JOINT POWER AND ADMISSION CONTROL VIA LINEAR PROGRAMMING DEFLATION

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ABSTRACT

In an interference network, joint power and admission control aims to support a maximum number of links at their specified signal to interference plus noise ratio (SINR) targets while using a minimum total transmission power. Since this problem is NP-hard, convex approximation heuristics have been considered in the literature. In this work, we first reformulate the problem as a sparse ℓ_0-minimization problem and then relax it to a linear program (LP). Then, we derive an easily-checkable necessary condition for all links in the network to be simultaneously supported at their target SINR levels, and use it to iteratively remove strong interfering links (deflation). Numerical simulations show the proposed heuristic compares favorably with the existing approaches in terms of both the number of supported links and speed.

Index Terms— Admission control, convex approximation, power control, sparse optimization.

1. INTRODUCTION

Power control has been studied extensively in the contexts of cellular, ad-hoc, and cognitive underlay networks [1, 2, 3, 4, 5]. Its aim is to use the minimum total transmission power to support all links in an interference network at their SINR targets. In this way, the network can enjoy a high spectral efficiency.

A longstanding issue associated with power control is that it often becomes infeasible, i.e., not all links in the network can be simultaneously supported at their SINR targets. This paper focuses on the infeasibility issue and examines efficient ways to selectively reject links so that the remaining ones can be simultaneously supported at their desired SINR levels. The goal is to maximize the number of links simultaneously supportable at their required SINR targets while using the minimum total transmission power.

This joint power and admission control problem can be solved to global optimality by checking the simultaneous supportability of every subset of links. However, the computational complexity of this enumeration approach grows exponentially with the total number of links. Theoretically, the problem is known to be NP-hard [1], so various heuristic algorithms have been proposed for this problem. Among them, the GRN-DCPC algorithm is proposed in [3], where the power is updated by a modified version of Foschini-Miljanic algorithm [4]. A convex approximation-based algorithm is derived in [1] for the joint power and admission control in cognitive underlay networks. Instead of directly solving the original NP-hard problem, the idea of the proposed linear programming deflation (LPD) algorithm is to approximate the problem by an LP whose solution can be used to iteratively remove interfering links. The recent work [2] also develops a removal-based algorithm for this problem.

In this paper, we reformulate the joint power and admission control problem as a sparse ℓ_0-minimization problem. We then use the ℓ_1-relaxation to derive a linear program (different from that in [1]) whose solution can be used to guide an iterative link removal procedure (deflation). Numerical results show that the proposed algorithm compares favorably with the existing approaches [1, 2, 3] in terms of both the number of supported links and speed.

Notation: We use $\|\mathbf{x}\|_0$ to denote the number of nonzero entries in a vector $\mathbf{x}$. For any subset $\mathcal{I} \subseteq \mathcal{K}$, we use $\mathbf{A}_{\mathcal{I}}$ to denote the matrix formed by the rows of $\mathbf{A}$ indexed by $\mathcal{I}$. For a vector $\mathbf{x}$, the notation $\mathbf{x}_{\mathcal{I}}$ is similarly defined. We use $\mathbf{e}$ to represent the vector with all components being one.

2. PROBLEM FORMULATION

Consider a K-link (K transmitter and receiver pairs) single-input single-output interference channel with channel gains $g_{kj} \geq 0$ (from the transmitter of link j to the receiver of link k), noise power $\eta_k > 0$, SINR target $\gamma_k > 0$, and power budget $p_k^{\max} > 0$ for $k, j \in \mathcal{K} := \{1, 2, \cdots, K\}$. Denote the power allocation vector by $\mathbf{p} = (p_1, p_2, \cdots, p_K)^T$ and the power budget vector by $\mathbf{p}^{\max} = (p_1^{\max}, p_2^{\max}, \cdots, p_K^{\max})^T$. The joint power and admission control problem can be mathematically formulated as a two-stage optimization problem, i.e., the first stage maximizes the number of admitted links:

$$\begin{aligned} \max_{\mathbf{p},\, \mathcal{S}} \quad & |\mathcal{S}| \\ \text{s.t.} \quad & \text{SINR}_k \geq \gamma_k,\ k \in \mathcal{S} \subseteq \mathcal{K}, \\ & \mathbf{0} \leq \mathbf{p} \leq \mathbf{p}^{\max}, \end{aligned} \tag{1}$$

