L_0 NORM BASED AFFINE PROJECTION SIGN ALGORITHM FOR SPARSE UNDERWATER ACOUSTIC CHANNEL ESTIMATION UNDER SYMMETRIC α-STABLE NOISE

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ABSTRACT

A new framework is proposed for deriving adaptive algorithms for sparse channel estimation under the presence of Symmetric α-Stable (SαS) noise. The algorithmic framework employs the natural gradient and incorporates both the L_p norm of the channel prediction error and the L_0 norm of the complex-valued channel taps. Based on this framework, a novel affine projection sign algorithm is derived and compared against the improved proportionate affine projection sign algorithm (IPAPSA) by estimating an experimental underwater acoustic (UWA) channel under the presence of simulated SαS noise. Enhanced convergence rate and tracking performance is demonstrated at the expense of a slight increase in computational complexity.

***Index Terms*—** impulsive noise, underwater acoustic communications, fractionally lower-order moment algorithms.

1. INTRODUCTION

Symmetric α-Stable (SαS) distributions model many random phenomena, such as fluctuations in gravitational fields, stock prices, low-frequency atmospheric noise, and man-made noises [1]. In underwater acoustics, Chitre *et al* [2] showed that the impulsive ambient noise due to snapping shrimp can be characterized as SαS. It is well known that, second and higher order moments of SαS random variables do not exist. Hence, any adaptive filter for system identification that is based on L_2 norm minimization will suffer poor performance.

Underwater acoustic (UWA) communication channels exhibit long, time-varying, and often sparse impulse responses [3]. Recently, an improved proportionate algorithm that employs the L_p norm, $p \in [1, 2)$, has shown robust performance under SαS noise [4]. Although this algorithm has linear complexity, it suffers from slow convergence rate when colored input signals are used. A notable algorithm that is robust under both impulsive noise and colored input signals and manages to exploit channel sparseness is the improved proportionate affine projection sign algorithm (IPAPSA) [5]. Although the IPAPSA employes a proportionate matrix to exploit sparseness, faster convergence will be possible if the L_0 norm of the filter taps is included in its cost function.

This paper is an incremental work of [4] and proposes an enhancement of the IPAPSA with a slight increase in complexity. The proposed algorithm is based on natural gradient adaptation [6] and incorporates an L_p norm of the channel prediction error and a differentiable L_0 norm of the complex-valued filter taps. The performance of the new algorithm is demonstrated by identifying an experimental sparse UWA channel under simulated SαS noise.

Notation and definitions: Superscripts $^\intercal$, †, and * stand for transpose, Hermitian transpose, and conjugate, respectively. Column vectors (matrices) are denoted by boldface lowercase (uppercase) letters. Let $z \in \mathbb{C}$ and $p \succeq 1$. The L_p norm of z is defined as $|z|_p \triangleq (|\operatorname{Re}\{z\}|^p + |\operatorname{Im}\{z\}|^p)^{1/p}$. The sign function of z is defined as $\operatorname{csgn}(z) \triangleq \operatorname{sgn}(\operatorname{Re}\{z\}) + j \cdot \operatorname{sgn}(\operatorname{Im}\{z\})$, where $\operatorname{sgn}(\cdot)$ stands for the sign function of a real scalar. Let $\mathbf{z} \in \mathbb{C}^N$. The sign function of $\mathbf{z}$ is given by the column vector $\operatorname{csgn}(\mathbf{z})$ with elements $\operatorname{csgn}(z_i)$, $i = 0, \dots, N-1$. The L_p norm of $\mathbf{z}$ is defined as $||\mathbf{z}||_p \triangleq (\sum_{i=0}^{N-1} |z_i|_p^p)^{1/p}$. The L_0 norm of $\mathbf{z}$, denoted as $||\mathbf{z}||_0$, equals to the number of the non-zero entries of $\mathbf{z}$.

