RELAY POWER ALLOCATION AND PRICING IN MULTI-USER RELAY NETWORKS USING GAME THEORY

Qian Cao, H. Vicky Zhao, and Yindi Jing

ECE Department, University of Alberta, Edmonton, AB Canada T6G 2V4
Email: qcao1@ualberta.ca, vzhao@ece.ualberta.ca, yindi@ualberta.ca

ABSTRACT

This paper considers a multi-user single-relay wireless network, where the relay gets paid for helping the users forward signals, and the users pay to receive the relay service. We study the relay power allocation and pricing problem, and model the interaction between the users and the relay as a two-level Stackelberg game. In this game, the relay, modeled as the service provider and the leader of the game, sets the relay price to maximize its revenue; while the users are modeled as customers and the follower who buy power from the relay. For the relay power allocation among users, we use a bargaining game model to achieve a fair allocation. Based on the proposed fair relay power allocation rule, we then analyze the optimal relay power price that maximizes the relay's revenue, and derive the analytical solution. Simulation shows that the proposed power allocation scheme achieves a higher network sum-rate than the even power allocation, and is fairer than the sum-rate-optimal allocation. We also show that the proposed pricing and power allocation solution is consistent with the laws of supply and demand.

1. INTRODUCTION

Cooperative communication has been shown to be a promising concept in future wireless networks. The basic idea is to have multiple nodes in the network help each other's transmission to achieve diversity. Numerous cooperative strategies aiming at optimizing the global network performance have been proposed, e.g., [1]. Two widely used ones are amplify-and-forward (AF) and decode-and-forward (DF). For a multi-user relay network, one prominent issue is the resource allocation among the users. There have been numerous work on this issue, including the relay selection and relay power allocation [2–4], most of which assume that nodes are altruistic and always willing to cooperate to optimize the overall network performance. In many practical applications, however, nodes are selfish and aim to optimize their own benefit or quality-of-service. This inspires the use of game theory to model the selfish behavior of nodes in a wireless network [5–8].

In [5] and [6], a two-user network where each user can also work as a relay for the other is studied. By employing a two-user bargaining game, fair bandwidth allocation [5] and power allocation [6] are found from the Nash bargaining solution. The work in [7] studies the user power control and pricing problems in a multi-user single-relay network, where the relay sets the price to maximize its revenue. In their work, a non-cooperative game is used to model the user behaviour, in which each user adjusts its transmit power to maximize its own utility. A distributed iterative scheme is proposed to achieve the unique Nash equilibrium point. In [8], for a single-user multi-relay network, the relay selection and relay power control are investigated using a two-level Stackelberg game. In this game, the relays compete to provide service to the user to gain revenue.

In this paper, we consider a multi-user single-relay network and use game theory to model and analyze the user and relay selfish behavior. Pricing mechanism is used where the relay gets paid for signal forwarding and users pay for the relay service. We model the interaction between the relay and the users as a two-stage Stackelberg game, in which the relay is the leader who sets the unit power price for the relay service, and the users are the followers who decide how much power to purchase from the relay for a given relay price. Different to [7], we consider the relay power allocation among users, instead of the user power control; and also the relay power competition among users is modeled as a cooperative bargaining game. The Kalai-Smorodinsky bargaining solution (KSBS) of the bargaining game is employed for a fair relay power allocation. Based on the KSBS relay power allocation, we analytically find the optimal relay price that maximizes the relay revenue. It is shown via simulations that the proposed KSBS power allocation is fairer than the sum-rate-optimal power allocation with a small penalty in the network sum-rate. It achieves higher network sum-rate than the even power allocation and with a similar fairness. We also show that the proposed pricing and power allocation solution is consistent with the laws of supply and demand.

