REDUCED COMPLEXITY MULTIMODE ANTENNA SELECTION WITH BIT ALLOCATION FOR ZERO-FORCING RECEIVER

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ABSTRACT

In this paper, we investigate multimode antenna selection for zero forcing receiver to maximize the overall data rate. The optimal selection scenario can be achieved by exhaustive search. However, antenna selection using exhaustive search leads to complicated computational burden. To reduce the complexity, we propose a greedy search algorithm for antenna selection. Using the proposed algorithm, the computations can be greatly reduced while the achievable data rate is nearly the same with exhaustive search. Moreover, generally fixed bit budgets are used in practical design. Hence, we propose to use water-filling bit allocation to further improve the performance of the proposed antenna selection scheme. Simulation results are provided to show the advantages of the proposed multimode antenna selection with bit allocation.

***Index Terms*—** MIMO systems, multimode antenna selection, zero-forcing receiver, bit allocation.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques have been widely employed in wireless systems to increase channel capacity without increasing transmit power and bandwidth. The transmission data rate can be increased by transmitting different data streams in different transmit antennas. Researches have been conducted to increase transmission data rate in MIMO systems. For instance, MIMO precoders were designed to maximize the achievable data rate in [1]. Also, for a fixed number of data streams, maximizing data rate via antenna selection has been extensively discussed.

To maximize transmission data rate, intuitively we should use all transmit antennas and each antenna bears different data stream. However, to relax the design effort for power amplifier, equal power assignment for individual data streams is usually used in practical design; in equal power scenario, using all transmit antennas does not always lead to the best performance. That is, some transmit antennas may have bad conditions; in this case, redistributing the power originally for antennas with bad conditions to the antennas with good conditions can significantly improve the overall performance, thus selecting proper transmit antennas is needed. This concept was first discovered in [2]. Similar concept to select proper transmit antennas to minimize vector symbol error rate (VSER) was proposed by [3]; and the concept to select proper transmit antennas was called *multimode antenna selection* in [3].

Multimode antenna selection provides additional array gain. The selection criteria in [3] minimizes the nearest neighbor union bound (NNUB) so as to minimize VSER for linear receivers. Several related topics have been discussed for different criteria. In [1], precoding with codebook design together with multimode antenna selection was proposed to minimize the probability of error and maximize the mutual information for independent and identically distributed (i.i.d.) Gaussian signaling.

In this paper, we investigate multimode antenna selection so as to maximize the achievable data rate for the popular zero-forcing (ZF) receiver with equal power scenario. We found that the optimal performance can be achieved by using the exhaustive search to select the best transmit antenna set. However, search exhaustively leads to extremely high computational complexity when the number of transmit antenna is large. To relax the computational burden, we propose a greedy search algorithm for multimode antenna selection in ZF receiver. With the proposed algorithm, the complexity can be greatly reduced while the achievable data rate remains nearly the same compared to the exhaustive search scheme. Moreover, in practical systems, predetermined integer bit budgets are usually used rather than using arbitrary bit rates. Instead of using the equal bit allocation for individual data streams as that in [3], we propose to use water-filling bit allocation in the proposed greedy multimode antenna selection for ZF receiver. The proposed bit allocation can effectively equalize the symbol error rate for individual data streams. As a result, the proposed scheme outperforms the scheme in [3] in terms of vector symbol error rate. Simulation results are provided to show the complexity advantage and performance improvement of the proposed greedy multimode antenna selection and the bit allocation scheme for ZF receiver.

2. SYSTEM MODEL

Consider a MIMO system with M_t transmit antennas and M_r receive antennas shown in Fig. 1. The system consists of a transmitter, a matrix that is a function of wireless environment, and a receiver. The transmitter consists of a spatial multiplexer that produces M-dimensional symbol vector and a symbol mapper that maps the M-dimensional symbol vector to the selected antennas. The receiver consists of a ZF receiver and a symbol detector. The low-rate feedback path sends back the information of the number M of substreams and the index p of selected antenna set to spatial multiplexer and symbol mapper respectively.