THIS WORK IS SUPPORTED IN PART BY THE CHINA NFDYS, GRANT NO. 11125107, THE NSFC, GRANT NO. 10831006, THE CAS GRANT KJCX-YW-S7, AND IN PART BY THE NSF, GRANT NO. DMS-1015346. THE AUTHORS WISH TO THANK N. D. SIDIROPOULOS FOR HIS HELP IN NUMERICAL SIMULATIONS.

where the SINR value at the k-th receiver is

$$\text{SINR}_k = \frac{g_{kk} p_k}{\eta_k + \sum_{j \neq k} g_{kj} p_j};$$

the second stage minimizes the total transmission power required to support these admitted links:

$$\begin{array}{ll} \min\limits_{\mathbf{p}} & \sum_{k \in \mathcal{S}_0} p_k \\ \text{s.t.} & \text{SINR}_k \geq \gamma_k,\ 0 \leq p_k \leq p_k^{\max},\ k \in \mathcal{S}_0, \end{array} \tag{2}$$

where $\mathcal{S}_0$ is the maximum admissible set for the problem (1).

3. A NEW LINEAR PROGRAMMING DEFLATION ALGORITHM

Similar to [1], we develop a new LPD algorithm for the joint control problem (1) and (2) in this section. We first define some notations. Let us denote

$$\mathbf{c} = \left(\frac{\gamma_1 \eta_1}{g_{11} p_1^{\max}}, \frac{\gamma_2 \eta_2}{g_{22} p_2^{\max}}, \cdots, \frac{\gamma_K \eta_K}{g_{KK} p_K^{\max}} \right)^T > \mathbf{0}, \tag{3}$$

define a matrix $\mathbf{A}$ with its (k, j)-th entry

$$a_{kj} = \begin{cases} \dfrac{\gamma_k g_{kj} p_j^{\max}}{g_{kk} p_k^{\max}}, & \text{if } k \neq j, \\ 0, & \text{if } k = j. \end{cases} \tag{4}$$

Denote $\mathbf{B} = \mathbf{I} - \mathbf{A}$, and let $\mathbf{q} = (q_1, q_2, \cdots, q_K)^T$ with

$$q_k = p_k / p_k^{\max},\ \forall\, k \in \mathcal{K}.$$

Then the constraints in (1) can be equivalently rewritten as

$$\begin{cases} (\mathbf{B}\mathbf{q} - \mathbf{c})_k \geq 0,\ \forall\, k \in \mathcal{S} \subseteq \mathcal{K}, \\ \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}. \end{cases} \tag{5}$$

In particular, link k is supported at its SINR level ($\text{SINR}_k \geq \gamma_k$) if and only if $(\mathbf{B}\mathbf{q} - \mathbf{c})_k \geq 0$.

3.1. ℓ_0-minimization reformulation

Lemma 3.1 (Balancing Lemma [5]) *Suppose there exists a vector $\tilde{\mathbf{q}} \geq \mathbf{0}$ such that $\tilde{\mathbf{q}} \geq \mathbf{A}\tilde{\mathbf{q}} + \mathbf{c}$, where $\mathbf{A}$ and $\mathbf{c}$ are defined in* (4) *and* (3). *Then, there exists a vector $\bar{\mathbf{q}}$ satisfying $\mathbf{c} \leq \bar{\mathbf{q}} \leq \tilde{\mathbf{q}}$, $\bar{\mathbf{q}} = \mathbf{A}\bar{\mathbf{q}} + \mathbf{c}$, and the vector $\bar{\mathbf{q}}$ solves problem*

$$\begin{array}{ll} \min\limits_{\mathbf{q}} & \mathbf{e}^T \mathbf{q} \\ \textit{s.t.} & \mathbf{q} \geq \mathbf{A}\mathbf{q} + \mathbf{c},\ \mathbf{q} \geq \mathbf{0}. \end{array}$$