2. THE IPAPSA

The original IPAPSA was applied to network echo cancellation (NEC) adaptive filters with real-valued coefficients [5]. For our purposes, we modify IPAPSA to include complex-valued coefficients. Let us consider an UWA channel, which is described by the unknown K-tap vector $\mathbf{h}[n] \triangleq [h_0[n]\, h_1[n] \dots h_{K-1}[n]]^\intercal$ at discrete time n. The channel output signal is given by

$$y[n] = \mathbf{h}[n]^\dagger \mathbf{x}[n] + w[n], \tag{1}$$

where $\mathbf{x}[n] \triangleq [x[n]\, x[n-1] \dots x[n-K+1]]^\intercal$ is the vector of the K most recent input samples and $w[n]$ is a complex SαS random variable. Let us denote $\hat{\mathbf{h}}[n]$ the estimate of $\mathbf{h}[n]$ and let L be the projection order of the IPAPSA. The a priori and a posteriori error vectors are defined as

$$\mathbf{e}[n]^* = [e[n]^* \, e[n-1]^* \ldots e[n-L+1]^*]^\intercal \quad (2)$$
$$= \mathbf{y}[n]^* - \mathbf{X}[n]^\dagger \hat{\mathbf{h}}[n-1] \quad (3)$$
$$\bar{\mathbf{e}}[n]^* = [\bar{e}[n]^* \, \bar{e}[n-1]^* \ldots \bar{e}[n-L+1]^*]^\intercal \quad (4)$$
$$= \mathbf{y}[n]^* - \mathbf{X}[n]^\dagger \hat{\mathbf{h}}[n], \quad (5)$$

where $\mathbf{X}[n]$=$[\mathbf{x}[n] \, \mathbf{x}[n-1] \ldots \mathbf{x}[n-L+1]]$ is the $K \times L$ matrix of input samples and $\mathbf{y}[n]$=$[y[n] \, y[n-1] \ldots y[n-L+1]]^\intercal$ contains the L most recent output samples. The IPAPSA channel update is given by [5]

$$\hat{\mathbf{h}}[n] = \hat{\mathbf{h}}[n-1] + \mu \frac{\mathbf{x}_{gs}[n]}{\sqrt{\delta + \|\mathbf{x}_{gs}[n]\|_2^2}}, \quad (6)$$

where $\mathbf{x}_{gs}[n]$=$\mathbf{G}[n-1]\mathbf{X}[n]\mathrm{csgn}(\mathbf{e}[n])^*$, $\mu \in (0,1)$ is the step-size parameter, and $\delta > 0$ is a regularization constant. The role of the proportionate matrix $\mathbf{G}[n]$ is to assign a time-varying step-size to each filter tap in proportion to the tap's magnitude so that active taps converge faster than inactive (zero or close to zero) ones. The most popular form of $\mathbf{G}[n]$ is given by the $K \times K$ diagonal matrix of the IPNLMS algorithm [7]. For complex-valued taps, we modify $\mathbf{G}[n]$ as follows

$$\frac{1-\beta}{2K} + (1+\beta)\frac{\left|\hat{h}_k[n]\right|_1}{2\left\|\hat{\mathbf{h}}[n]\right\|_1 + \epsilon}, \; 0 \le k \le K-1, \quad (7)$$

where ϵ is a small fixed constant to avoid division by zero, and $\beta \in [-1,1]$ is the parameter that controls the sparseness of the solution. Highly sparse channels should have β close to one while non-sparse channels should have $\beta = -1$.

3. NEW ALGORITHMIC FRAMEWORK

Here we consider the problem of channel identification under the presence of SαS noise when the characteristic exponent $\alpha \in (1,2)$. We focus on channel estimates that can be expressed as $\hat{\mathbf{h}}[n]$=$\hat{\mathbf{h}}[n-1] + \mathbf{r}[n]$, $\mathbf{r}[n]$ being the channel update vector. According to [8], an efficient adaptive algorithm must be conservative (avoid radical changes of $\hat{\mathbf{h}}[n]$ from one iteration to the other) and corrective (ensure better channel estimate if the same input and output were to be observed at two consecutive times). Towards this end, the new algorithm is derived by minimizing the following cost function:

