n, q), \mathbf{S}_{-n}^t, \mathbf{a}_{-n}^t, \mathbf{T}_n^t)$.
For the rest of users $m \neq n$, set $\mathbf{J}_m^{t+1} = \mathbf{J}_m^t$.

S4) **BS Utility Maximization**: Each BS $q \in \mathcal{Q}$ updates its price matrices by $\mathbf{T}_{q,n}^{t+1} = \sum_{m\in\mathcal{N}_q(\mathbf{a}^{t+1}), m\neq n} \nabla_{\mathbf{S}_n} w_m R_m(\mathbf{S}_m^{t+1}, \mathbf{S}_{-m}^{t+1}), \forall\, n \in \mathcal{N}$

S5) **Continue**: Set $t = t + 1$, go to S2) unless some stopping criteria is met.

Table 1. The Proposed Algorithm

In Table 1, we propose an algorithm that allows the entities in the network to distributedly reach the NEs of $\mathcal{G}^J$.

The optimization in S3) can be performed locally by user n in the following steps: a) for each BS $q \in \mathcal{Q}$, obtain the current interference levels $\sigma_q^2 \mathbf{I}_q + \sum_{m\neq n} \mathbf{H}_{q,m}\mathbf{S}_m^t\mathbf{H}_{q,m}^H$; b) solve Q inner optimization problems using this interference information as well as the current total price $\sum_{q\in\mathcal{Q}} \mathbf{T}_{q,n}^t$ (note that each of these problems is again a MAXDET problem); c) pick the best BS in terms of the optimal value of the inner problem.

We have the following result regarding to the convergence of the above algorithm.

Theorem 3 *The sequence* $\{R(\mathbf{S}^t, \mathbf{a}^t)\}_{t=1}^{\infty}$ *generated by the proposed algorithm is monotonically increasing and always converges. Any limit point of* $\{\mathbf{S}^t, \mathbf{a}^t, \mathbf{T}^t\}_{t=1}^{\infty}$ *is a NE of the game* $\mathcal{G}^J$.

4. NUMERICAL RESULTS

In this section, we compare the performance of the proposed algorithm with the WMMSE algorithm proposed in [4]. We consider a network with 7 BSs and 16 users. The distance between adjacent BSs is 200 meters (representing a dense network of small cell size). Let $d_{q,n}$ be the distance between BS q and user n. The entries of the channel $\mathbf{H}_{q,n}$ are generated from distribution $\mathcal{CN}(0, \sigma_{q,n}^2)$. The standard deviation is calculated by $\sigma_{q,n} = (200/d_{q,n})^{3.5} L_{q,n}$, where $10\log 10(L_{q,n}) \sim \mathcal{N}(0, 8)$ models the shadowing effect. We fix the environment noise power as $\sigma_q = 1$ for all $q \in \mathcal{Q}$, and define the SNR as $10\log_{10} \bar{p}_n$.

We focus on the situation where the users are all located at the cell edges, and one of the BSs is congested. We place half of the users uniformly at the cell edges of BS 1, which is within $d_{m,i} \in [90,\ 100]$ meters of BS 1. We place the rest of the users randomly at the cell edges of other BSs. For the WMMSE algorithm, we let the users associate to the BSs with the highest channel magnitude (in terms of the 2-norm of the channel matrices). For our proposed algorithm, we limit the users to be able to choose their association among the three strongest BSs.

Fig. 1 demonstrates the performance of the proposed algorithm in terms of the system sum rate (with $w_n = 1$ for all n). Each point on this figure is averaged over 100 random generation of users' positions and channel coefficients. It can be seen that the proposed algorithm achieves higher sum rate than the WMMSE algorithm. Fig. 2 compares the CDF of the individual rates of the two algorithms when SNR=30dB. Our simulation shows that the joint linear precoder optimization and BS selection can achieve higher degree of user fairness than the WMMSE algorithm.

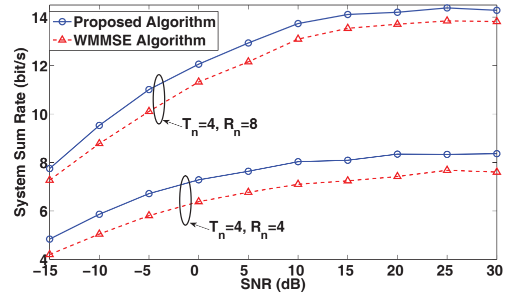

Fig. 1. Comparison of the sum rate of the proposed algorithm and the WMMSE algorithm.

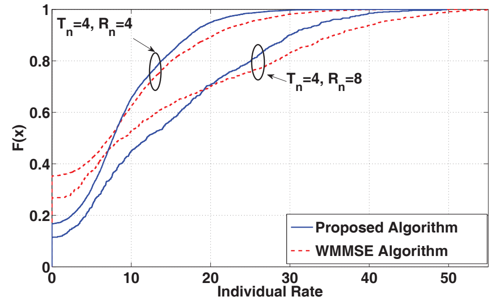

Fig. 2. Comparison of the CDF of the users' rates of the proposed algorithm and the WMMSE algorithm.

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