Based on Lemma 3.1, we can reformulate the two-stage joint power and admission control problem (1) and (2) as a single-stage sparse optimization problem

$$\begin{array}{ll} \min\limits_{\mathbf{x},\, \mathbf{q}} & \|\mathbf{x}\|_0 + \alpha\, \mathbf{e}^T \mathbf{q} \\ \text{s.t.} & \mathbf{x} = \mathbf{B}\mathbf{q} - \mathbf{c},\ \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}, \end{array} \tag{6}$$

where $0 < \alpha < 1/|\mathcal{K}|$ is a constant.

Theorem 3.1 *The optimal value of problem* (1) *is M if and only if $\|\mathbf{x}^*\|_0 = K - M$, where $(\mathbf{x}^*, \mathbf{q}^*)$ is the solution to problem* (6).

Proof: We first show that the optimal value of problem (1) is M if and only if the minimum value of the following problem

$$\begin{array}{ll} \min\limits_{\mathbf{x},\, \mathbf{q}} & \|\mathbf{x}\|_0 \\ \text{s.t.} & \mathbf{x} = \mathbf{B}\mathbf{q} - \mathbf{c},\ \mathbf{0} \leq \mathbf{q} \leq \mathbf{e} \end{array} \tag{7}$$

is $K - M$.

Let us first show the "if" direction. Suppose that the optimal value of (7) is $K - M$ and $(\tilde{\mathbf{x}}, \tilde{\mathbf{q}})$ is an optimal solution. Then $\|\tilde{\mathbf{x}}\|_0 = K - M$, implying $\tilde{\mathbf{x}}_{\mathcal{I}} = \mathbf{0}$ for some index set $\mathcal{I} \subseteq \mathcal{K}$ with $|\mathcal{I}| = M$. Since $\tilde{\mathbf{x}}_{\mathcal{I}} = \mathbf{B}_{\mathcal{I}}\tilde{\mathbf{q}} - \mathbf{c}_{\mathcal{I}} = \mathbf{0}$, it follows from the definition of $\mathbf{B}$ and $\mathbf{c}$ that $\text{SINR}_k = \gamma_k$ for all $k \in \mathcal{I}$. Thus, all links in $\mathcal{I}$ are supported at their target SINR levels, implying that the optimal value of (1) is at least M.

We now show the "only if" direction. Suppose M links can be supported in (1), we know from Lemma 3.1 that there exists a feasible $\bar{\mathbf{q}}$ such that M components of $\bar{\mathbf{x}} = \mathbf{B}\bar{\mathbf{q}} - \mathbf{c}$ are zero; hence the optimal value of problem (7) is at most $K - M$. This establishes the equivalence between (1) and (7).

Now denote $(\mathbf{x}^*, \mathbf{q}^*)$ and $(\tilde{\mathbf{x}}, \tilde{\mathbf{q}})$ to be the optimal solutions to problems (6) and (7), respectively. Since $(\mathbf{x}^*, \mathbf{q}^*)$ is feasible for problem (7), it is easy to see that $\|\tilde{\mathbf{x}}\|_0 \leq \|\mathbf{x}^*\|_0$. We also conclude that $\|\mathbf{x}^*\|_0 \leq \|\tilde{\mathbf{x}}\|_0$. Otherwise, if $\|\mathbf{x}^*\|_0 \geq \|\tilde{\mathbf{x}}\|_0 + 1$, we can get from this and the fact that $\alpha\, \mathbf{e}^T \mathbf{q} < 1$ for any $\mathbf{0} \leq \mathbf{q} \leq \mathbf{e}$ that