$$J[n] = \|\bar{\mathbf{e}}[n]\|_p^p + \delta \mathbf{r}[n]^\dagger \mathbf{A}[n-1]\mathbf{r}[n] + \gamma \left\|\hat{\mathbf{h}}[n]\right\|_0, \quad (8)$$

where δ, γ positive parameters and $p < \alpha$ ensures convergence of the algorithm since any moment of the form $E[\|\bar{\mathbf{e}}[n]\|_p^p]$, $p \ge \alpha$, is infinite. The term $\mathbf{r}[n]^\dagger \mathbf{A}[n-1]\mathbf{r}[n]$ in (8) is the Riemannian distance between $\hat{\mathbf{h}}[n]$ and $\hat{\mathbf{h}}[n-1]$. The Riemannian metric tensor $\mathbf{A}[n-1]$ is a $K \times K$ positive definite matrix that describes the curvature of the K-dimensional parameter space at $\hat{\mathbf{h}}[n-1]$. The regularizing term $||\hat{\mathbf{h}}[n]||_0$ in (8) is used to further accelerate the convergence of the zero filter taps. Following the line of thought in [9], $||\hat{\mathbf{h}}[n]||_0$ is approximated by the differentiable function

$$\left\|\hat{\mathbf{h}}[n]\right\|_0 \simeq \sum_{k=0}^{K-1} 1 - e^{-\eta|\hat{h}_k[n]|_1}, \eta > 0. \quad (9)$$

Let ζ denote the smallest non-zero tap of the channel, then the parameter η may be chosen as $\eta \simeq 5/\zeta$. In practice, an estimate of ζ may be acquired by cross-correlating a short pulse before transmitting the actual data signal.

Taking the complex gradient of $J[n]$ with respect to $\mathbf{r}[n]^*$, we have the following terms:

$$\nabla_{\mathbf{r}[n]^*}\left(\|\bar{\mathbf{e}}[n]\|_p^p\right) = \frac{-p}{2}\sum_{k=0}^{L-1} |\bar{e}[n\text{-}k]|_{p-1}^{p-1} \mathrm{csgn}(\bar{e}[n\text{-}k])^* \mathbf{x}[n\text{-}k] \quad (10)$$

$$\nabla_{\mathbf{r}[n]^*}\left(\delta \mathbf{r}[n]^\dagger \mathbf{A}[n-1]\mathbf{r}[n]\right) = \delta \mathbf{A}[n-1]\mathbf{r}[n]. \quad (11)$$

In addition, the gradient of (9) with respect to $r_k[n]^*$, k=0, $\ldots$, $K-1$ is equal to

$$\nabla_{r_k[n]^*}\left(\gamma \left\|\hat{\mathbf{h}}[n]\right\|_0\right) = \frac{\gamma\eta}{2} e^{-\eta|\hat{h}_k[n]|_1} \mathrm{csgn}\left(\hat{h}_k[n]\right)$$
$$= \frac{\gamma\eta}{2}\nu_k[n] \quad (12)$$

Setting $\nabla_{\mathbf{r}[n]^*} J[n]$=0, and combining terms from (10), (11), and (12) we note that the resulting equation is non-linear with respect to $\mathbf{r}[n]$. To solve for $\mathbf{r}[n]$, it is plausible to assume that at steady-state $\mathbf{e}[n] \simeq \bar{\mathbf{e}}[n]$. Thus, we now have

$$\mathbf{r}[n] = \mathbf{A}[n-1]^{-1}\left(\frac{p}{2\delta}\mathbf{X}[n]\mathbf{s}[n] - \frac{\gamma\eta}{2\delta}\boldsymbol{\nu}[n-1]\right) \quad (13)$$

