INTERFERENCE LEAKAGE MINIMIZATION FOR CONVOLUTIVE MIMO INTERFERENCE CHANNELS

Ó. González, C. Lameiro, J. Vía, I. Santamaria, Robert W. Heath Jr.†

Dept. of Communications Engineering
University of Cantabria, 39005 Santander, Cantabria, Spain
†Dept. of Electrical and Computer Engineering
The University of Texas at Austin, Austin, TX, 78712-0240, USA

ABSTRACT

An alternating optimization algorithm was recently proposed for the K-user multiple-input multiple-output (MIMO) interference channel. For flat-fading channels and feasible problems, this algorithm successfully aligns the interfering signals exploiting the spatial dimensions. In this paper, we consider the case in which all pairwise MIMO channels are frequency-selective (convolutive), and the users transmit broadband signals using a single-carrier scheme. Unlike the flat-fading case, for frequency-selective channels it is necessary to add a spectral mask in the frequency response of the precoders and decoders to avoid trivial solutions. We show in the paper that each step of the alternating minimization algorithm can be reformulated as a convex optimization problem in which the autocorrelation function of the precoders or decoders is obtained. Upon convergence, a final spectral factorization stage must be applied to obtain the precoders and decoders from their autocorrelation functions. Simulation results are provided to illustrate the performance of the proposed algorithm.

Index Terms— Interference MIMO channel, convex optimization, interference alignment.

1. INTRODUCTION

Alternating minimization algorithms are typically used to find interference alignment (IA) solutions for the K-user multiple-input multiple-output (MIMO) interference channel when closed-form solutions are not available [1, 2]. These algorithms minimize the interference leakage (IL) by fixing at each step either the precoders or decoders and optimizing over the remaining variables. For flat-fading MIMO interference channels, the algorithms in [1, 2] are able to find an IA solution when the system is feasible.

In this paper, we consider a single-beam MIMO interference channel, in which the MIMO channels are frequency-selective (convolutive) and the users transmit broadband signals using a single-carrier modulation scheme. Notice that for orthogonal frequency division multiplexing (OFDM) transmissions, the broadband channel can be decomposed into a set of non-overlapping flat-fading channels and, therefore, the IA problem can be easily solved using the algorithms in [1, 2] in a per-subcarrier basis, or doing symbol extension over different subcarriers. Nevertheless, these approaches would require some external synchronization among the users, since the interference alignment must be performed after the Fast Fourier Transform block. If the users transmit in an uncoordinated fashion, the high levels of interference present at the input of the receiver can impair the carrier and timing synchronization performance. With uncoordinated broadband single-carrier transmissions, the precoders and decoders can operate at the sample level in the time-domain [3], and thus are able to mitigate the interference before synchronization takes place. Furthermore, for frequency-selective channels the IL minimization problem is even more challenging than in the flat-fading case because it is necessary to add power spectrum constraints for the precoders and decoders in order to avoid trivial solutions for which every user transmits over a different frequency band. At each step of the alternating minimization algorithm, these nonconvex power spectrum constraints can be rewritten as linear constraints of the autocorrelation function of either the precoders or the decoders [4]. Exploiting this idea, the proposed algorithm for convolutive MIMO channels at a given step solves a convex optimization problem whose solution is the autocorrelation function of the precoders (or decoders). After convergence of the alternating minimization algorithm, a final spectral factorization stage is applied to get the time-domain minimum-phase precoders and decoders from their autocorrelation functions.

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2. INTERFERENCE MIMO CHANNELS WITH ISI

We consider the design of space-time precoders and decoders for frequency-selective MIMO interference channels. Specifically, we consider a K-user interference channel where each user transmits one data stream using a single-carrier modulation scheme. Let N_t and N_r denote the number of antennas at each transmitter and receiver, respectively. The convolutive MIMO channel from transmitter j to receiver i is represented as $\mathbf{H}_{ij}[n] \in \mathbb{C}^{N_r \times N_t}$, $n = 0, \ldots, L_h - 1$; where the MIMO channel order is taken as the maximum among those of the different pairwise channels.

At the i-th transmitter we apply a space-time precoder with L coefficients denoted as $\mathbf{v}_i[n] \in \mathbb{C}^{N_t \times 1}$, $n = 0, \ldots, L-1$. The received signal at receiver i is given as

$$\mathbf{y}_i[n] = \mathbf{H}_{ii}[n] * \mathbf{v}_i[n] * s_i[n] + \sum_{j \neq i} \mathbf{H}_{ij}[n] * \mathbf{v}_j[n] * s_j[n] + \mathbf{n}_i[n],$$

where $s_i[n]$ is the desired signal for the i-th user and $\mathbf{n}_i$ is the additive spatially white Gaussian noise at the i-th receiver.