Each time R bits is demultiplexed into M substreams, and modulated independently using quadrature-amplitude modulation (QAM). The number of allocated bits for individual substreams can be different. The symbol vector $\mathbf{s}$ is $(s_1\ s_2 \ldots\ s_M)^T$. Assume the total transmit power is normalized to one, and each substream has equal power, *i.e.* $E_s[\mathbf{s}\mathbf{s}^H] = (1/M)\mathbf{I}_M$, where $\mathbf{I}_M$ is an $M \times M$ identity matrix. The symbol vector $\mathbf{s}$ is mapped to M antennas. Let $\mathcal{W}_M$ be a set of $\binom{M_t}{M}$ submatrices obtained by selecting M columns from the $M_t \times M_t$ identity matrix. For each M, $\mathcal{W}_M$ can be written as $\left\{\mathbf{W}_{M,1}, \cdots, \mathbf{W}_{M,\binom{M_t}{M}}\right\}$, where $\mathbf{W}_{M,p}$ is the symbol mapper [3] which maps the substreams to the selected antennas; the sub-

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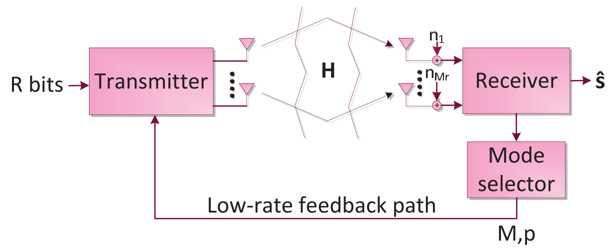


Fig. 1. A block diagram of system model.

scripts p represents the index of selected transmit antennas and M represents the number of substreams or called *mode* in [3].

All elements of the MIMO channel $\mathbf{H}$ are i.i.d. complex Gaussian distributed with zero mean. Define the equivalent channel as $\mathbf{H}_{eq} = \mathbf{H}\mathbf{W}_{M,p}$, the received symbol vector is

$$\mathbf{y} = \mathbf{H}_{eq} \cdot \mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is the noise vector. At the receiver, ZF receiver $\mathbf{G} = (\mathbf{H}\mathbf{W}_{M,p})^{\dagger}$ is used to process the received vector $\mathbf{y}$ [4]. Then, the input of symbol detector is $\mathbf{G} \cdot \mathbf{y}$ and the output is $\hat{\mathbf{s}} = \mathbf{s} + \mathbf{G} \cdot \mathbf{n}$.

3. MULTIMODE ANTENNA SELECTION TO MAXIMIZE ACHIEVABLE DATA RATE

We select the optimal transmit antenna set to maximize the achievable data rate. Suppose that a $2b$-bit-QAM symbol with power $\mathcal{E}_s$ is transmitted through a zero-mean complex additive white Gaussian noise (AWGN) channel with noise variance N_0. A bound for SER of QAM symbol is then given by [4]

$$P_{e,2b-\mathrm{QAM}} \leq 4(1 - \frac{1}{2^b})Q(\sqrt{\frac{3\mathcal{E}_s}{(2^{2b}-1)N_0}}), \tag{2}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp\left(-\frac{\tau^2}{2}\right) d\tau$ is Gaussian-Q function. For large number of 2^{2b} and high SNR per bit, the upper bound given in (2) is quite tight. By rearranging (2), we obtain the data rate of a QAM symbol as [4]

$$b = \log_2\left(1 + \frac{\gamma_0}{\Gamma}\right), \tag{3}$$

where $\gamma_0 = \mathcal{E}_s/N_0$ and Γ is the SNR gap defined as $\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{\mathrm{SER}}{4}\right)\right]^2$, for small enough symbol error rate (SER). In our system model, the maximum achievable data rate for a substream in (3) is a function of the post-processing SNR, which is the SNR at the detector. For the ZF receiver, the post-processing SNR for the i-th substream can be expressed as [5]

$$\mathrm{SNR}_i^{(ZF)} = \gamma_0 \frac{1}{\left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]_{i,i}^{-1}}, \tag{4}$$

where $\mathbf{A}_{i,i}^{-1}$ is entry (i,i) of $\mathbf{A}^{-1}$. Using (4) and under the high bit rate assumption ($b_i \gg 1$), we have

$$\begin{aligned} b_i &= \log_2\left(\frac{\mathcal{E}_s}{\Gamma\left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]_{i,i}^{-1} N_0}\right) \\ &= \log_2 \mathcal{E}_s - \log_2 \Gamma - \log_2 \left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]_{i,i}^{-1} - \log_2 N_0 \\ &= \log_2 \mathcal{E}_s - \log_2 N_0\Gamma - \log_2 \left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]_{i,i}^{-1}. \end{aligned} \tag{5}$$

