DIVERSITY ASPECTS OF POWER DELAY PROFILE BASED LOCATION FINGERPRINTING

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ABSTRACT

Although most of the conventional localization algorithms rely on Line of Sight (LOS) conditions, fingerprinting allows positioning in multipath and even in Non-LOS (NLOS) environments. In contrast to the traditional Received Signal Strength (RSS), the Power Delay Profile (PDP) fingerprint may allow positioning on the basis of a single link if the multipath is rich enough. Fingerprinting is a pattern matching technique for which a performance analysis may be difficult in general. In this paper we focus on a global performance indicator, in the form of Pairwise Error Probability (PEP). Similarly to PEP analysis in communication over fading channels, we find that the PEP for PDP fingerprinting exhibits a certain diversity order, linked to the number of paths. We investigate and show the results for Gaussian Maximum Likelihood (GML) based approaches for the Rayleigh fading path amplitude case.

Index Terms— fingerprinting, localization, pairwise error probability, diversity, least-squares, Rayleigh, Gaussian, maximum likelihood

1. INTRODUCTION

Location fingerprinting (LF) (introduced by U.S. Wireless Corp. of San Ramon, Calif.) relies on signal structure characteristics [1, 2]. It exploits the multipath nature of the channel hence the NLOS conditions. By using multipath propagation pattern, the LF creates a signature unique to a given location. The position of the mobile is determined by matching measured signal characteristics from the BS-MT link to an entry of the database. The location corresponding to the highest match of the database entry is considered as the location of the mobile. For LF, it is enough to have only one BS-MT link (multiple BSs are not required) to determine the location of the mobile. Also LF is classified among Direct Location Estimation (DLE) techniques. Ahonen and Eskelinen suggest using the measured Power Delay Profiles (PDPs) in the database [3] for fingerprints. In [4], the authors provide deterministic and Bayesian methods for PDP-F based localization. The Gaussian Maximum Likelihood (GML) based PDP-F introduced there is the main technique that we analyze in this paper.

What is meant by PEP is the same as in the PEP analysis for digital communication channels. In that case the aim is to find the probability of error when a vector of symbols $\mathbf{s_i}$ is transmitted but another vector $\mathbf{s_j}$ is detected at the receiver. We will pursue a similar approach for PDP-F PEP analysis. However its analysis is not as straightforward as for the digital communication channel case. The difficulty arises from the structure of the problem as will be clear soon. The objective is to determine the probability of error (the probability that wrong entry in the database is selected instead of the true position) when the channel estimates from the MT-BS link is matched with a wrong entry of the database. Hence position estimation error occurs as a result. We will investigate two different algorithms under different path amplitude modeling.

Notations: upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^T$ and $(.)^H$ represent the transpose and the transpose-conjugate operators. $E\{.\}$ is the statistical expectation, $\Re\{.\}$ is the real part and $tr\{.\}$ is the trace operator defined for square matrices.

2. POWER DELAY PROFILE FINGERPRINTING

Is multipath a curse or a blessing?

- Curse:
  - LOS case: additional paths hamper the estimation of LOS Time of Arrival (ToA) and other parameters,
  - NLOS case: introduces bias on LOS ToA.
- Blessing:
  - richer location information: may allow single anchor based localization!

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– each path providing as much info as a separate anchor in the LOS only case.

As is illustrated in Figure 1, the PDP exploits the ToA of all the multipath, creating a unique position dependent fingerprint that obvisates the need for multi-anchor reception.

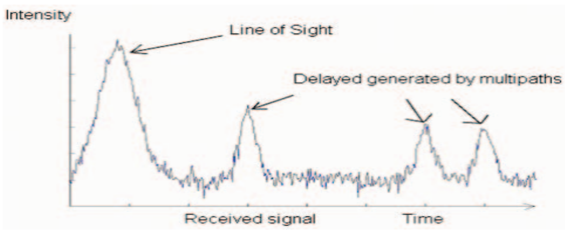

Fig. 1. Power Delay Profile.