The receiver also applies a space-time decoder with L coefficients:[1] $\mathbf{u}_i[n] \in \mathbb{C}^{N_r \times 1}$, $n = 0, \ldots, L-1$; and its output is given by

$$z_i[n] = h_{ii}[n] * s_i[n] + \sum_{j \neq i} h_{ij}[n] * s_j[n] + \check{n}_i[n], \qquad (1)$$

where $h_{ij}[n] = \mathbf{u}_i^H[-n] * \mathbf{H}_{ij}[n] * \mathbf{v}_j[n]$ is the equivalent frequency-selective single-input single-ouput (SISO) channel from transmitter j to receiver i, which has $L_h + 2L - 2$ coefficients, and $\check{n}_i[n]$ is now a colored Gaussian noise. We assume that each transmitter has perfect knowledge of the multipath MIMO channels corresponding to its direct link and the cross-links. In the frequency domain, Eq. (1) can be written as

$$Z_i(\omega) = |\mathbf{u}_i(\omega)^H \mathbf{H}_{ii}(\omega) \mathbf{v}_i(\omega)|^2 S_i(\omega) + \sum_{j \neq i} |\mathbf{u}_i(\omega)^H \mathbf{H}_{ij}(\omega) \mathbf{v}_j(\omega)|^2 S_j(\omega) + N_i(\omega), \qquad (2)$$

where $\mathbf{v}_j(\omega) = \sum_{n=0}^{L-1} \mathbf{v}_j[n] e^{-j\omega n}$, is the frequency response of the precoder, $\mathbf{u}_i(\omega)^H \mathbf{H}_{ii}(\omega) \mathbf{v}_i(\omega)$ is the frequency response of the i-th equivalent SISO link, $S_i(\omega)$ is the power spectral density of the i-th user and the rest of terms are defined analogously.

3. INTERFERENCE LEAKAGE MINIMIZATION

3.1. Problem statement

As can be observed in Eq. (1), the design of the precoders and decoders involves a tradeoff between the interference leakage coming from other users and the intersymbol interference (ISI) in the direct links. In this paper, we consider the problem of finding the precoders and decoders of length L minimizing the IL (ideally we would like to have $h_{ij}[n] = 0$, $\forall i \neq j$ and $\forall n$, while transmitting over the whole bandwidth). The equivalent SISO channels after precoding and decoding, $h_{ii}[n]$, will have length $L_h + 2L - 2$ and, therefore, the ISI originally provoked by the direct MIMO channel increases. Nevertheless, this additional ISI can be eliminated (or at least reduced) using a single-channel equalizer in a second stage. Furthermore, a penalty term trying to shorten the length of the equivalent channel and thus reducing the complexity of the single-channel equalizer can also be easily incorporated, similarly to [5].

Our interest here is to shed some light on the existence of space-time IA precoders and decoders of given length L for this problem, and point out the main differences with the flat-fading case. More precisely, our problem is the following: to find length-L precoders, $\mathbf{v}_i$, and decoders, $\mathbf{u}_i$, for $i = 1, \ldots, K$, such that

$$\underset{\mathbf{u}_j, \mathbf{v}_i}{\text{minimize}} \quad \sum_{i \neq j} \left\| \mathbf{u}_i^H[-n] * \mathbf{H}_{ij}[n] * \mathbf{v}_j[n] \right\|^2, \qquad (3)$$

$$\text{subject to} \quad |\mathbf{u}_i(\omega)^H \mathbf{H}_{ii}(\omega) \mathbf{v}_i(\omega)|^2 > 0, \ \forall \omega. \qquad (4)$$

Eq. (4) forces a non-zero frequency response $\forall \omega$ in the direct links, since otherwise we would end up with solutions in which each user tries to transmit over a different frequency band. Actually, when L tends to ∞, we would get zero IL by applying frequency-division multiplexing, i.e., by dividing the spectrum among the users. While these solutions obviously minimize the interference, they do not achieve the maximum degrees of freedom (DoF) for the K-user MIMO ISI channel.

[1]We use precoders and decoders with the same number of taps only for notational simplicity.