where $\boldsymbol{\nu}[n]$ is the vector with entries $\nu_k[n]$ and $\mathbf{s}[n]$ is the vector with elements $|e[n\text{-}k]|_{p-1}^{p-1}\mathrm{csgn}(e[n\text{-}k])^*$, k=0, $\ldots$, $L-1$. To further simplify the algorithm and avoid matrix inversion issues, we choose $\mathbf{A}[n]$=$\mathbf{G}[n]^{-1}$. Moreover, we introduce a step size parameter, $\mu \in (0,1]$, that exercises control over the change of the tap values from one iteration to the next. Thus, the channel update vector is deduced as follows:

$$\hat{\mathbf{h}}[n] = \hat{\mathbf{h}}[n-1] + \mu \mathbf{G}[n-1]\left(\frac{p}{2\delta}\mathbf{X}[n]\mathbf{s}[n] - \frac{\gamma\eta}{2\delta}\boldsymbol{\nu}[n-1]\right). \quad (14)$$

Equations (14) and (7) will be called the L$_0$-IPAPSA hereafter. The algorithm is initialized for $\hat{\mathbf{h}}[0]$=$\mathbf{0}$. Since $\mathbf{G}[n]$ is diagonal, the number of complex multiplications required for L$_0$-IPAPSA is $K(L+6)$+2L, slightly increased compared to that of IPAPSA. Note that if L=1, the L$_0$-IPAPSA boils down to a new L$_0$ norm constrained sign-error IPNLMS algorithm (we name it L$_0$-sIPNLMS hereafter) while if L=1 and γ=0, then the L_0-IPAPSA reduces to the sIPNLMS algorithm of [4]. Moreover, when L=1, γ=0, and β=-1 then L_0-IPAPSA boils down to the Least Mean p-Norm (LMP) algorithm [1]. Two final remarks are in order: (a) equation (8) provides a platform for generating new sparse, affine projection, L_p norm based algorithms if different matrices $\mathbf{A}[n]$ and L_0 norm proxies are used and (b) using (9) in (8) renders $J[n]$ a non-convex cost function and therefore, the algorithm could in principle stall at a local minimum. However, if γ is chosen sufficiently small (as we show below), then the algorithm always converges to meaningful solutions.

4. SIMULATION RESULTS

We now compare L_0-IPAPSA against IPAPSA, IPAPA [11], L_0-sIPNLMS, and sIPNLMS [4] by performing two computer generated experiments. The colored input signal is generated by a 6250 symbols/ s-rate Q-PSK modulated m-sequence that is pulse-shaped by a raised cosine filter with roll-off factor 0.25 and truncation length $\pm$ 4 symbol intervals. The output signal is generated in baseband at 12.5 kHz rate (2 samples/symbol) by using (1). The impulse responses to be identified were measured during the Focused Acoustic Fields (FAF) experiment off the coast of Pianosa Island, Italy in 2005. Fig. 1(a) illustrates the sparse, time-varying channel to be identified. To generate this figure, 156 FAF successive impulse responses are employed where each response is kept fixed for 9.6 ms during transmission. The channel length in samples is K=438 (35 ms). The performance measure is the normalized misadjustment (in dB), $20\log_{10}(||\mathbf{h}[n] - \hat{\mathbf{h}}[n]||_2/||\mathbf{h}[n]||_2)$ and is computed after averaging 200 independent runs. In all simulations below, p=1, β=0.5, η=100, γ=$0.01(2\delta/\mu\,\eta)$ and δ_{IPAPA}=$c(1-\beta)\sigma_x^2/2K$, c being a fixed parameter and $\sigma_x^2 = 1$ is the power of the input signal.