Conventional location techniques use a two step procedure. In a first step given physical parameters of the transmitted signal (ToA, TDoA, AoA, signal strength...) are measured. In a second step the multiple measurements from a convenient number of base stations (BSs) are combined to estimate the mobile position. In this approach, the localization parameters are estimated separately and independently at each BS, the constraint that all measurements must correspond to the same source is ignored. Hence this conventional approach is suboptimal, nonlinearities introduce a breakdown behavior at low SNR.

In contrast, direct position determination was introduced by Anthony Weiss (see [5] and references therein) as a one step procedure in which each BS transfers the observed signal to a central processing unit and the position is computed as the best match to all the data simultaneously. This is for classical LOS multi-link positioning. The GML PDP-F technique that we consider in this paper is an example of a direct approach that is applicable in the case of NLOS and multipath. In PDP-F, the position is determined by maximizing the Gaussian likelihood (GML) of channel impulse response estimates, using their position dependent covariance matrix that is stored in (or computed from) a database. The GML approach to PDP-F actually exploits more than just the PDP; it exploits the whole channel impulse response covariance matrix, of which the PDP is just the diagonal.

3. SPECULAR CHANNEL MODEL

We start with the channel model because the PDP is just the magnitude squared version of the channel impulse response (CIR). But before using the measured PDPs, it is classically averaged over some time duration. However, if the mobile moves rapidly and/or some paths are not resolvable (due to the limited bandwidth of the pulse-shape $p(t)$, path contributions can overlap), the averaging gives a poor PDP estimation. The channel imulse response is

$$h(t,\tau) = \sum_{i=1}^{N_p} A_i(t)\, p(\tau - \tau_i(t)) \qquad (1)$$

where N_p denotes the number of paths (rays), $p(t)$ is the convolution of the transmit and receive filters (pulse shape), $\tau_i(t)$, $A_i(t)$ denote delay and complex attenuation coefficient (amplitude and phase of the ray) of the i^{th} path respectively. We can write the complex path amplitude of path i in polar form as $A_i(t) = a_i(t)e^{j\phi_i(t)}$. The delays τ_i are only slowly time-varying. Let us now consider sampling the CIR with a sampling period of τ_s leading to N_τ samples and stacking them in a vector as follows:

$$\mathbf{h}(t) = \begin{bmatrix} h(\tau_s, t) \\ h(2\tau_s, t) \\ \vdots \\ h(N_\tau \tau_s, t) \end{bmatrix} = \sum_{i=1}^{N_p} A_i(t)\, \mathbf{p}_{\tau_i}, \qquad (2)$$

where $\mathbf{p}_\tau$ is defined as: $\mathbf{p}_\tau = \begin{bmatrix} p(\tau_s - \tau) \\ p(2\tau_s - \tau) \\ \vdots \\ p(N_\tau \tau_s - \tau) \end{bmatrix}$ which is the sampled complex pulse shape vector having a delay equal to the delay of the path in samples and has N nonzero samples. If we write Equation (2) in matrix notation and include the channel estimation noise, we obtain the estimated CIR vector as:

$$\hat{\mathbf{h}}(t) = \underbrace{\left[\mathbf{p}_{\tau_1} \cdots \mathbf{p}_{\tau_{N_p}}\right]}_{\mathbf{P}_\tau} \underbrace{\begin{bmatrix} A_1(t) \\ \vdots \\ A_{N_p}(t) \end{bmatrix}}_{\mathbf{a}(t)} + \mathbf{v}(t). \qquad (3)$$

where $\mathbf{v}(t)$ is the complex additive white Gaussian noise vector with covariance matrix $\sigma_v^2 \mathbf{I}$. The PDP, being another vector having the same length as the CIR, could be estimated as:

$$\widehat{\mathbf{PDP}} = \frac{1}{T} \sum_{t=1}^{T} \left| \hat{\mathbf{h}}(t) \right|^2 \qquad (4)$$

where T is the number of channel observations. and for a vector argument, $|.|^2$ is to be interpreted element-wise.