Notice also that in the flat-fading case there is not need to constraint the direct links since, for any set of random precoders and decoders and for MIMO channels with i.i.d. entries drawn from a continuous distribution, the direct links $|\mathbf{u}_i^H \mathbf{H}_{ii} \mathbf{v}_i| > 0$ with probability one. For MIMO ISI channels, however, the same condition on the direct links must be imposed at every frequency. This can only be achieved by imposing some spectral mask on the frequency response of the precoders and decoders. Specifically, if the modulus of the frequency response of the precoders and decoders is strictly larger than zero (i.e., $\|\mathbf{v}_i(\omega)\|^2 \geq \alpha$, with $\alpha > 0$) then with probability one the frequency response of the direct channels will not be zero at any frequency.

To solve the above optimization problem, we will resort to an alternating minimization procedure similar to that used in the flat-fading case [1, 2]. At each step, for fixed precoders or decoders the cost function in (3) becomes convex, however, the constraint (4) remains nonconvex. To circumvent this problem we apply some ideas previously proposed in the context of filter design with convex optimization techniques [4].

3.2. A convex framework for IL minimization

First, notice that the nonconvex constraint, $\|\mathbf{v}_i(\omega)\|^2 \geq \alpha$, which is quadratic on $\mathbf{v}_i(\omega)$, can also be written as a linear constraint on the autocorrelation function of $\mathbf{v}_i[n]$. In particular, let us define

$$\mathbf{R}_{v_i}[m] = \mathbf{v}_i[m] * \mathbf{v}_i^H[-m], \qquad (5)$$

$$\mathbf{S}_{v_i}(\omega) = \sum_{m=-L+1}^{L-1} \mathbf{R}_{v_i}[m] e^{-j\omega m}. \qquad (6)$$

Second, we use a discretization technique to approximate the infinite constraint $\|\mathbf{v}_i(\omega)\|^2 = \mathrm{Tr}\left(\mathbf{S}_{v_i}(\omega)\right) \geq \alpha$, $\forall \omega$, by samplig it uniformly at N points in frequency. Finally, let us also notice that the IL cost function can be written in terms of the autocorrelation of the equivalent SISO channel between transmitter j and receiver i. In particular, by defining

$$r_{ij}[n] = \mathrm{Tr}\left(\mathbf{R}_{v_j}[n] * \mathbf{H}_{ij}^H[-n] * \mathbf{R}_{u_i}[n] * \mathbf{H}_{ij}[n]\right) \qquad (7)$$

then, it is easy to see that

$$\left\| \mathbf{u}_i^H[-n] * \mathbf{H}_{ij}[n] * \mathbf{v}_j[n] \right\|^2 = r_{ij}[0]. \qquad (8)$$

With all these ingredients, the autocorrelation function for the precoders (assuming fixed decoders) can be obtained by solving the following convex optimization problem

$$\begin{aligned} \underset{\mathbf{R}_{v_j}[n]}{\text{minimize}} \quad & \sum_{i \neq j} r_{ij}[0] \qquad (9) \\ \text{subject to} \quad & \mathrm{Tr}\left(\mathbf{S}_{v_j}(\omega_m)\right) \geq \alpha, \ \forall m \\ & \mathbf{S}_{v_j}(\omega_m) \succeq 0, \ \forall m \\ & \mathbf{S}_{v_j}(\omega_m) = \sum_{n=-L+1}^{L-1} \mathbf{R}_{v_j}[n] e^{-j\omega_m n}, \ \forall m \\ & \mathbf{R}_{v_j}[n] = \mathbf{R}_{v_j}^H[-n], \ n = 0, \ldots, L-1 \\ & \mathrm{Tr}\left(\mathbf{R}_{v_j}[0]\right) = 1. \end{aligned}$$

Note that the rank of the power spectral density, $\mathbf{S}_{v_j}(\omega_m)$, is the number of data streams that each user is allowed to transmit. More data streams being transmitted increases the total interference

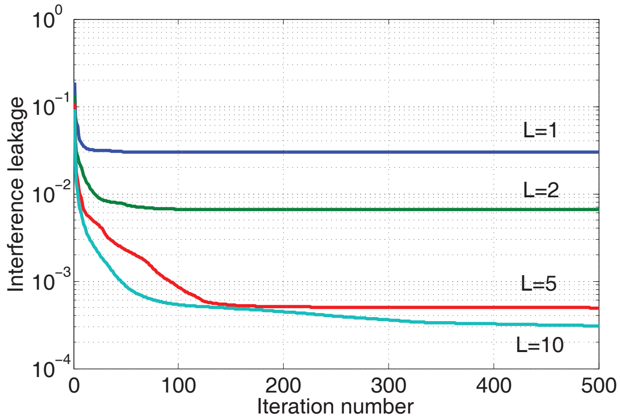

Fig. 1. IL vs. number of iterations for different orders, L, of the space-time precoders and decoders and parameter $\alpha = 0.5$.

