ITERATIVE NETWORK/CHANNEL DECODING FOR THE NOISY MULTIPLE-ACCESS RELAY CHANNEL (MARC)

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ABSTRACT

In this paper, we study the three–node multiple–access relay channel under realistic operating conditions. More specifically, in our setup all wireless links are subjected to Rayleigh fading and Gaussian noise, and the relay node can be located at different distances from source and destination nodes. We assume that coded bits are fully-interleaved, so that the transmitted symbols are mutually independent. By exploiting factor graph representation, we propose a joint network/channel decoding algorithm, which takes into account decoding errors introduced by the relay node. The convergence of the proposed iterative decoding algorithm is studied through EXIT chart analysis. The proposed decoder is compared with a recently proposed algorithm, which employs the demodulate-and-forward protocol and works on coded bits, and it is shown that it provides better performance. More specifically, for a bit error rate (BER) equal to 1e-3, it is 1.5dB better when the relay is located near the destination and 2.0dB better when the relay is in between source and destination.

Index Terms— Network coding, joint network/channel decoding, factor graph, cooperative relaying.

1. INTRODUCTION

Network Coding (NC) is receiving much attention in electrical engineering and computer science because it can improve performance and throughput of cooperative networks. Unlike classical network routing techniques, NC allows the transmission of linear combinations of packets received at the relay nodes [1], [2]. However, when used in noisy and faded channels, NC is affected by the so-called error propagation problem: corrupted packets at the intermediate nodes might propagate through the network and result in bad performance at the destination [3]–[4]. It is shown in [3], [4] that error propagation can dramatically degrade performance and diversity gain if it is not treated carefully.

Joint Network/Channel Decoding (JNCD) is a recently proposed approach that takes advantage of channel and network coding to improve the end-to-end performance [5]. By exploiting the inherent redundancy of network and channel codes, it has been shown that JNCD can improve the performance of canonical two-way and multiple-access relay channels if compared to conventional distributed turbo coding and separate network and channel decoding [5]. However, in [5] it is assumed that the relayed signal is the combination of the signals emitted from sources, which is true only if the relays perfectly decode the incoming data. Unfortunately, this is not always true in a fading environment. Thus, designing a decoder at the destination that takes into account possible decoding errors introduced by the relays is essential to overcome the error propagation problem.

The error propagation problem has been widely addressed in cooperative wireless networks [6]. The idea is that, if the destination is informed about source-relay channel state information (CSI), it can mitigate possible detection errors at the relay. In these strategies, the relay always performs NC regardless of decoding successfully or not. It was shown in [7] that a channel-aware receiver design can significantly improve the performance of NC. However, no channel coding is considered in [7]. The authors in [8] propose a Multiple–Input–Multiple–Output (MIMO) cooperative communication scheme where the destination is aware of decoding errors introduced at the relays. A similar method can be found in [9].

In this paper, we study JNCD for the multiple-access Rayleigh fading channel by considering a realistic operating scenario with Additive White Gaussian Noise (AWGN) and channel fading over all the wireless links [4]. By exploiting factor graph representation, we propose a joint network/channel decoding algorithm, which takes into account decoding errors introduced by the relay node. Unlike the methods above, where error probability of coded bits is employed to modify exchanged information between decoders, our algorithm employs error probability of information bits, which can be easily approximated in closed–form. The idea of exploiting error probability of information bits can also be found in [10], which studied turbo-like decoding for the relay channel. However, no NC is consider therein. The convergence of the proposed iterative decoding algorithm is studied through EXIT chart analysis. It is shown by simulation that, compared with another solution which employs the Demodulate-and-Forward protocol (DMF) and works on coded bits [9], the proposed decoding algorithm is 1.5dB better when the relay is near the destination and almost 2dB better when the relay is at half distance between source and destination.

This paper is organized as follows. Section II describes the system model. Section III introduces the JNCD algorithm. Convergence analysis and numerical results are given in Section IV. Finally, Section V concludes this paper.

