UTILITY MAXIMIZATION IN THE HALF-DUPLEX TWO-WAY MIMO RELAY CHANNEL

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ABSTRACT

This paper addresses utility maximization problems in the half-duplex two-way multiple-input multiple-output (MIMO) relay channel, where the relay uses the decode-and-forward strategy. Perfect channel information at all nodes and a time division duplex communication protocol with per node peak power constraints for every protocol phase are assumed. For this scenario, we show how solutions to the considered class of problems can efficiently be determined by means of a dual decomposition approach.

Index Terms— Two-way relay channel, MIMO, half-duplex, decode-and-forward, utility maximization.

I. INTRODUCTION

The two-way relay channel models the scenario where two terminals want to exchange information with the aid of a relay. It was introduced in [1], where the authors showed that a significant portion of the loss in spectral efficiency suffered in the one-way relay channel due to the half-duplex constraint can be compensated when bidirectional communication is considered. As a result, many other scientific papers have since addressed the half-duplex two-way relay channel in combination with various communication protocols and relay strategies that include decode-and-forward (DF), compress-and-forward (CF), and amplify-and-forward (AF). For a short and very incomplete list of references, see for example [2]–[5] and references therein.

A general outer bound $\mathcal{C}$ on achievable rate regions for the half-duplex two-way MIMO relay channel was established in [6], assuming time division duplex (TDD) communication protocols with per node peak power constraints for every protocol phase. In addition, an achievable rate region $\mathcal{R}_{\text{DF}}$ based on the relay using DF was presented, which is a superset of all previously known rate regions that can be achieved with the decode-and-forward scheme. The main contribution of [6], then, was to derive parameterizations of $\mathcal{C}$ and $\mathcal{R}_{\text{DF}}$ that allow to efficiently evaluate these regions by means of a dual decomposition approach.

In this work, we consider utility maximization problems on the above-mentioned achievable rate region $\mathcal{R}_{\text{DF}}$, where the utility is a nondecreasing and concave function of the rate vector. It is demonstrated how the parameterization of $\mathcal{R}_{\text{DF}}$ and the dual decomposition approach proposed in [6] can also be used to efficiently determine solutions to this class of problems. Moreover, we take a closer look at the problems for specific utilities that are associated with various well-known fairness criteria: max-min fairness [7], proportional fairness [8], and α-fairness [9], which is a generalization of the first two fairness measures.

The remainder of this paper is organized as follows. Sec. II introduces the system model for the half-duplex two-way MIMO relay channel. In Sec. III, we address the utility maximization problems to be solved and show that optimal solutions can be obtained in an efficient manner using a dual decomposition approach. Numerical results are presented in Sec. IV before we conclude in Sec. V.

II. SYSTEM MODEL

We consider the *restricted* half-duplex two-way relay channel in this paper, i.e., the bidirectional communication is restricted in the sense that the encoders of the two terminal nodes can neither cooperate, nor are they able to use previously decoded information to encode their messages. The most general communication protocol for this channel is composed of all six phases (network states) where either one or two nodes transmit [10]. Obviously, no information is conveyed when no or all nodes transmit at the same time (the latter due to the half-duplex constraint). The six phases are characterized as follows:

1) Node 1 transmits to node 2 and the relay:
$$\boldsymbol{y}_{\text{R}}^{(1)} = \boldsymbol{H}_{1\text{R}}\boldsymbol{x}_1^{(1)} + \boldsymbol{n}_{\text{R}}^{(1)}, \quad \boldsymbol{n}_{\text{R}}^{(1)} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathbf{I}_{N_{\text{R}}}),$$
$$\boldsymbol{y}_2^{(1)} = \boldsymbol{H}_{12}\boldsymbol{x}_1^{(1)} + \boldsymbol{n}_2^{(1)}, \quad \boldsymbol{n}_2^{(1)} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathbf{I}_{N_2}).$$
2) Node 2 transmits to node 1 and the relay:
$$\boldsymbol{y}_{\text{R}}^{(2)} = \boldsymbol{H}_{2\text{R}}\boldsymbol{x}_2^{(2)} + \boldsymbol{n}_{\text{R}}^{(2)}, \quad \boldsymbol{n}_{\text{R}}^{(2)} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathbf{I}_{N_{\text{R}}}),$$
$$\boldsymbol{y}_1^{(2)} = \boldsymbol{H}_{21}\boldsymbol{x}_2^{(2)} + \boldsymbol{n}_1^{(2)}, \quad \boldsymbol{n}_1^{(2)} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathbf{I}_{N_1}).$$
3) Node 1 and node 2 transmit to the relay:
$$\boldsymbol{y}_{\text{R}}^{(3)} = \boldsymbol{H}_{1\text{R}}\boldsymbol{x}_1^{(3)} + \boldsymbol{H}_{2\text{R}}\boldsymbol{x}_2^{(3)} + \boldsymbol{n}_{\text{R}}^{(3)}, \; \boldsymbol{n}_{\text{R}}^{(3)} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathbf{I}_{N_{\text{R}}}).$$
4) The relay transmits to node 1 and node 2:
$$\boldsymbol{y}_1^{(4)} = \boldsymbol{H}_{\text{R}1}\boldsymbol{x}_{\text{R}}^{(4)} + \boldsymbol{n}_1^{(4)}, \quad \boldsymbol{n}_1^{(4)} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathbf{I}_{N_1}),$$
$$\boldsymbol{y}_2^{(4)} = \boldsymbol{H}_{\text{R}2}\boldsymbol{x}_{\text{R}}^{(4)} + \boldsymbol{n}_2^{(4)}, \quad \boldsymbol{n}_2^{(4)} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathbf{I}_{N_2}).$$

5) The relay and node 2 transmit to node 1:

$$\boldsymbol{y}_1^{(5)} = \boldsymbol{H}_{\mathrm{R}1}\boldsymbol{x}_{\mathrm{R}}^{(5)} + \boldsymbol{H}_{21}\boldsymbol{x}_2^{(5)} + \boldsymbol{n}_1^{(5)},\ \boldsymbol{n}_1^{(5)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I}_{N_1}).$$

6) The relay and node 1 transmit to node 2:

$$\boldsymbol{y}_2^{(6)} = \boldsymbol{H}_{\mathrm{R}2}\boldsymbol{x}_{\mathrm{R}}^{(6)} + \boldsymbol{H}_{12}\boldsymbol{x}_1^{(6)} + \boldsymbol{n}_2^{(6)},\ \boldsymbol{n}_2^{(6)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I}_{N_2}).$$

Here, $\boldsymbol{H}_{\mathrm{AB}}$ denotes the channel gain matrix from node A to node B, where we have assumed that all channels remain constant for all six network states in order to simplify the notation. However, this is without loss of generality since we require all channels to be perfectly known at all nodes as well as perfect synchronization between all nodes for the discussions below. The circularly symmetric additive white Gaussian noise $\boldsymbol{n}_{\mathrm{A}}^{(i)}$ received at node A during phase i is assumed to be independent of the noise $\boldsymbol{n}_{\mathrm{B}}^{(j)}$ received at another node B for all phases $j = 1, \ldots, 6$ and independent of $\boldsymbol{n}_{\mathrm{A}}^{(j)}$ for all $j \neq i$. A transmit covariance matrix

$$\boldsymbol{R}_{\mathrm{A}}^{(i)} = \mathrm{E}\left[\boldsymbol{x}_{\mathrm{A}}^{(i)}\boldsymbol{x}_{\mathrm{A}}^{(i),\mathrm{H}}\right] \tag{1}$$

is associated with each node A that transmits in the i-th phase. This node A is then subject to a peak power constraint of the form $\mathrm{tr}\big(\boldsymbol{R}_{\mathrm{A}}^{(i)}\big) \leq P_{\mathrm{A}}^{(i)}$. Furthermore, if nodes A and B transmit simultaneously during phase i, we have a joint transmit covariance matrix