2. NETWORK MODEL

Consider a wireless network with N users communicating with their destinations with the help of one relay as shown in Fig. 1. Denote the channel gain from User i to Destination i as h_i, the channel gain from User i to the relay as f_i, and the channel gain from the relay to Destination i as g_i. The relay and destinations are assumed to have global and perfect channel state information (CSI) $f_1, \cdots, f_N, g_1, \cdots, g_N$ through training and feedback. No CSI is required at the users. User i uses transmit power Q_i and the maximum transmit power of the relay is P.

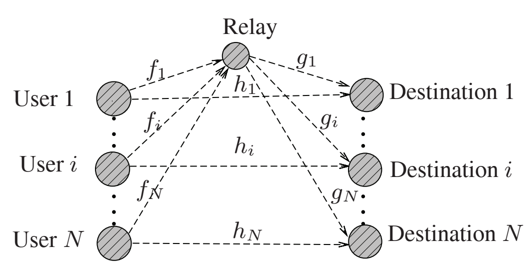


Fig. 1. Multi-user single-relay network.

Frequency division multiple access (FDMA) is used, so transmissions of different users are orthogonal and interference-free. Without loss of generality, we elaborate the transmission of User i's message on Channel i. We use the popular half-duplex two-step AF relaying protocol. Let s_i be the information symbol of User i. It is normalized as $\mathbf{E}(|s_i|^2) = 1$, where $\mathbf{E}$ stands for the average. In the first step, User i transmits $\sqrt{Q_i}s_i$, The signals received by the relay and Destination i are $y_{iR} = \sqrt{Q_i}s_i f_i + n_{iR}$ and $y_{iD} = \sqrt{Q_i}s_i h_i + n_{iD}$, respectively, where n_{iR} and n_{iD} are the additive noises at the relay and the destination in the first step, respectively. In the second step, the relay amplifies its received signal and forwards it to Destination i. Denote the power the relay uses to help User i in forwarding information as P_i. Since the relay has perfect CSI, coherent power coefficient is used for better performance than non-coherent power coefficient [2]. The signal received at Destination i can be shown to be $y_{Ri} = \sqrt{\frac{Q_i P_i}{Q_i|f_i|^2+1}}s_i f_i g_i + \sqrt{\frac{P_i}{Q_i|f_i|^2+1}}g_i n_{iR} + n_{RD}$, where n_{RD} is the additive noise at the destination in the second step. All noises are assumed to be i.i.d. additive circularly symmetric complex Gaussian with zero-mean unit-variance.

After maximum-ratio combining of both the direct path and the relay path, the effective received signal-to-noise-ratio (SNR) of User i's transmission can be shown to be

$$\mathrm{SNR}_i = \frac{Q_i P_i |f_i g_i|^2}{P_i|g_i|^2 + Q_i|f_i|^2 + 1} + Q_i|h_i|^2. \quad (1)$$

If User i's transmission is not helped by the relay and only the direct transmission is active, the received SNR of User i's transmission becomes $\mathrm{SNR}_{iD} = Q_i|h_i|^2$.

3. RELAY POWER ALLOCATION AND PRICING USING GAME THEORY

In this section, we employ the Stackelberg game to model the interaction between the users and the relay. To address the conflicts among the users, we use the bargaining game for a fair allocation of the relay power.

3.1. Stackelberg Game Model

We consider the relay as the leader who sets the price of its power in helping the users. The relay revenue, denoted as u_R, is the total payment from the users. We use a simple pricing model by assuming that the relay revenue is linear in the amount of power it sells, i.e., $u_R = \sum_{i=1}^{N} \lambda P_i$, where λ is the normalized unit price of the relay power and P_i is the power the relay uses to help User i.