2. SYSTEM MODEL

The system model under analysis is given by the canonical multiple-access relay channel, where two mobile stations, MS1 and MS2, communicate to a base station (BS) with the help of a relay R [5]. We study the realistic situation where all the channels are subject to Rayleigh fading and AWGN. In order to avoid mutual interference, we consider that transmissions are scheduled in time-orthogonal time-slots [2]. The system model is the same as in [11, Fig. 1]. The source node S_j, $j \in \{1,2\}$, emits the information message $\mathbf{u}_j = (u_{j1}, u_{j2}, \ldots, u_{jK})$, where $u_{jk} \in \{0,1\}$ is the *k*th information bit, and K is the number of information bits in $\mathbf{u}_j$. At each source, each information message $\mathbf{u}_j$ is processed as follows: i) first, it is encoded using a recursive convolutional code, which produces the codeword $\mathbf{c}_j = (c_{j1}, c_{j2}, \ldots, c_{jN})$, where $c_{jn} \in \{0,1\}$ is the *n*th coded bit, $N = K/R$ is length of coded word, R is the code rate; ii) then, the codedword is mapped into the modulated signal $\mathbf{x}_j = (x_{j1}, x_{j2}, \ldots, x_{jN})$ by using, for simplicity but without loss of generality, Binary Phase Shift Keying (BPSK) modulation, where $x_{jn} \in \{-1,1\}$ is the *n*th modulated bit.

We assume that coded bits are ideally interleaved, so that transmitted symbols are mutually independent. The relay employs the Decode–and–Forward (DF) strategy with hard decision. After decoding, the relay network-encodes the estimated information messages to get $\mathbf{w}^r = \mathbf{u}_1^r \oplus \mathbf{u}_2^r$ (binary plus-XOR of corresponding elements)and channel re-encodes $\mathbf{w}^r$ to get coded word $\mathbf{c}_r$, which is then mapped into BPSK signal $\mathbf{x}_r$. In this setup, we always network code the estimated bits ($\mathbf{u}_1^r$ and $\mathbf{u}_2^r$) regardless of having a decoding error. The received *k*th coded bit are as follows:

$$
\begin{aligned}
y_{jk,d} &= h_{jk,d} x_{jk} + n && (1)\\
y_{jk,r} &= h_{jk,r} x_{jk} + n && (2)\\
y_{rk,d} &= h_{rk,d} x_{rk} + n; k = 1,2,...,N && (3)
\end{aligned}
$$

where $y_{jk,d}, y_{jk,r}, y_{rk,d}$ denote the *k*th received bit from source S_j at the destination, from source S_j at the relay, and from the relay at the destination, respectively; $h_{jk,d}, h_{jk,r}$ and $h_{rk,d}$ are *k*th Rayleigh fading coefficients of source-to-destination, source-to-relay, and relay-to-destination channels, respectively; n (we avoid index of noise) is zero-mean Gaussian random variables with power spectral density $\frac{N_0}{2}$.

3. JOINT NETWORK/CHANNEL DECODING ALGORITHM FOR NOISY MARC

To develop our JNCD algorithm, we start with the maximum a posteriori probability decoding rule, and then derive the iterative algorithm based on factor graph representation [12]. At the destination, the maximum a posteriori probability decision rule is given as follows:

$$\hat{u}_{1k}, \hat{u}_{2k} = \arg \max_{u_{1k}, u_{2k}} P[u_{1k}, u_{2k} | \mathbf{y}_{1d}, \mathbf{y}_{2d}, \mathbf{y}_{rd}] \quad (4)$$

where $P[.]$ denotes probability and $P[a|b]$ denotes probability of a conditioned on b.