$$\boldsymbol{R}^{(i)} = \mathrm{E}\left[\begin{bmatrix}\boldsymbol{x}_{\mathrm{A}}^{(i)} \\ \boldsymbol{x}_{\mathrm{B}}^{(i)}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_{\mathrm{A}}^{(i)} \\ \boldsymbol{x}_{\mathrm{B}}^{(i)}\end{bmatrix}^{\mathrm{H}}\right] = \begin{bmatrix}\boldsymbol{R}_{\mathrm{A}}^{(i)} & \boldsymbol{R}_{\mathrm{AB}}^{(i)} \\ \boldsymbol{R}_{\mathrm{AB}}^{(i),\mathrm{H}} & \boldsymbol{R}_{\mathrm{B}}^{(i)}\end{bmatrix}. \tag{2}$$

III. UTILITY MAXIMIZATION

The general utility maximization problems we consider in this paper read as

$$\max_{\boldsymbol{z}}\ u(\boldsymbol{z}) \quad \mathrm{s.t.} \quad \boldsymbol{z} \in \mathcal{R}_{\mathrm{DF}} \subset \mathbb{R}_+^2, \tag{3}$$

where $u(\boldsymbol{z})$ is nondecreasing and concave in $\boldsymbol{z}$. $\mathcal{R}_{\mathrm{DF}}$ denotes the rate region that is achievable in the half-duplex two-way relay channel with a communication protocol being composed of all the six phases specified in Sec. II (performed in exactly that order) and the relay using the decode-and-forward strategy [10]. Assuming both perfect channel state information (CSI) and perfect synchronization, all rate vectors $\boldsymbol{z} \in \mathcal{R}_{\mathrm{DF}}$ can be achieved with a jointly Gaussian input distribution that factors as $\prod_{i=1}^{6} p_{X_1^{(i)}X_2^{(i)}X_{\mathrm{R}}^{(i)}}$, where $p_{X_1^{(3)}X_2^{(3)}} = p_{X_1^{(3)}}p_{X_2^{(3)}}$ [6]. Since the Gaussian distribution is completely determined by its mean and covariance, the optimal zero mean input for phase i is specified by $\boldsymbol{R}^{(i)}$. Moreover, a convenient parameterization of $\mathcal{R}_{\mathrm{DF}}$ is given by

$$\mathcal{R}_{\mathrm{DF}} = \Big\{\boldsymbol{z} \in \mathbb{R}_+^2 : \boldsymbol{A}\boldsymbol{z} \leq \sum_{i=1}^{6}\tau_i\boldsymbol{B}_i\boldsymbol{r}_i, \sum_{i=1}^{6}\tau_i = 1, \\ \tau_i \geq 0, \boldsymbol{r}_i \in \mathcal{R}_i, \forall i = 1, \ldots, 6\Big\}. \tag{4}$$

Here, $\mathcal{R}_i \subset \mathbb{R}_+^2$ is a compact convex set that is parameterized by means of the (joint) transmit covariance matrix $\boldsymbol{R}^{(i)}$ and associated with the i-th phase of the transmission protocol, whose duration is denoted by τ_i. For example, the set $\mathcal{R}_1$ corresponding to the first phase of the protocol is given by

$$\mathcal{R}_1 = \Big\{\boldsymbol{r} \in \mathbb{R}_+^2 : r_1 \leq \log\det\big(\mathbf{I} + \boldsymbol{H}_{1\mathrm{R}}\boldsymbol{R}^{(1)}\boldsymbol{H}_{1\mathrm{R}}^{\mathrm{H}}\big), \\ r_2 \leq \log\det\big(\mathbf{I} + \boldsymbol{H}_{12}\boldsymbol{R}^{(1)}\boldsymbol{H}_{12}^{\mathrm{H}}\big), \\ \boldsymbol{R}^{(1)} \succeq \mathbf{0},\ \mathrm{tr}\big(\boldsymbol{R}^{(1)}\big) \leq P_1^{(1)}\Big\}. \tag{5}$$

Each row of $\boldsymbol{A} = \left[\begin{smallmatrix}1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1\end{smallmatrix}\right]^{\mathrm{T}}$ selects one of the constraints on $\boldsymbol{z}$ as defined in $\mathcal{R}_{\mathrm{DF}}$, and the corresponding rows of the matrices $\boldsymbol{B}_i \in \{0,1\}^{5\times 2}$ specify the structures of these constraints with regard to the sets $\mathcal{R}_i$. For all details on the parameterization of $\mathcal{R}_{\mathrm{DF}}$ and its properties, we refer the reader to [6].