The first experiment tests the misadjustment of all algorithms by estimating the channel of Fig. 1(a) under SαS noise. The simulated SαS noise is generated based on the signal parameters used in the FAF experiment, i.e., 7.81 kHz bandwidth, 37.5 kHz sampling rate, and 12 kHz carrier frequency. The SαS noise parameter α=1.65. The noise simulator can be found in [12]. Fig. 1(b) reports on the misadjustment when the signal-to-noise ratio (denoted as SNR_α and defined in [4]) is $30\,dB$. Clearly, the L_0-IPAPSA shows the lowest misadjustment from all other algorithms. Note that both L_0 norm-based algorithms exhibit better tracking than IPAPSA validating the use of the $||\hat{\mathbf{h}}[n]||_0$. Fig. 1(c) reports on the misadjustment when SNR_α=15 dB. Observe that the L_0 norm-based algorithms still have better tracking than IPAPSA but L_0-IPAPSA shows only $0.5dB$ better tracking than L_0-sIPNLMS after 1 s. Finally, we see that the IPAPA has poor performance for both high and low SNR_α, corroborating that L_2 norm minimization criterion is not a judicious choice for SαS noise.

The second experiment tests the misadjustment of all al-

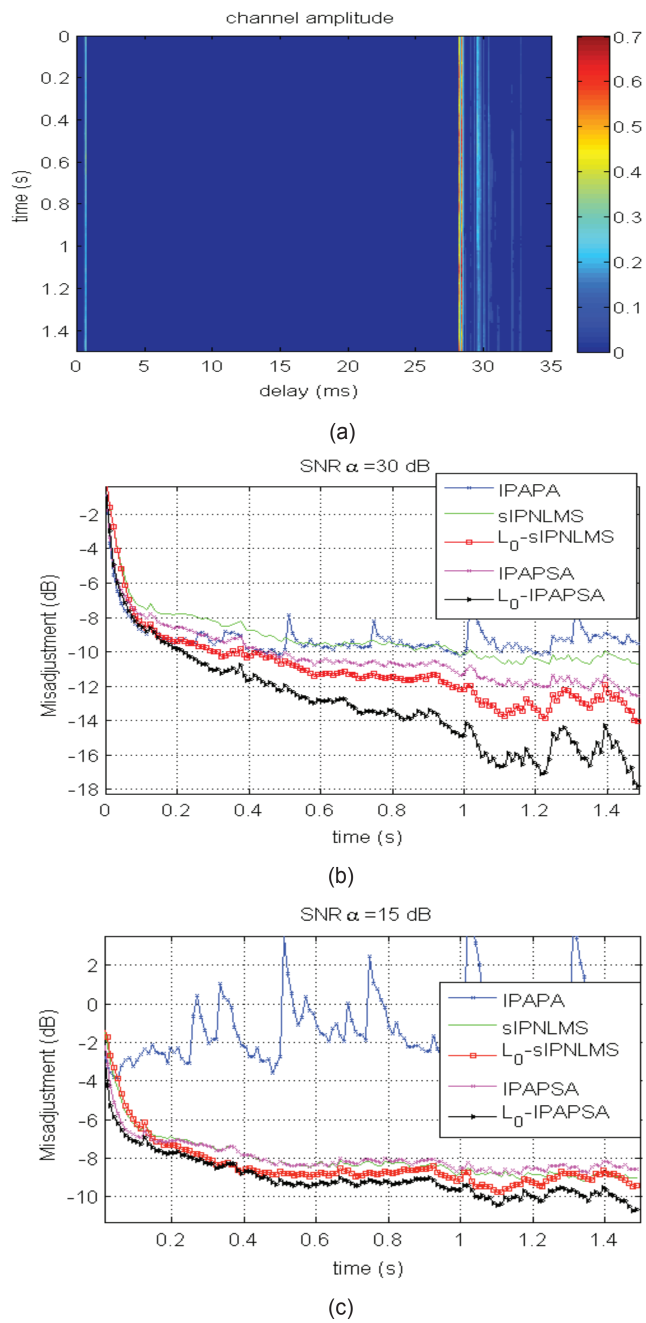

Fig. 1. (a) Channel used in simulations. The x-axis shows multipath delay, the y-axis shows absolute time and the colorbar shows amplitude in linear scale. (b) Misadjustment for all algorithms when SNR_α=30 dB, $\mu_{L_0-IPAPSA}$=0.1, μ_{IPAPSA}= 0.03, $\mu_{sIPNLMS}$= $\mu_{L_0-sIPNLMS}$= 0.4, μ_{IPAPA}= 0.2, c= 10, $\delta_{L_0-IPAPSA}$= $\delta_{sIPNLMS}$= $\delta_{L_0-sIPNLMS}$= 2, δ_{IPAPSA}= 0.00013, L= 20. (c) Misadjustment for all algorithms when SNR_α= 15 dB, μ_{IPAPSA}= 0.005, $\delta_{L_0-IPAPSA}$=$\delta_{sIPNLMS}$=$\delta_{sIPNLMS-L_0}$= 3. All other parameters are the same as those in (b).