The probability in (4) is the marginal probability of the whole sequence, which can be re-written as follows:

$$\hat{u}_{1k}, \hat{u}_{2k} = \arg \max_{u_{1k}, u_{2k}} \sum_{\substack{\mathbf{u}_2 \sim \{u_{2k}\} \\ \mathbf{u}_1 \sim \{u_{1k}\}, \mathbf{w}^r}} \quad (5)$$
$$P[\mathbf{y}_{1d}|\mathbf{u}_1] \times P[\mathbf{y}_{2d}|\mathbf{u}_2] \times P[\mathbf{y}_{rd}|\mathbf{w}^r] \times P[\mathbf{w}^r|\mathbf{u}_1, \mathbf{u}_2]$$

where $\sum_{\mathbf{u}_1 \sim \{u_{1k}\}, \mathbf{u}_2 \sim \{u_{2k}\}, \mathbf{w}^r}(.)$ is sum over all the bits of $\mathbf{u}_1, \mathbf{u}_2, \mathbf{w}^r$ except bits $\{u_{1k}, u_{2k}\}$.

The last term in (5) accounts for possible decoding errors at the relay node. To derive the factor graph representation of the joint decoding algorithm, we assume, for the sake of simplicity, that the network coded information bits are independent. Thus, we have: $P[\mathbf{w}^r] = \prod_{k=1}^{K} P[w_k^r]$. This assumption leads to a suboptimal JNCD algorithm. In addition, since the transmitted information bits are independent, we have: $P[\mathbf{w}^r|\mathbf{u}_1, \mathbf{u}_2] = \prod_{k=1}^{K} P[w_k^r|u_{1k}, u_{2k}]$. We define variable $w_k = u_{1k} \oplus u_{2k}$ as correct network coded bits. We note that w is based on the codebook and w^r is based on actual estimates at the relays. Thus, the uncertainty of the decoding process at the relay is given by $P[w_k^r|w_k]$. The decision rule in (5) becomes:

$$\sum_{\mathbf{u}_1 \sim \{u_{1k}\}} P[\mathbf{y}_{1d}|\mathbf{u}_1] \times \sum_{\mathbf{u}_2 \sim \{u_{2k}\}} P[\mathbf{y}_{2d}|\mathbf{u}_2] \times \quad (6)$$
$$\sum_{\mathbf{w}^r} P[\mathbf{y}_{rd}|\mathbf{w}^r] \times \prod_{l=1}^{K} P[w_l^r|w_l] P[w_l|u_{1l}, u_{2l}]$$

From (6), the factor graph of the proposed JNCD algorithm is illustrated in Fig. 1. The graph has three subgraphs corresponding to three SISO (Single-Input-Single-Output) decoders for the two sources and the single relay. Each subgraph consists of input variable nodes, output variable nodes, and state variable nodes which are linked to local check nodes (see [12] for details). These subgraphs are interconnected via network check nodes. More specifically, the variable nodes u_{1k}, u_{2k} from the two sources are directly linked to the network check node, while the variable node w_k^r (estimated value) from the relay is connected to the network check function via the correct network variable node w_k. There is also a decoding (at the relay) check node between w_k^r and w_k which controls the uncertainty of the decoding process at relay. Iterative decoding is carried out by means of a message-passing method which links to backward/forward recursive computation over a portion of graph corresponding to one SISO decoder (SISO decoder 1, 2 and SISO decoder R) and exchange (Log-Likelihood Ratio) LLR-value messages (*extrinsic information* in turbo-decoding literature) between SISO decoders and network check node (see graph in Fig. 1). Let $\eta_1(.), \eta_2(.)$ and $\eta(.)$ be LLR extrinsic information sent by network check node to variable node u_{1k}, u_{2k} and node w_k, respectively; and $\mu_1(.), \mu_2(.)$ and $\mu(.)$ be LLR extrinsic information that reach network check node from variable node u_{1k}, u_{2k} and correct network variable node w_k, respectively. Furthermore, let $\mu_r(.)$ be extrinsic information sent from variable node w_k^r to decoding check node, and $\eta_r(.)$ be extrinsic information that reaches variable node w_k^r from decoding check node.