$$\|\mathbf{x}^*\|_0 + \alpha\, \mathbf{e}^T \mathbf{q}^* \geq \|\mathbf{x}^*\|_0 \geq \|\tilde{\mathbf{x}}\|_0 + 1 > \|\tilde{\mathbf{x}}\|_0 + \alpha\, \mathbf{e}^T \tilde{\mathbf{q}},$$

which is a contradiction to the optimality of $(\mathbf{x}^*, \mathbf{q}^*)$. Thus we must have $\|\mathbf{x}^*\|_0 \leq \|\tilde{\mathbf{x}}\|_0$ and hence $\|\mathbf{x}^*\|_0 = \|\tilde{\mathbf{x}}\|_0$. This completes our proof. **Q.E.D.**

From the above proof, we see that problem (6) can serve the same role as (7) in finding the maximum admissible set for problem (1) while minimizing the total power required to support these links. Indeed, if there are more than one maximum admissible set (i.e., the solution for (1) is not unique), the formulation (6) is capable of picking the one with minimum total transmission power.

As a whole, the reformulation (6) takes both admission control and power control into consideration. The first term in the objective function of (6) can be regarded as the admission control term since the more zero components $\mathbf{x}$ has, the more links will be supported. The second term is the power control term. The parameter α is used to balance the admission control term $\|\mathbf{x}\|_0$ and the power control term $\mathbf{e}^T \mathbf{q}$. In general, a small value of α is preferable since it gives priority to maximizing the number of supported links. The choice of α will be discussed later.

3.2. Linear programming relaxation

Since ℓ_0-optimization problem (6) is still NP-hard, it is natural to consider its ℓ_1-convex relaxation

$$\begin{array}{ll} \min\limits_{\mathbf{x},\, \mathbf{q}} & \|\mathbf{x}\|_1 + \alpha\, \mathbf{e}^T \mathbf{q} \\ \text{s.t.} & \mathbf{x} = \mathbf{B}\mathbf{q} - \mathbf{c},\ \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}. \end{array} \tag{8}$$

Denote $\mathcal{K}^{+}= \{k \mid \tilde{x}_k > 0\}$, $\mathcal{K}^{=}= \{k \mid \tilde{x}_k = 0\}$, and $\mathcal{K}^{-} =\{k \mid \tilde{x}_k < 0\}$, where $(\tilde{\mathbf{x}}, \tilde{\mathbf{q}})$ is the solution to problem (8). We claim $|\mathcal{K}^{+}| = 0$. Assume the contrary that $|\mathcal{K}^{+}| \geq 1$. Then by Lemma 3.1, we can appropriately reduce the power of links in $\mathcal{K}^{+} \cup \mathcal{K}^{=}$ so that both the first term and the second term in the objective of (8) are strictly decreased. Therefore, the ℓ_1-relaxation problem (8) is *equivalent* to the following linear program

$$\begin{array}{ll} \min\limits_{\mathbf{q}} & \mathbf{e}^T(\mathbf{c}-\mathbf{B}\mathbf{q}) + \alpha\,\mathbf{e}^T\mathbf{q} \\ \text{s.t.} & \mathbf{c}-\mathbf{B}\mathbf{q} \geq \mathbf{0},\ \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}. \end{array} \tag{9}$$

Notice that the LPD algorithm in [1] is also based on the LP relaxation. However, the LP approximation (9) and the one in [1] are different. For instance, the *reversed* SINR constraints make LP (9) always feasible ($\mathbf{q} = \mathbf{0}$ is one feasible solution); while some extra parameters need to be introduced to make LP in [1] feasible. In addition, the auxiliary admission variables double the number of unknowns in [1], which makes the number of the unknown variables in their LP twice as large as ours.