In the following, we will make use of the aforementioned parameterization of $\mathcal{R}_{\mathrm{DF}}$ in order to solve problem (3) for different utility functions. First, max-min fair rate allocation is addressed (the problem can be tackled more directly than the general one), and subsequently, we consider general utilities with a closer look at proportional and α-fairness. We do not elaborate on weighted sum rate maximization over $\mathcal{R}_{\mathrm{DF}}$ here because this is already covered in [6].

III-A. Max-Min Fairness

One of the most common fairness criteria is max-min fairness [7]. The rate vector $\boldsymbol{z} \in \mathcal{R}_{\mathrm{DF}} \subset \mathbb{R}_+^2$ is max-min fair if it maximizes the utility $u(\boldsymbol{z}) = \max\min\{z_1, z_2\}$ over the rate region $\mathcal{R}_{\mathrm{DF}}$. For our system model, the max-min fair rate vector is obtained as the solution of the following utility maximization problem:

$$\max_{\boldsymbol{z},\tau_i,\boldsymbol{r}_i}\ \min\{z_1, z_2\}\ \mathrm{s.t.}\ \mathbf{0} \leq \boldsymbol{A}\boldsymbol{z} \leq \sum_{i=1}^{6}\tau_i\boldsymbol{B}_i\boldsymbol{r}_i, \sum_{i=1}^{6}\tau_i = 1, \\ \tau_i \geq 0, \boldsymbol{r}_i \in \mathcal{R}_i, \forall i = 1, \ldots, 6. \tag{6}$$

Noting that $\boldsymbol{A}\boldsymbol{z} = \left[\begin{smallmatrix}z_1 & z_1 & z_2 & z_2 & z_1+z_2\end{smallmatrix}\right]^{\mathrm{T}}$, we define the vector $\boldsymbol{c} = \left[\begin{smallmatrix}1 & 1 & 1 & 1 & 2\end{smallmatrix}\right]^{\mathrm{T}}$ and reformulate (6) as

$$\max_{y,\tau_i,\boldsymbol{r}_i}\ y \quad \mathrm{s.t.} \quad \mathbf{0} \leq y\boldsymbol{c} \leq \sum_{i=1}^{6}\tau_i\boldsymbol{B}_i\boldsymbol{r}_i, \sum_{i=1}^{6}\tau_i = 1, \\ \tau_i \geq 0, \boldsymbol{r}_i \in \mathcal{R}_i, \forall i = 1, \ldots, 6. \tag{7}$$

This is a convex optimization problem for which strong duality holds so that we can equivalently solve the dual problem. To this end, we use the (vector-valued) Lagrangian multiplier $\boldsymbol{\lambda}$ to incorporate the constraints $y\boldsymbol{c} \leq \sum_{i=1}^{6}\tau_i\boldsymbol{B}_i\boldsymbol{r}_i$ into the objective function. This leads to the Lagrangian function

$$L(y, \boldsymbol{r}_i, \tau_i, \boldsymbol{\lambda}) = y - \boldsymbol{\lambda}^{\mathrm{T}}\Big(y\boldsymbol{c} - \sum_{i=1}^{6}\tau_i\boldsymbol{B}_i\boldsymbol{r}_i\Big) \tag{8}$$

and the corresponding dual function

$$\Theta(\boldsymbol{\lambda}) = \begin{cases}\max\limits_{i=1,\ldots,6}\Big(\max\limits_{\boldsymbol{r}_i\in\mathcal{R}_i}\boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{B}_i\boldsymbol{r}_i\Big) & \text{if } \boldsymbol{c}^{\mathrm{T}}\boldsymbol{\lambda} \geq 1, \\ +\infty & \text{otherwise.}\end{cases} \tag{9}$$