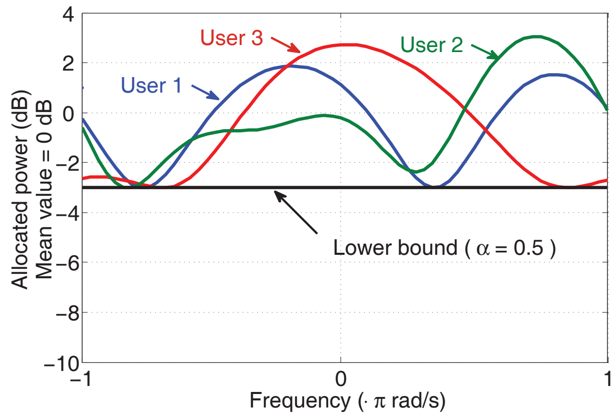

Fig. 2. Power allocation over frequency for each of the three users. Results after algorithm convergence at iteration 500 with $L = 5$ and $\alpha = 0.5$.

leakage, and thus the optimal solution of (9) is rank-one, i.e., a space-time SIMO filter.

The autocorrelation function for the decoders, $\mathbf{R}_{u_j}[n]$, (assuming fixed precoders) can be obtained by solving the same convex optimization problem in which the roles of the precoders and decoders are exchanged.

3.3. Spectral factorization step

Once $\mathbf{R}_{v_i}[n]$ and $\mathbf{R}_{u_i}[n]$ have been obtained, a spectral factorization algorithm must be applied to get the precoder $\mathbf{v}_i[n]$ and the decoder $\mathbf{u}_i[n]$ from their autocorrelation functions. For simplicity, we will focus on obtaining the precoder. In particular, the spectral factorization problem which we want to solve is

$$\mathbf{S}_{v_i}(\omega) = \mathbf{v}_i(\omega)\mathbf{v}_i^H(\omega),$$

where $\mathbf{S}_{v_i}(\omega)$ is the power spectral density function in (6) and $\mathbf{v}_i(\omega)$ is the precoder frequency response. According to [6], if $\mathbf{S}_{v_i}(\omega)$ is a rational $m \times m$ spectral density matrix of rank 1, then there exist a rational $m \times 1$ vector function $\mathbf{k}_i(\omega)$ that is causal, stable, minimum-phase and unique up to a unitary constant such that

$$\mathbf{S}_{v_i}(\omega) = \mathbf{k}_i(\omega)\mathbf{k}_i^H(\omega).$$

Hence, all the solutions to our spectral factorization problem verify

$$\mathbf{v}_i(\omega) = e^{j\phi}\mathbf{k}_i(\omega).$$

This result allow us to obtain the minimum-phase factor $\mathbf{v}_i(\omega)$ from $\mathbf{S}_{v_i}(\omega)$ in the following way: firstly, N_t independent SISO factorizations are computed to obtain N_t single-antenna filters whose power spectral densities match exactly those of the diagonal elements of $\mathbf{S}_{v_i}(\omega)$. Secondly, the filter of the first antenna is fixed and the rest of the filters are multiplied by an unitary constant, $e^{j\phi_t}$, $t = 1, \ldots, N_t - 1$, such that the cross spectral density terms (outside the diagonal) match those of $\mathbf{S}_{v_i}(\omega)$.