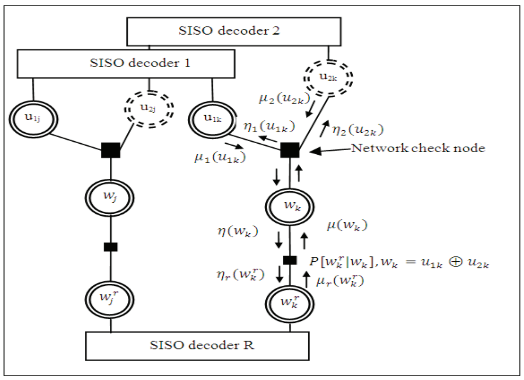

Fig. 1. Factor graph of JNCD algorithm.

Each SISO decoder 1,2,R runs its own sum-product algorithm [12], it sends extrinsic information to network check node. The network check node computes network extrinsic information as follows:

$$\eta_i(u_{ik}) = \log \frac{1 + e^{\mu_j(u_{jk}) + \mu(w_k)}}{e^{\mu_j(u_{jk})} + e^{\mu_r(w_k)}}; i \neq j = 1,2 \quad (7)$$

$$\eta(w_k) = \log \frac{1 + e^{\mu_1(u_{1k}) + \mu_2(u_{2k})}}{e^{\mu_1(u_{1k})} + e^{\mu_1(u_{1k})}} \quad (8)$$

Finally, network input node w_k^r and correct network input node w_k exchange messages controlled by the uncertainty function $P[w_k^r|w_k] = Pe_r = Pe_1 + Pe_2 - 2Pe_1Pe_2$, which is the error probability introduced by the relay, and Pe_1, Pe_2 are the error probabilities of S1-to-relay and S2-to-relay links. We assume that the destination knows all channel coefficients, thus these error probabilities can be estimated in closed–form [14]: $Pe_j \approx \frac{w(d)}{K} C_{2d-1}^{d} (4RSNR_r)^{-d}$, $j = 1, 2$, where d is minimum distance of the code and $w(d)$ is distance spectrum of the code:

$$\eta_r(w_k^r) = \log \frac{(1 - Pe_r)e^{\mu(w_k)} + Pe_r}{Pe_r e^{\mu(w_k)} + 1 - Pe_r} \quad (9)$$

$$\mu(w_k) = \log \frac{(1 - Pe_r)e^{\mu_r(w_k^r)} + Pe_r}{Pe_r e^{\mu_r(w_k^r)} + 1 - Pe_r} \quad (10)$$

4. PERFORMANCE EVALUATION

4.1. EXIT Chart Analysis

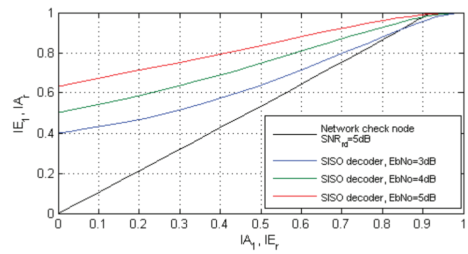

Fig. 2. EXIT chart of JNCD algorithm over Fully-Interleaved fading channels

Let us study the convergence properties and mutual information evolution by using extrinsic information transfer (EXIT) chart analysis [13]. The EXIT chart is built from mutual information between variable and a priori input, and between variable and extrinsic output of channel decoder and network decoder (network check node in Fig. 1). In fully–interleaved Rayleigh channel, the a priori information of SISO decoder is modeled by a Gaussian distribution [13]. Fig. 2 shows the EXIT chart of the JNCD algorithm in fully–interleaved Rayleigh channel. We can see that the crossing point between EXIT function of SISO decoder and network check node is very close to point (1,1). This confirms the good convergence property of the proposed decoding algorithm. From the EXIT chart, we see that mutual information can be almost maximized after 4 iterations. This is the reason why we set the maximum number of iterations equal to 4 in our numerical simulations.