In order to determine the max-min fair rate vector, we thus need to solve the dual problem

$$\min_{\boldsymbol{\lambda}} \max_{i=1,\ldots,6} \left(\max_{\boldsymbol{r}_i \in \mathcal{R}_i} \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{B}_i \boldsymbol{r}_i \right) \text{ s.t. } \boldsymbol{\lambda} \geq \boldsymbol{0},\ \boldsymbol{c}^{\mathrm{T}} \boldsymbol{\lambda} \geq 1. \quad (10)$$

This can be done using the cutting plane method [11] as described in [6]. In each iteration of the cutting plane algorithm, the dual function $\Theta(\boldsymbol{\lambda})$ must be evaluated, which requires to solve six weighted sum rate (WSR) maximization problems, one over each of the convex sets $\mathcal{R}_i$. For this purpose, standard SDP solvers like SDPT3 that are capable of dealing with log-det terms in the objective function may be applied [6]. The max-min fair rate vector and the optimal time shares τ_i are finally obtained by primal reconstruction.

III-B. General Utility Functions

In this section, we consider general nondecreasing and concave utility functions. Using the parameterization of $\mathcal{R}_{\mathrm{DF}}$, the following utility maximization problem results:

$$\max_{\boldsymbol{z},\tau_i,\boldsymbol{r}_i} u(\boldsymbol{z}) \quad \text{s.t.} \quad \boldsymbol{0} \leq \boldsymbol{A}\boldsymbol{z} \leq \sum_{i=1}^{6} \tau_i \boldsymbol{B}_i \boldsymbol{r}_i, \sum_{i=1}^{6} \tau_i = 1,$$
$$\tau_i \geq 0, \boldsymbol{r}_i \in \mathcal{R}_i, \forall i = 1, \ldots, 6. \quad (11)$$

Since $u(\boldsymbol{z})$ is assumed to be concave, strong duality holds again. Like in the previous subsection, we incorporate the constraints $\boldsymbol{A}\boldsymbol{z} \leq \sum_{i=1}^{6} \tau_i \boldsymbol{B}_i \boldsymbol{r}_i$ into the objective function, which eventually yields the dual function

$$\Theta(\boldsymbol{\lambda}) = \sup_{\boldsymbol{z} \geq \boldsymbol{0}} \left\{ u(\boldsymbol{z}) - \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{A}\boldsymbol{z} \right\} + \max_{i=1,\ldots,6} \left(\max_{\boldsymbol{r}_i \in \mathcal{R}_i} \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{B}_i \boldsymbol{r}_i \right). \quad (12)$$

The dual function decomposes as $\Theta_1(\boldsymbol{\lambda}) + \Theta_2(\boldsymbol{\lambda})$, where $\Theta_2(\boldsymbol{\lambda})$ is equal to the maximum weighted sum rate over all sets $\mathcal{R}_i$ as known from the max-min problem, and $\Theta_1(\boldsymbol{\lambda}) = \sup_{\boldsymbol{z} \geq \boldsymbol{0}} \{ u(\boldsymbol{z}) - \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{A}\boldsymbol{z} \}$ is obtained from a convex optimization problem that only depends on $\boldsymbol{z}$. As the utility is assumed to be nondecreasing, the supremum may be infinity for general $\boldsymbol{\lambda}$. Without changing the original problem, this can be prevented by adding the constraint $\boldsymbol{z} \leq \boldsymbol{d}$ if it is guaranteed that $\boldsymbol{d} \geq \boldsymbol{z}$, $\forall \boldsymbol{z} \in \mathcal{R}_{\mathrm{DF}}$. We may for example choose any $\boldsymbol{d}$ that is larger than the vector of optimal unidirectional rates. The dual problem is then given by

$$\min_{\boldsymbol{\lambda}} \left\{ \max_{\boldsymbol{0} \leq \boldsymbol{z} \leq \boldsymbol{d}} \left\{ u(\boldsymbol{z}) - \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{A}\boldsymbol{z} \right\} + \max_{i=1,\ldots,6} \left(\max_{\boldsymbol{r}_i \in \mathcal{R}_i} \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{B}_i \boldsymbol{r}_i \right) \right\}$$
$$\text{s.t.} \quad \boldsymbol{\lambda} \geq \boldsymbol{0}. \quad (13)$$