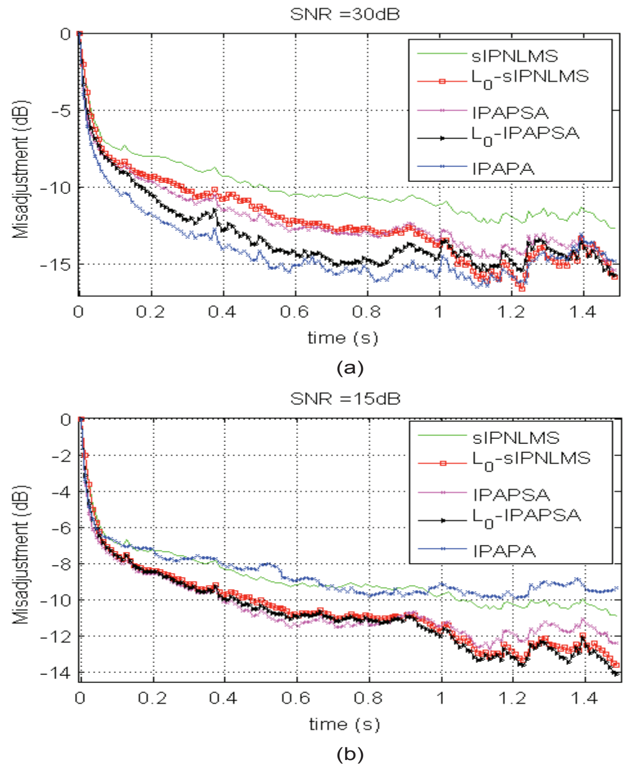

Fig. 2. (a) Misadjustment for all algorithms when SNR= $30\,dB$, μ_{IPAPSA}= 0.07, μ_{IPAPA}= 0.9, c = 20, $\delta_{L_0-IPAPSA}$= $\delta_{sIPNLMS}$= $\delta_{sIPNLMS-L_0}$= 1. Other parameters are the same as those in Fig. 1(b). (b) Misadjustment for all algorithms when SNR= $15\,dB$, c= 100. Other parameters are the same as those in (a).

gorithms by using the channel of Fig. 1(a) in the presence of complex Gaussian noise. Recall that Gaussian noise is a class of SαS noise when $\alpha = 2$. Fig. 2(a) is generated for SNR=$30\,dB$. We see that IPAPA has the lowest misadjustment for $1.1\,$s but after it shows similar performance with that of the L_0 norm-based algorithms. Note also that L_0-IPAPSA offers better tracking than IPAPSA for most of the time. Fig. 2(b) shows the misadjustment for all algorithms when SNR=$15\,dB$. Surprisingly, all the affine projection sign algorithms demonstrate better tracking than IPAPA. We also see that L_0-IPAPSA shows better tracking than IPAPSA after $1\,$s.

5. CONCLUSION

A novel algorithm, the L_0-IPAPSA, was proposed for sparse channel identification in the presence of SαS noise. The proposed algorithm was based on the L_p norm minimization criterion and used both the natural gradient and the L_0 norm of the channel response to exploit channel sparseness. The L_0-IPAPSA was compared against the IPAPSA by estimating an experimental sparse, time-varying acoustic link under simulated SαS noise. The clear superiority of L_0-IPAPSA was demonstrated in high and low SNR.

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