For the path amplitudes, there can be two possibilities:

- deterministic model: $A_i(t)$ deterministic unknowns
- Gaussian model: $A_i(t)$ Gaussian with zero mean, characterized by a power (variance) i.e. $var(A_i) = \sigma_i^2$, which corresponds to Rayleigh fading for the magnitudes.

As we are interested in investigating the robustness of PDP fingerprinting to fading channel elements, we shall consider the Rayleigh model.

4. ANALYTICAL EXPRESSIONS OF PEP FOR THE GML TECHNIQUE FOR RAYLEIGH FADING MODELING OF THE PATH AMPLITUDES

In this part, we investigate the PEP analysis for the GML based PDP-F technique.

4.1. Ergodic Case

The Gaussian loglikelihood for T i.i.d. channel estimates $\widehat{\mathbf{h}}_i$ at a given position with channel estimate covariance matrix $\mathbf{C}_{\widehat{\mathbf{h}}_i\widehat{\mathbf{h}}_i}$ is

$$\mathcal{LL} \propto -\ln\left(\det\left(\mathbf{C}_{\widehat{\mathbf{h}}\widehat{\mathbf{h}}}\right)\right) - tr\left(\widehat{\mathbf{C}}\,\mathbf{C}_{\widehat{\mathbf{h}}\widehat{\mathbf{h}}}^{-1}\right) \quad (5)$$

where $\widehat{\mathbf{C}} = \frac{1}{T}\sum_{i=1}^{T}\widehat{\mathbf{h}}_i\widehat{\mathbf{h}}_i^H$ is the sample covariance matrix. So we have an error when the loglikelihood for a false position is larger than that for the true position:

$$\text{PEP} = \Pr\{\mathcal{LL}_T < \mathcal{LL}_F\}. \quad (6)$$

Hence, using (5), we get

$$\text{PEP} = \Pr\{tr(\widehat{\mathbf{C}}\,\mathbf{A}) < \ln\det(\mathbf{C}_T\mathbf{C}_F^{-1})\} \quad (7)$$

where $\mathbf{A} = \mathbf{C}_F^{-1} - \mathbf{C}_T^{-1}$. In the (extremely) ergodic case, we shall assume that $\widehat{\mathbf{h}}_i = \mathbf{h}_i\mathbf{v}_i$ where both the channel vectors $\mathbf{h}_i$ and the channel estimation error vectors $\mathbf{v}_i$ are i.i.d. and mutually independent. So $\widehat{\mathbf{h}}_i \sim \mathcal{CN}(0, \mathbf{C}_T)$. Let $\xi = tr(\widehat{\mathbf{C}}\,\mathbf{A}) = \frac{1}{T}\sum_{i=1}^{T} x_i$ where $x_i = \widehat{\mathbf{h}}_i^H\mathbf{A}\widehat{\mathbf{h}}_i$, with mean $m_\xi = m_{x_i}$ and $\sigma_\xi^2 = \frac{1}{T}\sigma_{x_i}^2$. As T increases, we can invoke the Central Limit Theorem (CLT) to state that asymptotically $\zeta = \frac{\xi - m_\xi}{\sigma_\xi}$ is a standard normal variable. Now,

$$m_{x_i} = \mathrm{E}\,\widehat{\mathbf{h}}_i^H\mathbf{A}\widehat{\mathbf{h}}_i = tr\{\mathbf{C}_T\,\mathbf{A}\} = tr\{\mathbf{C}_T\,\mathbf{C}_F^{-1} - \mathbf{I}\}. \quad (8)$$