We can of course apply the standard cutting plane method to this problem again. However, instead of approximating the whole dual function $\Theta(\boldsymbol{\lambda})$, it is also possible to introduce a cut each for $\Theta_1(\boldsymbol{\lambda})$ and $\Theta_2(\boldsymbol{\lambda})$ in every iteration [12]. While this increases the complexity of the master program, fewer iterations are usually needed to approximate $\Theta(\boldsymbol{\lambda})$ accurately. As the complexity is dominated by the evaluation of the dual function, this is a worthwhile tradeoff here.

Let us now have a closer look at $\Theta_1(\boldsymbol{\lambda})$ for the utility associated with α-fairness [9], which is a generalization of max-min and proportional fairness [8]. In particular, a proportional fair rate allocation is obtained by considering α-fairness with $\alpha = 1$, and as α becomes large, α-fairness converges to that of max-min. The utility function associated with α-fairness is given by

$$u_\alpha(\boldsymbol{z}) = \begin{cases} \sum_{k=1}^{2} \log(z_k) & \text{if } \alpha = 1, \\ \sum_{k=1}^{2} (1-\alpha)^{-1} z_k^{1-\alpha} & \text{if } \alpha > 0, \alpha \neq 1. \end{cases} \quad (14)$$

Note that $u_\alpha(\boldsymbol{z})$ is differentiable and concave for all $\alpha > 0$. The necessary and sufficient condition for $\boldsymbol{z}^*$ to maximize $u_\alpha(\boldsymbol{z}) - \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{A}\boldsymbol{z}$ over the set $\boldsymbol{0} \leq \boldsymbol{z} \leq \boldsymbol{d}$ is [11]

$$\left(\nabla u_\alpha^{\mathrm{T}}(\boldsymbol{z}^*) - \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{A} \right) (\boldsymbol{z} - \boldsymbol{z}^*) \leq 0, \ \ \forall \boldsymbol{0} \leq \boldsymbol{z} \leq \boldsymbol{d}. \quad (15)$$

Since $u_\alpha(\boldsymbol{z})$ is additive and increasing in every component, it hence follows for $\boldsymbol{\lambda} \geq \boldsymbol{0}$ that

$$z_k^* = \min \left\{ (\boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{A} \boldsymbol{e}_k)^{-1/\alpha}, d_k \right\}, \quad (16)$$

where $\boldsymbol{e}_k$ is the unit vector with a one as the k-th entry and zeros elsewhere, and d_k is the k-th entry of $\boldsymbol{d}$.

We therefore have a closed-form expression for $\Theta_1(\boldsymbol{\lambda})$ and its maximizer, which means that we do not have to approximate $\Theta_1(\boldsymbol{\lambda})$ in the dual problem (13). The downside of this approach, however, is that the master program is not a linear program but a general convex optimization problem. Since the evaluation of $\Theta_1(\boldsymbol{\lambda})$ is very cheap, it might thus be preferable to approximate both $\Theta_1(\boldsymbol{\lambda})$ and $\Theta_2(\boldsymbol{\lambda})$. In any case, the complexity of evaluating the dual function is again dominated by the six WSR maximization problems.

IV. NUMERICAL RESULTS

In this section, numerical results for the utilities associated with max-min and proportional fair rate allocation are provided. The example scenario we consider is the line network depicted in Fig. 1. This is a commonly used geometry where $d_{12} = 1$ is fixed and the relay is positioned on the line connecting the two terminals such that $d_{1\mathrm{R}} = |d|$ and $d_{2\mathrm{R}} = |1 - d|$. The channel gain matrices $\boldsymbol{H}_{\mathrm{AB}}$ are assumed to be random and independent, where the entries of $\boldsymbol{H}_{\mathrm{AB}}$ are independent and identically distributed complex Gaussian random variables with zero mean and variance d_{AB}^{-4}. In addition, we assume that the channels are reciprocal, i.e., $\boldsymbol{H}_{\mathrm{AB}} = \boldsymbol{H}_{\mathrm{BA}}^{\mathrm{T}}$. The transmit power of node A is the same for every phase i, i.e., $P_{\mathrm{A}}^{(i)} = P_{\mathrm{A}}$. Finally, note that all results are averaged over a number of independent channel realizations, where perfect CSI is assumed for every realization.