Define $q_k^e = (\mathbf{c}-\mathbf{B}\mathbf{q})_k$, $\forall\, k \in \mathcal{K}$. This quantity measures the excess transmission power [1] that the transmitter of link k needs in order to be served with its SINR target, assuming other links keep their transmission powers unchanged. Therefore, (9) actually minimizes a weighted sum of the total excess transmission power and the total transmission power. With an appropriate choice of parameter α, linear program (9) enjoys a nice "never-over-removal" property.

Theorem 3.2 *Assume that there exists some vector $\bar{\mathbf{q}}$ such that $\mathbf{0} \leq \bar{\mathbf{q}} \leq \mathbf{e}$ and $\mathbf{B}\bar{\mathbf{q}} = \mathbf{c}$. Then, the matrix $\mathbf{B}$ must be invertible and there holds*

$$\alpha^* := 1/\max\{\mathbf{z}\} > 0, \tag{10}$$

where $\mathbf{z}$ satisfies $\mathbf{B}^T\mathbf{z} = \mathbf{e}$. Furthermore, the vector $\bar{\mathbf{q}}$ solves linear program (9) provided that α satisfies $0 \leq \alpha \leq \alpha^*$.

We give an outline proof of Theorem 3.2. According to [2], if there exists some vector $\bar{\mathbf{q}}$ satisfying $\mathbf{0} \leq \bar{\mathbf{q}} \leq \mathbf{e}$ and $\mathbf{B}\bar{\mathbf{q}} = \mathbf{c}$, then $\rho(\mathbf{A}) < 1$, which further implies $(\mathbf{B}^T)^{-1}\mathbf{e} > \mathbf{0}$. The second part of Theorem 3.2 can be shown by checking the KKT condition of (9).

Combining Theorems 3.1 and 3.2, we can provide the following reasonable choice for the parameter α in (9),

$$\alpha = \begin{cases} c/|\mathcal{K}|, & \text{if } \rho(\mathbf{A}) \geq 1, \\ \min\{c/|\mathcal{K}|,\, \alpha^*\}, & \text{if } \rho(\mathbf{A}) < 1, \end{cases} \tag{11}$$

where $c < 1$ is a constant and α^* is given in (10).

It is evident that problem (9) is not equivalent to problem (6). Nevertheless, the solution to (9) can provide us some insight into problem (6). By solving (9) with α given in (11), we know whether all links in the network can be simultaneously supported or not since the solution $\mathbf{q}$ satisfies $\mathbf{B}\mathbf{q} = \mathbf{c}$ if and only if all links can be supported simultaneously. More importantly, having obtained the solution of (9), we can use the efficient removal strategy in [1], i.e., drop link k_0 with

$$k_0 = \arg\max_{k\in\mathcal{K}} \left\{ \sum_{j\in\mathcal{K}} a_{jk} q_k^e + \sum_{j\in\mathcal{K}} a_{kj} q_j^e \right\}. \tag{12}$$

3.3. A necessary condition

We now derive a necessary condition for all links in the network to be simultaneously supportable. Suppose that all links can be simultaneously served. Then there exists a vector $\mathbf{q}$ such that $\mathbf{0} \leq \mathbf{q} \leq \mathbf{e}$ and

$$\mathbf{B}\mathbf{q} = (\mathbf{I}-\mathbf{A})\mathbf{q} = \mathbf{c}. \tag{13}$$