On the other hand, exploiting fourth order moments for complex Gaussian vectors, we get

$$\mathrm{E}x_i^2 = \mathrm{E}\widehat{\mathbf{h}}_i^H\mathbf{A}\widehat{\mathbf{h}}_i\widehat{\mathbf{h}}_i^H\mathbf{A}\widehat{\mathbf{h}}_i = (tr\{\mathbf{C}_T\,\mathbf{A}\})^2 + tr\{\mathbf{C}_T\mathbf{A}\,\mathbf{C}_T\mathbf{A}\} \quad (9)$$

hence $\sigma_{x_i}^2 = \mathrm{E}x_i^2 - (m_{x_i})^2 = tr\{\mathbf{C}_T\mathbf{A}\,\mathbf{C}_T\mathbf{A}\}$. So we get using the CLT, and the symmetry of the Gaussian distribution, that

$$\text{PEP} = Q\left(\frac{tr\{\mathbf{C_T}\mathbf{C_F^{-1}} - \mathbf{I}\} - \ln\det(\mathbf{C_T}\mathbf{C_F^{-1}})}{\sqrt{\frac{1}{T}tr\{(\mathbf{C_T}\mathbf{C_F^{-1}} - \mathbf{I})^2\}}}\right) \quad (10)$$

from which we will see that a mismatch in every path contributes separately to decreasing the PEP when the path delays are well separated. Note that the numerator of the argument of the Q function is a form of the Itakura-Saito distance between covariance matrices.

To explore this further, consider the specular path model with $\widehat{\mathbf{h}} = \mathbf{P}_T\,\mathbf{a} + \mathbf{v}$ with covariance matrix

$$\mathbf{C}_T = \mathbf{P}_T\mathbf{D}_T\mathbf{P}_T^H + \sigma_v^2\mathbf{I} \quad (11)$$

where the columns of $\mathbf{P}_T$ contain the delayed pulse shapes for the true channel, and the $N_p \times N_p$ diagonal $\mathbf{D}_T$ conatins the N_p path powers for the true channel. The channel estimation error level is reflected by σ_v^2. For the ease of exposition, we shall assume here that the pulse shape is energy normalized and that the path delays are well separated so that $\mathbf{P}^H\mathbf{P} = \mathbf{I}_{N_p}$. We can define the SNR in the channel estimates as $\rho = tr\{\mathbf{D}_T\}/\sigma_v^2$. We then get for

$$\mathbf{C}_T^{-1} = \frac{1}{\sigma_v^2}\mathcal{P}_{\mathbf{P}_T}^{\perp} + \mathbf{P}_T(\mathbf{D}_T + \sigma_v^2\mathbf{I})^{-1}\mathbf{P}_T^H \quad (12)$$

where $\mathcal{P}_\mathbf{X} = \mathbf{X}(\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H$ nd $\mathcal{P}_\mathbf{X}^{\perp} = \mathbf{I} - \mathcal{P}_\mathbf{X}$ denote the orthogonal projection matrices onto the column space of $\mathbf{X}$ (which is assumed here to be of full column rank), and its orthogonal complement respectively. For the false position hypothesis, let the channel estimate covariance matrix $\mathbf{C}_F$ be structured similary, $\mathbf{C}_F = \mathbf{P}_F\mathbf{D}_F\mathbf{P}_F^H + \sigma_v^2\mathbf{I}$ with a possibly different number of again well separated paths (σ_v^2 is assumed to be estimated separately, so σ_v^2 can be taken to be the same in $\mathbf{C}_T$ and $\mathbf{C}_F$). Now let $\mathbf{C}_F$ have N_c path delays in common with $\mathbf{C}_T$, with the remaining number of path delays being different. Then we can write

$$\mathbf{P}_T = [\mathbf{P}_1 \;\; \mathbf{P}_2] \quad (13)$$

where the N_c columns of $\mathbf{P}_2$ are in common with $\mathbf{P}_F$. We have a corresponding split in $\mathbf{D}_T = \text{blockdiag}\{\mathbf{D}_1, \mathbf{D}_2\}$. We then get up to first order in SNR:

$$\begin{aligned}\mathbf{C}_T\mathbf{C}_F^{-1} &= \tfrac{1}{\sigma_v^2}\mathbf{P}_T\mathbf{D}_T\mathbf{P}_T^H\,\mathcal{P}_{\mathbf{P}_F}^{\perp} + \mathcal{O}(\rho^0) \\ &= \tfrac{1}{\sigma_v^2}\mathbf{P}_1\mathbf{D}_1\mathbf{P}_1^H + \mathcal{O}(\rho^0)\,.\end{aligned} \quad (14)$$