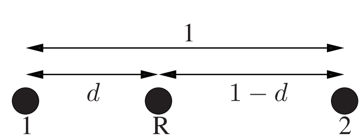
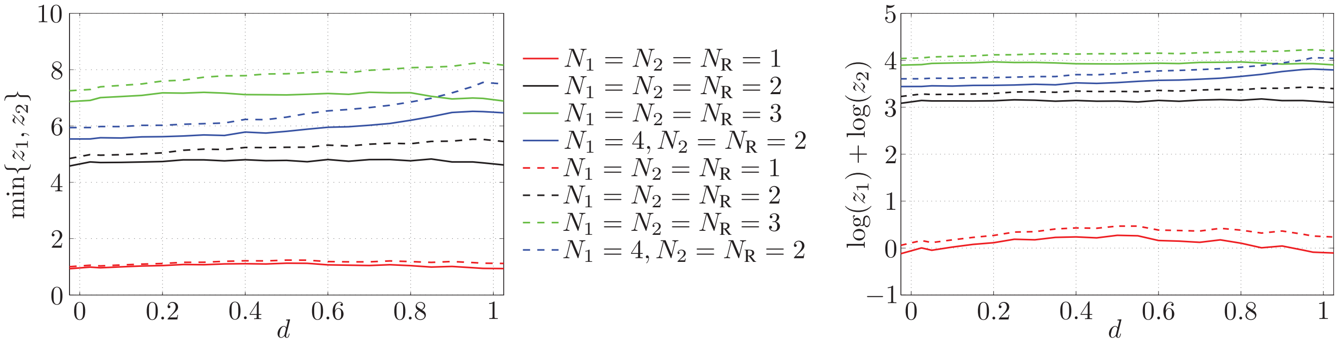


Fig. 1. Line network.



(a) Max-min fairness. (b) Proportional fairness.

Fig. 2. Comparison of optimal max-min and proportional fair utility values for different antenna and power configurations; solid curves: $P_1 = P_2 = P_{\mathrm{R}} = 10$, dashed curves: $P_1 = 30, P_2 = P_{\mathrm{R}} = 10$.

Fig. 2 compares the optimal max-min and proportionally fair utility values for different network configurations, where the results are averaged over 100 channel realizations (250 for $N_1 = N_2 = N_{\mathrm{R}} = 1$). Not surprisingly, substantial gains are achieved by equipping all nodes with multiple antennas. For all symmetric scenarios, we see that the utilities do not heavily depend on the relay position, but the best relay position clearly is in the middle between the terminals. If one terminal has more antennas and/or transmit power than the other one, the utility increases as the relay is moved closer to the terminal with less antennas/power, whereby the effect is stronger for max-min fairness.

For an absolute accuracy of $\varepsilon = 10^{-3}$, the average number of cutting plane iterations ranged from 5–12, depending on the choice of parameters and utility function. Note that both $\Theta_1(\boldsymbol{\lambda})$ and $\Theta_2(\boldsymbol{\lambda})$ were approximated in the case of proportional fairness. These relatively small numbers of required iterations confirm that the approach proposed in this paper allows to efficiently solve the considered class of utility maximization problems.

V. CONCLUSION

We presented a generic method to efficiently solve utility maximization problems in the half-duplex two-way MIMO relay channel when the relay uses the decode-and-forward strategy. To this end, a dual decomposition approach was proposed, and we discussed how the resulting dual problems can be tackled by means of the cutting plane algorithm. For max-min, proportional, and α-fairness, we concluded that the complexity of the proposed method is dominated by six weighted sum rate maximization problems, which can be solved using standard SDP tools. Finally, we remark that our approach is also applicable to other communication protocols that are often considered in the literature.

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