We consider the users as the followers who react in a rational way given the unit price of the relay power. In this work, we assume that users make agreements to cooperatively share the relay power, and use the bargaining game to model the negotiation among the users. The first step to formulate the power allocation problem as a bargaining game is to design the utility function. In this work, we define User i's utility function as

$$u_i \triangleq \frac{Q_i P_i |f_i g_i|^2}{P_i|g_i|^2 + Q_i|f_i|^2 + 1} + Q_i|h_i|^2 - \lambda P_i, \quad i = 1 \cdots N. \quad (2)$$

The first two terms in (2) correspond to the effective received SNR of User i given in (1) and represent the quality-of-service of the user. It is directly related to the performance of the communication, e.g., the achievable rate. The term λP_i represents the user's normalized cost in purchasing the relay service. If User i does not buy any power from the relay and uses the direct transmission only, i.e., $P_i = 0$, it receives the minimum utility that User i expects, which is $u_{i,0} = Q_i|h_i|^2$.

In the following, we solve the power allocation and pricing problem jointly using the backward induction method [9]. That is, we first solve the power bargaining game among users for a given price of the relay power, then find the optimal price of the relay power based on the derived user bargaining strategy.

3.2. User Game

The user game is to find the relay power allocation among the users for a given unit relay power price λ. We use the Kalai-Smorodinsky bargaining solution (KSBS) [10], which guarantees fairness in the sense of equal penalty, to find a power allocation among the users. For this purpose, we first define $b_i \triangleq \frac{Q_i|f_i g_i|^2}{Q_i|f_i|^2+1}$ and show the following lemma, whose proof can be found in [12].

Lemma 1. *Given the relay power price λ, the ideal power demand of User i that maximizes its utility u_i in (2) is*

$$P_i^I(\lambda) = \begin{cases} 0 & \text{if } \lambda \geq b_i \\ \frac{Q_i|f_i|^2}{\sqrt{b_i}}\left(\frac{1}{\sqrt{\lambda}} - \frac{1}{\sqrt{b_i}}\right) & \text{if } b_i > \lambda > b_i\left(\frac{b_i P}{Q_i|f_i|^2}+1\right)^{-2} \\ P & \text{if } \lambda \leq b_i\left(\frac{b_i P}{Q_i|f_i|^2}+1\right)^{-2} \end{cases}. \quad (3)$$

From Lemma 1, when the price is too high, User i will not purchase any power from the relay. When the price is low, ideally, User i wants to purchase all the relay power to maximize its utility. Otherwise, User i will ask for part of the relay power that gives the ideal balance between the SNR improvement and the cost.

Without loss of generality, we assume that the users are sorted in the descending order of their b_i values, that is $b_1 \geq b_2 \geq \cdots \geq b_N$. With the given price λ, let L be the number of users satisfying $b_i > \lambda$, i.e., $b_L \geq \lambda \geq \lambda_{L+1}$. From Lemma 1, for Users $L+1, \cdots, N$, their ideal power demand is 0 and they do not buy any power from the relay. The first L users will enter the bargaining game and purchase the relay service.

Given λ, for Users $1, \cdots, L$ to find the KSBS is equivalent to solving the following optimization problem [10]:

$$\max_{P_i} k \quad \text{s.t.} \quad \frac{b_i P_i}{(Q_i|f_i|^2)^{-1} b_i P_i + 1} - \lambda P_i = k\left(u_i^I - u_{i,0}\right),$$
$$\sum_{i=1}^{L} P_i \leq P, \quad \text{and} \quad 0 < P_i < Q_i|f_i|^2\left(\frac{1}{\lambda} - \frac{1}{b_i}\right), \quad (4)$$

where k is a constant independent of users, the constraints in (4) forces all players participating in the bargaining game to suffer the same quality penalty in the logarithmic scale and $u_i^I = u_i(P_i^I)$ is the ideal utility of User i that can be derived directly from (2) and (3). The second constraint in (4) is due to the total power constraint on the relay, and the last constraint is to ensure the feasibility of the solution and $u_i > u_{i,0}$ for $i = 1, \cdots, L$.