Focusing now on the dominant SNR terms in numerator and denominator of the Q function argument, we get for the PEP from (15)

$$\text{PEP} = Q\left(\sqrt{T}\,\frac{tr\{\mathbf{D}_1\}}{\sqrt{tr\{\mathbf{D}_1^2\}}}\right)\,. \quad (15)$$

If all path powers in $\mathbf{D}_1$ would be equal, then we get PEP $= Q\left(\sqrt{T\,(N_p - N_c)}\right)$ from which we observe a decreasing PEP as the multipath diversity $N_p - N_c$ increases.

4.2. Non-ergodic case

In the non-ergodic case, the channel $\mathbf{h}$ remains constant in the T estimates $\widehat{\mathbf{h}}_i$. In this case, $\widehat{\mathbf{h}}_i$ is not a zero mean vector (conditionally on $\mathbf{h}$), it is of the form: $\widehat{\mathbf{h}}_i = \mathbf{h} + \mathbf{v}_i$ where $\mathbf{h}$ represents the (conditional) mean. Now (7) becomes

$$\text{PEP} = \mathrm{E}_\mathbf{h}\Pr\{tr(\widehat{\mathbf{C}}\,\mathbf{A}) < \ln\det(\mathbf{C}_T\mathbf{C}_F^{-1})\} \quad (16)$$

where now the argument of Pr(.) is conditional on $\mathbf{h}$ and we recall that $tr\left(\hat{\mathbf{C}}\mathbf{A}\right) = \frac{1}{T}\sum_{i=1}^{T}\hat{\mathbf{h}_\mathbf{i}}^H\mathbf{A}\hat{\mathbf{h}_\mathbf{i}}$. The derivation is similar to the ergodic case. Let us call again $\hat{\mathbf{h}_\mathbf{i}}^H\mathbf{A}\hat{\mathbf{h}_\mathbf{i}} = x_i$, which are i.i.d.. Before using the CLT, mean and variance of x_i are required. For the mean of x_i, we obtain it easily:

$$\mu_{x_i} = \mathbf{h}^H \mathbf{A}\mathbf{h} + \sigma_v^2 \, tr(\mathbf{A}). \tag{17}$$

For the variance $\sigma^2_{x_i}$, we need $\mathrm{E}x_i^2$ again. We will exploit another identity for non-zero mean complex Gaussian vectors:

$$\mathrm{E}\{\hat{\mathbf{h}_i}^H \mathbf{A}\hat{\mathbf{h}_i} \, \hat{\mathbf{h}_i}^H \mathbf{A}\hat{\mathbf{h}_i}\} = \sigma_v^4 \, ||\mathbf{A}||_F^2 + (\sigma_v^2 \, tr(\mathbf{A}) + \mathbf{h}^H \mathbf{A}\mathbf{h})^2 + 2\,\sigma_v^2 \, \mathbf{h}^H \mathbf{A}^2 \mathbf{h}. \tag{18}$$

Consequently:

$$\sigma^2_{x_i} = \sigma_v^4 \, ||\mathbf{A}||_F^2 + 2\,\sigma_v^2 \, \mathbf{h}^H \mathbf{A}^2 \mathbf{h}, \tag{19}$$

so that using the CLT, (16) becomes

$$\mathrm{PEP} = \mathrm{E}_{\mathbf{h}} \, Q\left(\frac{\mathbf{h}^H \mathbf{A}\mathbf{h} + \sigma_v^2 tr(\mathbf{A}) - \ln\det(\mathbf{C_T}\mathbf{C_F^{-1}})}{\frac{1}{\sqrt{T}}\sqrt{\sigma_v^4 \, ||\mathbf{A}||_F^2 + 2\,\sigma_v^2 \mathbf{h}^H \mathbf{A}^2 \mathbf{h}}}\right). \tag{20}$$

Note that the use of the CLT is actually not really required here in order to elucidate the multipath diversity we shall investigate below. Indeed, for the fading analysis, the behavior near zero is what counts, and not the tail behavior. So it is not very important if the tail behavior of the argument of Pr(.) in (16) does not fit a Gaussian well. However, we shall take this simplified route here for ease of exposition.