Let us illustrate (5) in Fig. 2, which is the bit allocation for the ith substream under equal substream power arrangement. For description convenience, let $a_i = \log_2 N_0\Gamma + \log_2\left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]_{i,i}^{-1}$. It is like pouring water into a tank. The water level D in Fig. 2 is the logarithm of substream power, *i.e.* $\log_2 \mathcal{E}_s$. The subscripts of a and b represent the indices of substreams. With uneven floor a_i and the water level D, b_i bits are allocated to achieve water height D for the i-th substream. Therefore, for an arbitrary SER for each substream,

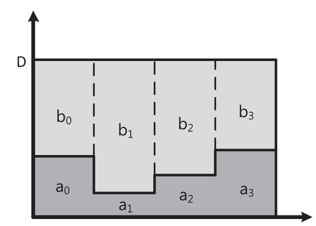


Fig. 2. A water-filling interpretation of the optimal bit allocation.

the problem of maximizing the achievable data rate subject to the ZF constraint can be written as follows:

$$\begin{aligned} B &= \max_{M=M^*,p=p^*} \sum_{i=1}^{M} b_i \\ &= \max_{M=M^*,p=p^*} \sum_{i=1}^{M} \log_2\left(1 + \frac{\gamma_0}{\Gamma\left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]_{i,i}^{-1}}\right). \end{aligned} \tag{6}$$

With transmit antenna selection in (6), the achievable transmission data rate can be more than that without antenna selection; the reason is that the transmit power for a data stream is limited to one, and the power of each substream is averaged and decreased as the number of transmit antennas is increased. To allocate the power efficiently, sometimes the worst few substreams should be dropped. Then, the power originally for the dropped antennas can be redistributed to all the selected antennas to improve the performance. Note that B is a function of M and p. The optimal selection can be achieved by performing exhaustive search.

3.1. Antenna Selection by Exhaustive Search

Exhaustive search is a general problem-solving technique that computes all possible sets for the solution and decide which set matches the objective function. For the optimal multimode antenna selection of maximizing achievable data rate in (6), we calculate the achievable data rates under the ZF constraint for all possible equivalent channels. The equivalent channels $\mathbf{H}_{eq}$ are calculated by all possible symbol mappers $\mathbf{W}_{M,p}$ from the sets $\mathcal{W}_M$, where $1 \leq M \leq M_t$.

The computational complexity is dominated by the number of transmit antennas. As the number of transmit antennas increases, the complexity soon becomes prohibiting. To overcome the complexity issue, we propose a simplified selection algorithm as follows.

3.2. Antenna Selection by Proposed Greedy Search

The proposed greedy search algorithm is described in Algorithm 1. In Step 1, let $\mathcal{S}$ be the index set of transmit antennas. The sum rate for mode M_t is obtained in Step 2. In Step 4 , we remove one antenna from current antenna set, and the corresponding sum rate is calculated to obtain the maximal sum rate for mode M. In Steps 5-7, if the maximal data rate for mode M is larger than mode $(M+1)$, the set $\mathcal{P}$ is replaced by $\mathcal{S}_m$, where m is the index of removed antenna. In Step 8, we remove one antenna according to the result in Step 4 that leads to the maximal sum rate in mode M, and $\mathcal{S}$ can be used in next iteration. Steps 3-9 are repeated until the number of elements in $\mathcal{S}$ is less than two. The transmit antenna set $\mathcal{P}$ is determined in Step 10.

Algorithm 1: Proposed greedy multimode antenna selection to maximize data rate.

1: Define the transmit antenna index set $\mathcal{S} = \{1, 2, \cdots, M_t\}$.
2: Obtain the initial sum rate $B(\mathcal{S})$ for mode $M = M_t$ by (3).
3: **while** $|\mathcal{S}| \geq 2$ **do**
4: $\quad m = \arg\max_r B(\mathcal{S}_r)$, where $\mathcal{S}_r = \mathcal{S} - \{r\}, r \in \mathcal{S}$.
5: $\quad$ **if** $B(\mathcal{S}_m) > B(\mathcal{S})$ **then**
6: $\quad\quad \mathcal{P} = \mathcal{S}_m$.
7: $\quad$ **end if**
8: $\quad \mathcal{S} = \mathcal{S}_m$.
9: **end while**
10: The desired transmit antenna set is $\mathcal{P}$.

Fig. 3 shows the comparison of computational complexity between exhaustive search and proposed greedy search. The computational complexity using Algorithm 1 (greedy search) is dramatically reduced compared to that using exhaustive search.