We now look at the last constraint in (4). It can be shown that u_i is a concave function of P_i [12]. Thus, for each value $u \in (u_{i,0}, u_i^I(\lambda))$, there are two possible P_i's in $\left(0, Q_i|f_i|^2(1/\lambda - 1/b_i)\right)$ that satisfy $u_i(P_i) = u$: one in the range $\left(0, P_i^I(\lambda)\right)$ and the other in the range $\left(P_i^I(\lambda), Q_i|f_i|^2(1/\lambda - 1/b_i)\right)$. Thus, we can shrink the feasible region of P_i to either of these two. We choose the first one, which results in a smaller P_i than the second region for the same u value, for two reasons. First, the users prefer to buy less power to gain the same utility. Second, smaller power consumption for each user saves relay power so that more users can be helped. With the last constraint being replaced by $0 < P_i < P_i^I(\lambda)$, (4) is equivalent to the following max-min problem

$$\max_{P_i} \min_i \left\{ \frac{\frac{b_i P_i}{(Q_i|f_i|^2)^{-1} b_i P_i + 1} - \lambda P_i}{u_i^I - u_{i,0}} \right\},$$

$$\text{s.t.} \quad \sum_{i=1}^{L} P_i \le P, \quad 0 < P_i < P_i^I(\lambda). \tag{5}$$

The proof of the equivalence is available in [12]. (5) is a convex optimization problem and can be efficiently solved using standard convex optimization techniques.

We call the solution of (5) the KSBS-based power allocation, which are the relay power requested by the first L users. We show the following properties of the KSBS-based power allocation, whose proof can be found in [12].

Lemma 2. *For a fixed λ, let the ideal power allocation of User i be $P_i^I(\lambda)$, which is given in (3); and let the KSBS-based power allocation be $P_i^K(\lambda)$. When $\sum_{i=1}^{L} P_i^I(\lambda) \le P$, we have $P_i^K(\lambda) = P_i^I(\lambda)$; when $\sum_{i=1}^{L} P_i^I(\lambda) > P$, i.e., the total ideal power demands exceeds the relay power constraint, we have $\sum_{i=1}^{L} P_i^K(\lambda) = P$.*

3.3. Relay Game

In the relay game, the relay set the price to gain the maximum revenue. When the unit price of the relay power is λ, the revenue of the relay is $u_R(\lambda) = \sum_{i=1}^{N} \lambda P_i^K(\lambda)$, where $P_i^K(\lambda)$ is the relay power allocated to User i based on the KSBS from (5). The relay pricing problem can be formulated as:

$$\max_{\lambda} \sum_{i=1}^{N} \lambda P_i^K(\lambda). \tag{6}$$

Note that the relay power constraint $\sum_{i=1}^{N} P_i^K(\lambda) \le P$ is always guaranteed by the KSBS, thus needs not to appear explicitly in (6).

To solve the above optimization problem, we first show the following lemma.

Lemma 3. *The optimal price is inside the interval $[b_{lb}, b_1)$, where b_{lb} satisfies $\sum_{i=1}^{N} P_i^I(b_{lb}) = P$.*

Proof. Note that b_i's are in non-increasing order. When $\lambda \ge b_1$, from (3), $P_i^I(\lambda) = 0$ for all i's, and no user purchases any relay power. Thus, the relay revenue is 0. This is obviously not optimal, which means that the optimal price is smaller than b_1. To prove the lower bound, we can show that $P_i^I(\lambda)$ in (3) is a non-decreasing function of λ, and thus $\sum_{i=1}^{N} P_i^I(\lambda) \ge P$ when $0 \le \lambda \le b_{lb}$. Consequently, from Lemma 2, all relay power will be allocated to the users and $\sum_{i=1}^{N} P_i^K(\lambda) = P$. Thus, in the price range $[0, b_{lb}]$, the relay revenue is $\lambda \sum_{i=1}^{N} P_i^K(\lambda) = \lambda P$, which reaches its maximum at $\lambda = b_{lb}$. It shows that the optimal price is no less than b_{lb}. □