4.2. Simulation Results

We assume a symmetric system scenario in which the distances from the two sources to the destination are the same. The relay node can be in different positions between sources and destination. Two scenarios are investigated. *Scenario 1*: the source-to-relay distance is three times the relay-to-destination distance, and *Scenario 2*: the source-to-relay distance is equal to the relay-to-destination distance. As a result: i) the SNR (Signal-to-Noise-Ratio) of the source-to-relay channel is 2.5dB and 6dB greater than the SNR of the source-to-destination channel in *Scenario 1* and *Scenario*

2, respectively; and ii) the SNR of the relay-to-destination channel is 12dB and 6dB greater than the SNR of the source-to-destination channel in *Scenario 1* and *Scenario 2*, respectively. In all cases, the proposed algorithm is compared with

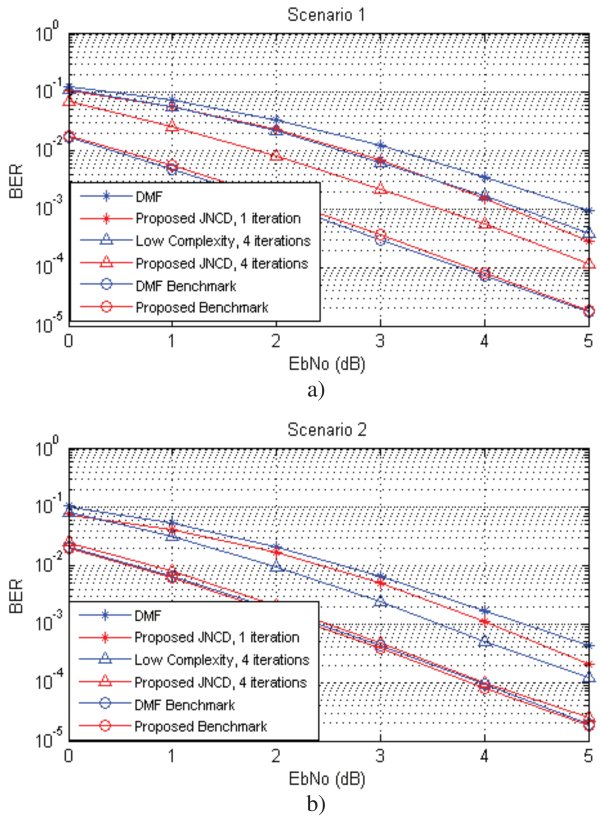

Fig. 3. Performance of the proposed JNCD algorithm compared with DMF algorithm in [9]

the algorithm proposed by [9], which implements the DMF protocol and works on coded bits. It is denoted by DMF in Fig. 3. We note that the DMF algorithm only implements network decoding and channel decoding once. To further improve the performance of the DMF protocol, we propose an iterative decoding algorithm based on [11] (*Low-Complexity* in the figure) which applies the same error propagation mitigation as in [9]. Furthermore two *Benchmark* scenarios, which assumes that the source-to-relay links are error free, *Proposed Benchmark* and *DMF Benchmark*, are also shown.

Fig. 3a shows the simulation results for *Scenario 1*. For a fair comparison in terms of complexity, we provide curves for a different number of iterations. It is shown that the proposed JNCD algorithm, after one iteration, is 0.5dB better than the DMF algorithm and has almost the same performance as the *Low-Complexity* implementation of [9] based on [11]. After 4 iterations, the proposed JNCD algorithm outperforms the DMF scheme in [9] of about 1.5dB at a BER equal to 1e-3.

The more interesting results are shown in Fig. 3b for *Scenario 2*. The proposed JNCD algorithm performs slightly better than DMF after one iteration and it outperforms DMF after 4 iterations of about 2dB at a BER equal to 1e-3. The *Low-Complexity* implementation of [9] based on [11], performs almost 1dB better than the DMF algorithm at a BER equal to 1e-3. Furthermore, the proposed JNCD algorithm is very close to the JNCD benchmark, while *Low-Complexity* and DMF are far from the DMF benchmark.

5. CONCLUSION

We have proposed a JNCD algorithm for the MARC based on factor graph. The proposed algorithm takes into account decoding errors introduced at the relay. The convergence of the proposed algorithm has been studied using EXIT chart analysis. We have shown that our algorithm outperforms a recently proposed solution that uses the DMF protocol and works on coded bits.

6. REFERENCES

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