Hence, $\mathbf{q} = \mathbf{A}\mathbf{q} + \mathbf{c} \geq \mathbf{c}$. Denote $\boldsymbol{\mu} = \mathbf{B}^T\mathbf{e}$, $\boldsymbol{\mu}_+ = \max\{\boldsymbol{\mu}, \mathbf{0}\}$, and $\boldsymbol{\mu}_- = \max\{-\boldsymbol{\mu}, \mathbf{0}\}$. It is obvious that $\boldsymbol{\mu} = \boldsymbol{\mu}_+ - \boldsymbol{\mu}_-$. Multiplying $\mathbf{e}^T$ from both sides of (13), we get that $(\boldsymbol{\mu}_+ - \boldsymbol{\mu}_-)^T\mathbf{q} = \mathbf{e}^T\mathbf{c}$. Moreover, we can obtain

$$\boldsymbol{\mu}_+^T\mathbf{e} \geq \boldsymbol{\mu}_+^T\mathbf{q} = \boldsymbol{\mu}_-^T\mathbf{q} + \mathbf{e}^T\mathbf{c} \geq (\boldsymbol{\mu}_- + \mathbf{e})^T\mathbf{c},$$

where the first inequality is due to $\mathbf{q} \leq \mathbf{e}$ and the last one is due to $\mathbf{q} \geq \mathbf{c}$. Therefore, the condition

$$\boldsymbol{\mu}_+^T\mathbf{e} - (\boldsymbol{\mu}_- + \mathbf{e})^T\mathbf{c} \geq 0 \tag{14}$$

is necessary for all links in the network to be simultaneously supported.

Condition (14) allows us to iteratively remove strong interfering links until it becomes true. In particular, we drop the link

$$k_0 = \arg\max_{k\in\mathcal{K}} \left\{ \sum_{j\in\mathcal{K}} a_{kj} + \sum_{j\in\mathcal{K}} a_{jk} + c_k \right\}. \tag{15}$$

Assuming $\mathbf{q} = \mathbf{e}$, the above operation can be interpreted as removing the link with the largest interference plus noise footprint in the network.

3.4. A new liner programming deflation algorithm

The basic idea of the proposed NLPD algorithm is to solve linear program (9) under necessary condition (14) and check whether all links are supported or not; if not, remove a link (mathematically, delete the corresponding row and column of $\mathbf{B}$ and the corresponding entry of $\mathbf{c}$) from the network, and solve a reduced linear program (9) again until all the remaining links are supported.

A New Linear Programming Deflation (NLPD) Algorithm

Step 1. Initialization: Input data $(\mathbf{B}, \mathbf{c})$.

Step 2. Preprocessing: Remove link k_0 iteratively according to (15) until condition (14) holds true.

Step 3. Power control: Compute parameter α by (11) and solve linear program (9); check whether all links are supported: if yes, terminate the algorithm; else go to **Step 4**.

Step 4. Admission control: Remove link k_0 according to (12), set $\mathcal{K} = \mathcal{K}/\{k_0\}$, and go to **Step 3**.

We compare the proposed NLPD algorithm and the LPD algorithm in [1] in terms of the computational complexity needed to drop one link from the network. Since both require solving a linear program, their asymptotic complexities are both equal to $O\left(|\mathcal{K}|^{3.5}\right)$, although the LPD algorithm solves a LP with twice as many variables. By comparison, the Algorithm II-B in [2] has a complexity of $O(|\mathcal{K}|^4)$, since it needs to solve $|\mathcal{K}|$ eigenvalue problems to check whether all links in the network can be supported.

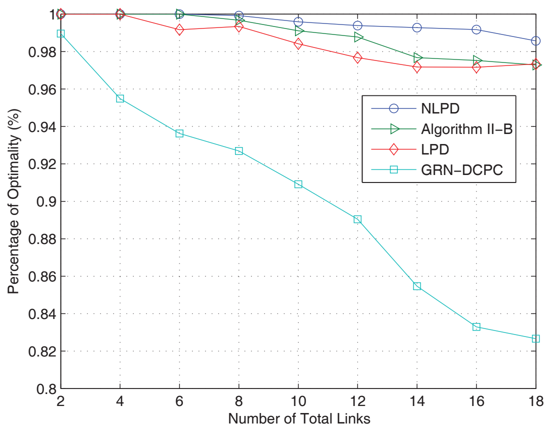

Fig. 1. Percentage of optimality versus the number of total links.