To find b_{lb}, from the definition of b_{lb} in Lemma 3, we need to solve the following equation:

$$\psi(b_{lb}) \triangleq \sum_{i=1}^{N} P_i^I(b_{lb}) = P. \tag{7}$$

Note that $\psi(b_{lb})$ in (7) monotonically decreases from ∞ to 0 as b_{lb} increases from 0 to b_1. To find the value of b_{lb}, we can first find the M such that $\psi(b_M) < P < \psi(b_{M+1})$. Thus, $b_{lb} \in [b_M, b_{M+1}]$. Within this price interval, from Lemma 1, $P_i^I(b_{lb}) = 0$ for $i = M+1, \cdots, N$. Therefore, by using (3), (7) becomes

$$\psi(b_{lb}) = \sum_{i=1}^{M} \frac{Q_i|f_i|^2}{\sqrt{b_i}} \left(\frac{1}{\sqrt{b_{lb}}} - \frac{1}{\sqrt{b_i}} \right) = P, \tag{8}$$

from which we have

$$b_{lb} = \left(\sum_{i=1}^{M} \frac{Q_i|f_i|^2}{\sqrt{b_i}} \right)^{-1} \left(P + \sum_{i=1}^{M} \frac{Q_i|f_i|^2}{b_i} \right). \tag{9}$$

In what follows, we solve the optimal relay power price analytically. Recall that M is the index such that $b_M \ge b_{lb} \ge b_{M+1}$. Define $\gamma_i \triangleq b_i$ for $i = 1, \cdots, M$ and $\gamma_{M+1} \triangleq b_{lb}$. Further define $\Gamma_1 \triangleq [\gamma_2, \gamma_1)$ and $\Gamma_i \triangleq [\gamma_{i+1}, \gamma_i]$ for $i =, 2 \cdots, M$. We thus have divided the possible price range $[b_{lb}, b_1)$ into the following M intervals:

$$[b_{lb}, b_1) = \Gamma_M \cup \Gamma_{M-1} \cdots \cup \Gamma_2 \cup \Gamma_1. \tag{10}$$

Because $\sum_{i=1}^{N} P_i^I(\lambda)$ is a non-increasing function of λ, inside this price range $[b_{lb}, b_1)$, we have $\sum_{i=1}^{N} P_i^I(\lambda) \le P$. Thus, from Lemma 2, $P_i^K(\lambda) = P_i^I(\lambda)$. For the price range Γ_i, we have $\lambda \ge b_{i+1}$, thus only Users $1, \cdots, i$ will purchase non-zero power. So the price optimization problem in (6) can be rewritten as

$$\max_{i=1,2,\cdots M} \underbrace{\max_{\lambda \in \Gamma_i} \sum_{j=1}^{i} \lambda P_j^I(\lambda)}_{\text{Sub-problem i}}. \tag{11}$$

In (11), we have decomposed the optimization problem into M subproblems, where the ith subproblem is to find the optimal price within the range Γ_i. This subproblem and thus the optimization problem in (11) are solved analytically in the following theorem, the proof of which can be found in [12].

Theorem 1. *Define $c_i \triangleq \left(\frac{\sum_{j=1}^{i} Q_j|f_j|^2/\sqrt{b_j}}{2\sum_{j=1}^{i} Q_j|f_j|^2/b_j} \right)^2, i = 1, \cdots, M$. The solution to Subproblem i is*

$$\lambda_i \triangleq \begin{cases} \gamma_{i+1} & \text{if } c_i < \gamma_{i+1}, \\ \gamma_i & \text{if } c_i > \gamma_i, \\ c_i & \text{if } \gamma_{i+1} \le c_i \le \gamma_i. \end{cases} \tag{12}$$

The optimal relay power price, denoted as λ^*, is

$$\lambda^* = \arg\max_{\lambda_i} \left\{ \left(\sum_{j=1}^{i} \frac{Q_j|f_j|^2}{\sqrt{b_j}} \right) \sqrt{\lambda_i} - \sum_{j=1}^{i} \frac{Q_j|f_j|^2}{b_j} \lambda_i \right\}. \tag{13}$$

With Theorem 1, we can find the optimal price for the relay power by solving the M sub-problems in (11) analytically, then find the optimal price among the M sub-problem solutions that results in the maximum relay revenue. We can see that its complexity is linear in the number of users in the network.