Consider now the same type of path delay distributions as in the previous subsection. Then we can write

$$\mathbf{h} = \mathbf{P}_T \, \mathbf{a} = \mathbf{P}_1 \mathbf{a}_1 + \mathbf{P}_2 \mathbf{a}_2 \; . \tag{21}$$

At high SNR (and with the $\mathbf{h}$ considered), the dominant terms to be considered for the PEP in (20) are

$$\mathrm{PEP} = \mathrm{E}_{\mathbf{h}} \, Q\left(\sqrt{\frac{T}{2\sigma_v^2}} \, \frac{\mathbf{h}^H \mathbf{A}\mathbf{h}}{\sqrt{\mathbf{h}^H \mathbf{A}^2 \mathbf{h}}}\right) \; . \tag{22}$$

On the other hand we get up to first order in SNR

$$\mathbf{A}\mathbf{h} = \mathbf{C}_F^{-1}\mathbf{h} + \mathcal{O}(\rho^0) = \frac{1}{\sigma_v^2}\mathcal{P}^{\perp}_{\mathbf{P}_F}\mathbf{h} + \mathcal{O}(\rho^0) = \frac{1}{\sigma_v^2}\mathbf{P}_1 \, \mathbf{a}_1 + \mathcal{O}(\rho^0). \tag{23}$$

This in turn leads to

$$\mathbf{h}^H \mathbf{A}\mathbf{h} = \frac{1}{\sigma_v^2}\|\mathbf{a}_1\|^2 + \mathcal{O}(\rho^0), \; \mathbf{h}^H \mathbf{A}^2 \mathbf{h} = \frac{1}{\sigma_v^4}\|\mathbf{a}_1\|^2 + \mathcal{O}(\rho) \tag{24}$$

so that we get at high SNR ρ, from (22)

$$\mathrm{PEP} = \mathrm{E}_{\mathbf{h}} \, Q\left(\sqrt{\frac{T}{2\sigma_v^2}} \, \|\mathbf{a}_1\|\right) \; . \tag{25}$$

Now exploiting the Gaussian distribution of $\mathbf{a}_1$, this leads to [6]

$$\mathrm{PEP} \; = \; \frac{c}{\det(T\,\mathbf{D}_1) \; \rho^{N_p - N_c}} \tag{26}$$

(for some constant c) which exhibits the well-known diversity behavior of probability of error for digital communication over fading channels. Again, $N_p - N_c$ are the number of path delays in which the mistaken PDP differes from the true PDP. Clearly, the richer the multipath, the smaller the PEP is likely to be, esp. at high SNR.

5. CONCLUSION

In this contribution we derived approximate analytic expressions for the Pairwise Error Probability (PEP) for Power Delay Profile Fingerprinting (PDP-F). Whereas the CRB analyzes local performance (such as local identifiability), the PEP allows to assess the more global error performance. Assuming Rayleigh fading channels, we have considered optimized PDP-F criteria in the form of the Gaussian likelihood of the measured channel impulse responses. We have seen that the number of measurements T boosts the SNR as can be expected. We have considered two types of channel estimation scenarios, the ergodic and non-ergodic cases. In both cases we have seen that a richer multipath leads to smaller PEP. Especially in the non-ergodic case, we have seen that the diversity present in the channel impulse response leads to the same SNR diversity order for PDP-F PEP as for probability of error in digital communications over fading channels.

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