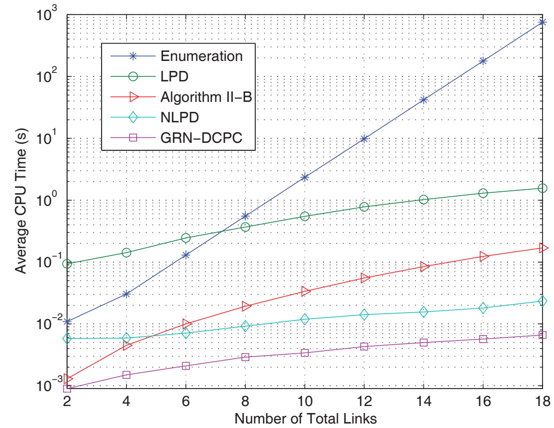

Fig. 2. Average CPU time versus the number of total links.

4. NUMERICAL SIMULATIONS

In our numerical simulations, we generate the channel parameters in the same way as in [1], i.e., each transmitter's location obeys the uniform distribution over a 2 Km $\times$ 2 Km square and the location of its corresponding receiver is uniformly generated in a disc with radius 400 m; channel gains are given by $g_{kj} = 1/d_{kj}^4$ $(\forall\ k, j \in \mathcal{K})$, where d_{kj} is the Euclidean distance from the link of transmitter j to the link of receiver k. Each link's SINR target is set to be $\gamma_k = 2$ dB $(\forall\ k \in \mathcal{K})$ and the noise power is set to be $\eta_k = -60$ dBm $(\forall\ k \in \mathcal{K})$. The power budget of the link of transmitter k is $p_k^{\max} = 4p_k^{\min}$ $(\forall\ k \in \mathcal{K})$, where $p_k^{\min}$ is the minimum power needed for link k to meet its SINR requirement without any interference from other links.

All figures are averaged over 200 Monte-Carlo runs. The parameter c in (11) is set to be 0.999. The number of supported links, total transmission power, and CPU time are employed as the metric to compare different algorithms, including the LPD algorithm in [1], the Algorithm II-B in [2], the GRN-DCPC algorithm in [3], and the proposed NLPD algorithm. In each case, the global optimal solution obtained by "brute force" enumeration is used as the benchmark.

The vertical axis in Fig. 1 shows the average percentage of global optimality achieved by different algorithms. Figs. 1, 2, and 3 indicate that the NLPD algorithm can support more links with less transmission power, and does so with substantially less CPU time than the existing algorithms (except for the GRN-DCPC algorithm). As shown in Fig. 3, the GRN-DCPC algorithm transmits the least power among the tested algorithms. This is because the GRN-DCPC algorithm supports the least number of links (Fig. 1).

5. CONCLUSIONS

In this paper, we have developed a new LP-approximation based algorithm for the joint power and admission control problem. Numerical simulations show the proposed algorithm outperforms the existing approaches. This performance improvement is a result of the new LP reformulation (9) and the use of necessary condition (14).

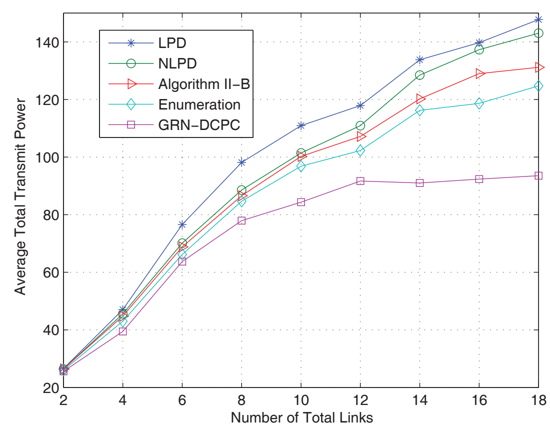

Fig. 3. Average transmission power versus the number of total links.

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