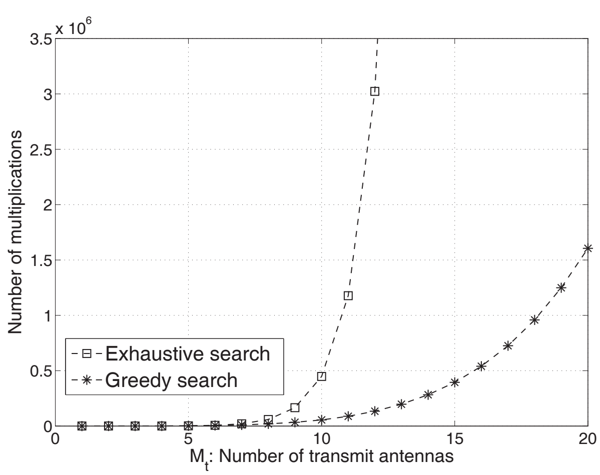


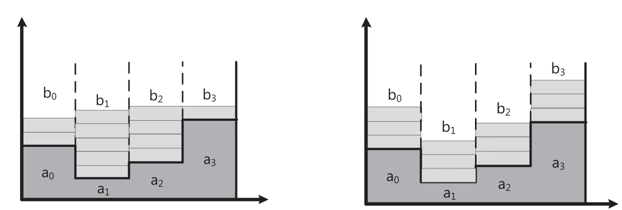
Fig. 3. Comparison of computational complexity for exhaustive search and proposed greedy selection.

4. BIT ALLOCATION FOR GREEDY SEARCH WITH FIXED BIT BUDGET

In previous section, we discuss how to select transmit antennas to maximize achievable data rate. The determined data rate can be non-integer value. In practice, however, we generally have an integer bit budget for the data rate. In this case, integer bit allocation is needed. Let us introduce how to allocate bits to all substreams for a given integer bit budget as follows: When the transmit antenna set is determined (exhaustive search or the proposed greedy search can be used), the available bits are allocated to substreams by water-filling algorithm [4] under a fixed bit budget instead of allocating equal number of bits to all substreams in [3]. The bit allocation aims to equalize the SER for individual substreams. It turns out the equalized SER of each substream significantly improve the VSER (vector SER), where the VSER is the probability that at least one substream is in error. The water-filling bit allocation is summarized in Algorithm 2. In Step 1, the height of floor a_i is obtained. In Step 3, the index of the substream which has the lowest floor height is obtained as j. Then in Step 4, the j-th substream is allocated one bit. After adding one bit to the j-th substream, the height of floor a_j is added one bit in Step 5. Steps 2-6 are repeated until the number of allocated bits is equal to the given bit budget R.

Algorithm 2: The water-filling algorithm for optimal bit allocation.

1: Let R be the bit budget and let a_i be calculated by
$\quad a_i = \log_2 N_0\Gamma + \log_2 \left[\mathbf{H}_{eq}^H \mathbf{H}_{eq}\right]_{i,i}^{-1}$.
2: **for** $k = 1 : R$ **do**
3: $\quad j = \arg\min_{1 \leq i \leq M^*} a_i$.
4: $\quad b_j \leftarrow b_j + 1$.
5: $\quad a_j \leftarrow a_j + 1$.
6: **end for**
7: b_j for $1 \leq i \leq M$ is the target bit allocation.



(a) (b)

Fig. 4. Interpretation of (a) water-filling bit allocation with $R = 12$, and (b) equal bit allocation with $R = 12$.

Fig. 4(a) and Fig. 4(b) illustrate water-filling bit allocation and equal bit allocation with $R = 12$, respectively. The power of each substream is equalized to $1/M$ no matter how the bits are allocated. From (5), we can see that the water level D which is related to the substream power is $(a_i + b_i)$. In Fig. 4(a), b_i is related to a_i. Using Algorithm 1, $(a_i + b_i)$ are nearly the same for all the substreams, which leads nearly the same SER for all substreams due to the assumption of equal substream power. However in Fig. 4(b), every substream has the same allocated bits and it results in different water levels, *e.g.* the fourth substream has much higher floor than that of the second substream. Thus the water level of the fourth substream is much higher than the second substream. In this case, the fourth substream leads to the highest SER among all the substreams and this degrades VSER.

5. SIMULATION RESULTS

We show the Monte Carlo simulation results of the proposed antenna selection with bit allocation. The channel coefficients have i.i.d. complex Gaussian distribution $CN(0,1)$. 10000 MIMO channel realizations were conducted in the simulations. QAM is applied for evaluating the vector symbol error performance.