4. SIMULATION RESULTS

In this section, we show the simulated performance of the proposed relay power allocation and pricing solution, and compare it with the sum-rate-optimal and even power solutions. The sum-rate-optimal solution is the relay power allocation among users that maximizes the network sum-rate without fairness consideration. The even power solution allocates $1/N$ of the relay power to every of the N users, and every user decides how much power to buy to maximize its own utility but no more than $1/N$ of the relay power. We consider Rayleigh flat-fading channels, where f_i, h_i, and g_i are generated as i.i.d. random variables following the distribution $\mathcal{CN}(0, 1)$. The transmit power of the users is set to be 10 dB. The simulation results follow the same trend for other values of user powers.

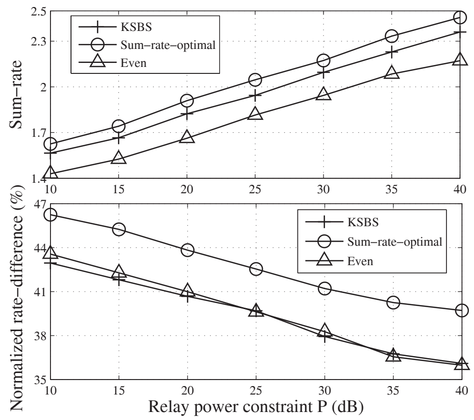


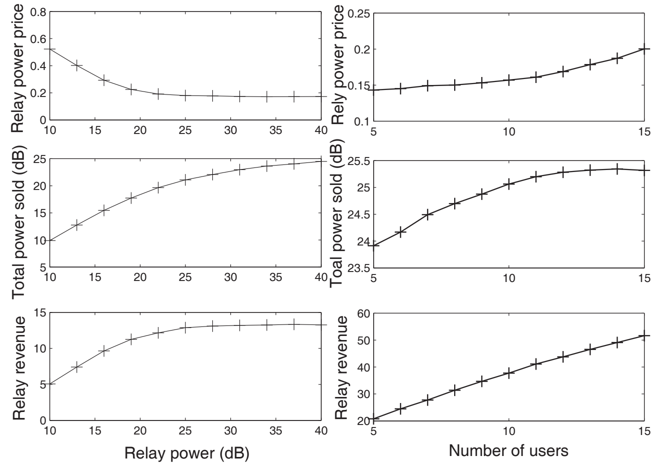
Fig. 2. System sum-rate and fairness of a three-user relay network.

In the first simulation, we assume that there are three users and the relay power ranges from 10 dB to 40 dB. We set the relay power price to be the optimal according to Theorem 1. Fig. 2 compares the network sum-rate and fairness of the proposed KSBS-based power allocation solution with those of the sum-rate-optimal and the even power solutions. It can be seen that for the system sum-rate, the difference between our algorithm and the sum-rate-optimal solutions is within 3.5%, while it is within 13% between the sum-rate-optimal and the even power solutions. The proposed solution is about 5 dB superior to the even power solution. For the fairness, we use the average value of the normalized difference: $[max_i(r_i) - min_i(r_i)]/max_i(r_i)$, where r_i is the achievable rate of User i. A smaller difference indicates a fairer solution. We can see that our solution has similar fairness to the even power solution and is fairer than the sum-rate-optimal one.