Example 1. In this example, we compare the sum rate between the systems with and without the proposed multimode antenna selection to maximize the achievable data rate for numbers of transmit antennas $M_t = 3$ and $M_t = 8$ respectively. The target SER for each substream is set to be 10^{-4}. The SNR (E/N_0) is the total power of a symbol vector over noise power. Fig. 5 shows the achievable data rates for 3×3 and 8×8 multimode antenna selection. We observe that with much simplified computational complexity, the performance with the proposed greedy search in Algorithm 1 is very close to the optimal exhaustive search scheme. Moreover, the performance with multimode antenna selection (for both greedy search and exhaustive search) greatly outperform the systems without antenna selection. The improvement becomes more pronounced when M_t is large. For instance, for an 8×8 MIMO system with SER$= 10^{-4}$, the achievable data rate difference with and without multimode antenna selection can be up to 12 bps/Hz.

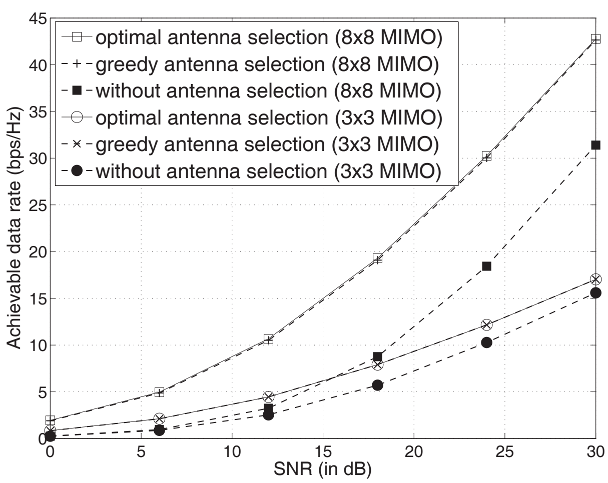


Fig. 5. Maximization of achievable data rate with antenna selection and without antenna selection.

Example 2. In this example, we compare the VSER performance of the proposed multimode antenna selection with the water-filling bit allocation in Algorithm 2 and the scheme that minimizes the NNUB Criterion 4 in [3]. In this example, we use SNR per bit, *i.e.* $\mathcal{E}_b/N_0 = E_s/(N_0(R/M))$, for a fair comparison. For the VSER of minimizing NNUB in [3], b_i is equal allocated for all substreams. Fig. 6 shows the VSER performance of these two schemes. We observe that the with water-filling bit allocation, the performance of multimode antenna selection can be further improved. The performance improvement is more pronounced for a large sum rate R. Take $R = 16$ for instance, the proposed antenna selection with water-filling bit allocation can achieve a better diversity than the scheme that minimizes the NNUB. This is reasonable since the proposed scheme efficiently allocates the bit budget to individual substreams according to the corresponding substream conditions; hence the SERs of individual streams are well equalized. The equalization in lager R is better than in small R.

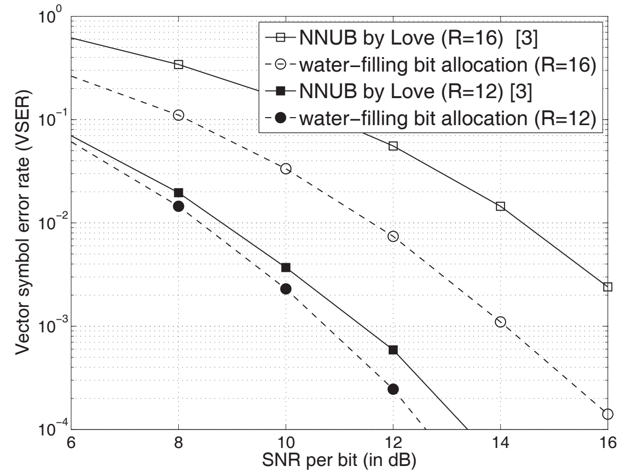


Fig. 6. Antenna selection with bit allocation and with NNUB minimization for 4×4 MIMO system.

6. CONCLUSIONS

In this paper, we presented multimode antenna selection to maximize the achievable data rate. Under the equal power constraint, better performance can be achieved by redistributing transmission power to the substreams with better channel conditions via selecting proper transmit antennas. To reduce the complexity of the optimal exhaustive selection scheme, we proposed a low-complexity greedy multimode antenna selection for ZF receiver. By greedy search, the computational complexity can be dramatically reduced while a near-optimal performance is achieved. In addition, we proposed bit allocation to efficiently equalize the SERs of individual substreams for the proposed selection scheme. Simulation results showed the advantages of the proposed antenna selection with bit allocation.

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