Fig. 3 shows the optimal relay power price, the relay power actually sold, and the maximum relay revenue with two different network settings. In Fig. 3(a), we use the same network setting as Fig. 2, which corresponds to the scenario where the total user demand is fixed while the relay power supply increases. We can see that when the relay has more power to sell, the optimal relay power price is lower, more relay power is sold, and the relay receives more revenue. This complies with one of the laws of supply and demand [11], which says that if supply increases and demand remains unchanged, it leads to lower equilibrium price and higher quantity. In Fig. 3(b), the relay power is fixed as 20 dB and the number of users varies from 5 to 15. This corresponds to the scenario where the relay power supply is fixed while the total user demand increases. From Fig. 3(b), we can see that as the the number of users increases, the optimal relay power price increases, the relay power actually sold increases, and the maximum relay revenue increases. This fits one of the laws of supply and demand, which says, if the supply is unchanged and demand increases, it leads to higher equilibrium price and quantity.

5. CONCLUSION

In this paper, we study the relay power allocation and pricing problems in a multi-user single-relay network. The Stackelberg game is used to model the interactions between the relay and the users, in which the relay is the leader and sets the price of its power to gain the maximum revenue, and the users are the followers who decide how much power to purchase for a given price. Bargaining game is used to model the interaction between the users and a KSBS-based solution is proposed to achieve a fair relay power allocation among users. Based on the KSBS solution, the optimal relay price is derived analytically. By comparing with the sum-rate-optimal and the even power allocations via simulations, we show that the proposed solution achieves the tradeoff between the network sum-rate and the fairness. It also reflects the laws of supply and demand.



(a) Increasing supply (b) Increasing demand

Fig. 3. Networks with increasing supply and demand.

6. REFERENCES

[1] L. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. on Information Theory*, vol. 50, pp. 3062-3080, Dec. 2004.

[2] R. Annavajjala, P. C. Cosman, and L. B. Milstein, "Statistical channel knowledge-based optimum power allocation for relaying protocols in the high SNR regime," *IEEE Trans. on Selected Areas in Communications*, vol. 25, pp. 292-305, Feb. 2007.

[3] K. T. Phan, Le-Ngoc Tho, S. A. Vorobyov, and C. Tellambura, "Power allocation in wireless multi-user relay networks," *IEEE Trans. on Wireless Communications*, vol. 8, pp. 1536-1276, May 2009.

[4] Y. Jing and H. Jafarkhani, "Relay power allocation in distributed space-time coded networks with channel statistical information," *IEEE Trans. on Wireless Communications*, vol. 10, pp. 443-449, Feb. 2011.

[5] Z. Zhang, J. Shi, H. Chen, M. Guizani, and P. Qiu, "A cooperation strategy based on Nash bargaining solution in cooperative relay networks," *IEEE Trans. on Vehicular Technology*, vol. 57, pp. 2570-2577, July 2008.

[6] G. Zhang, H. Zhang, L. Zhao, W. Wang, and L. Cong, "Fair resource sharing for cooperative relay networks using Nash bargaining solutions," *IEEE Communications Letters*, vol. 13, pp. 381-383, June 2009.

[7] S. Ren and M. van der Schaar, "Pricing and distributed power control in wireless relay networks," in *IEEE Trans. on Signal Processing*, vol. 59, pp. 2913-2926, June 2011.

[8] B. Wang, Z. Han, and K. J. R. Liu, "Distributed relay selection and power control for multiuser cooperative communication networks using Stackelberg Game," *IEEE Trans. on Mobile Computing*, vol. 8, pp. 975-990, July 2009.

[9] D. Fudenberg and J. Tirole, *Game Theory*. MIT Press, 1991.

[10] E. Kalai and M. Smorodinsky, "Other solutions to Nash's bargaining problem," *Econometrica*, vol. 43, pp. 513-518, May 1975.

[11] D. Besanko and R. R. Braeutigam, *Microeconomics*. Wiley Press, 2005.

[12] Q. Cao, H. Vicky Zhao, and Yindi Jing, "Power allocation and pricing in multi-user relay networks using Stackelberg and Bargaining games," submitted to *IEEE Transactions on Vehicular Technology*, available at http://arxiv.org/abs/1201.3056.