4. SIMULATION RESULTS

In this section we evaluate the performance of the proposed algorithm for a 3-user MIMO interference channel where each user is

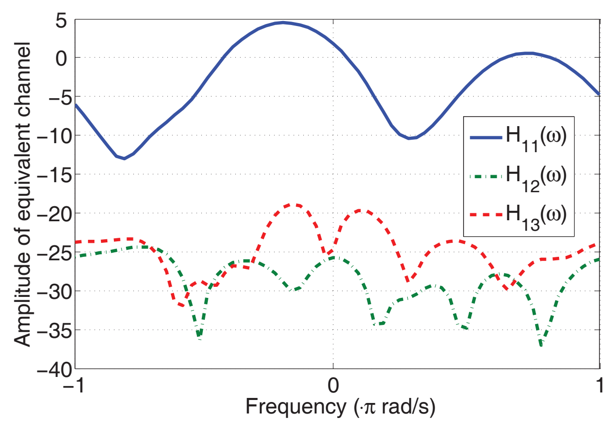

Fig. 3. SISO equivalent channels for the receiver 1: the direct channel (solid line) and two interfering channels (dashed line).

equipped with 2 antennas at both sides of the link and wishes to send one stream of data. This interference network is sometimes denoted as $(2 \times 2, 1)^3$. All pairwise MIMO channels have $L_h = 2$ taps and the entries of each MIMO matrix are i.i.d. zero-mean complex Gaussians with unit variance. Fig. 1 shows the convergence of the interference leakage averaged over 20 different channel realizations and for different orders, L, of the precoders and decoders. The spectral mask for the frequency-response of the precoders and decoders is $\alpha = 0.5$. As expected, the interference leakage decreases as L increases. Furthermore, it can be noticed that the relative improvement diminishes for high filter orders: in this example the improvement from $L = 5$ to $L = 10$ is not very significant.

Fig. 2 shows the power allocation over frequency for the 3 users for a particular channel realization and filters of order $L = 5$. We can observe that the mask constraint (i.e., $\text{Tr}(\mathbf{S}_{v_i}(\omega)) > 0.5$, $i = 1, \ldots, 3$) is active in several frequency bands. In this way, with the proposed algorithm the 3 users are transmitting over the whole bandwidth while minimizing the overall interference. Figure 3 depicts the three equivalent SISO channels (after precoding and decod-

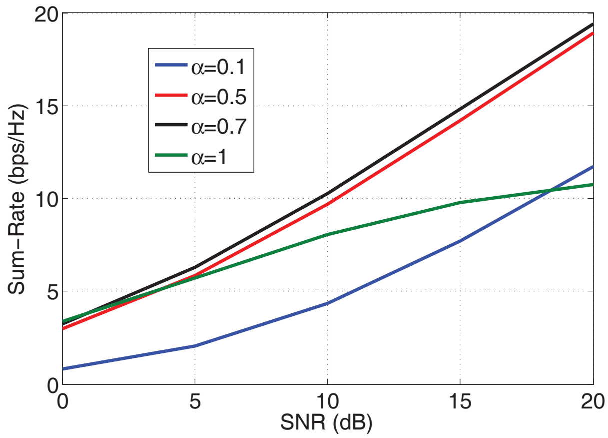

Fig. 4. Sum-Rate performance for $L = 5$ and different values of α.

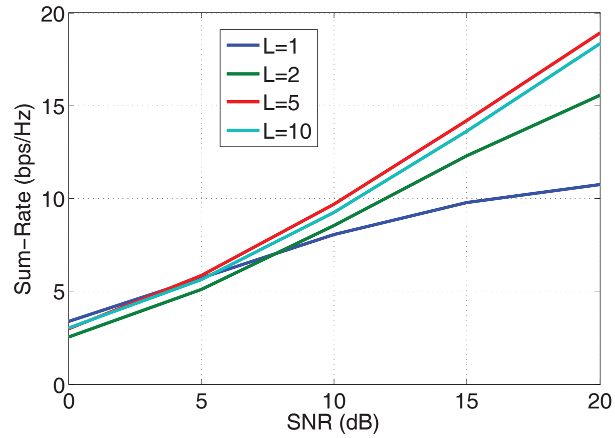

Fig. 5. Sum-Rate performance for $\alpha = 0.5$ and different orders, L, of the space-time precoders and decoders.

ing) from the 3 transmitters to the receiver of the first user: its direct channel in solid line and the two interfering channels in dashed line. It can be seen how this user achieves more than 20 dB of interference suppression over the whole frequency band.

An example of the sum-rate performance for $L = 5$ and different values of α is depicted in Fig. 4. When the value of α is too low, the resulting precoders and decoders minimize the interference not only aligning it, but also reducing the bandwidth of the transmitted signals. As a result, the sum-rate is dominated by the noise and the system is unable to achieve the maximum degrees of freedom of the network with a SNR lower than 20 dB. When α increases, the sum-rate is to be dominated by interference, and 2.8 degrees of freedom out of 3 are achieved. However, if the spectral constraint is too restrictive, the interference cannot be aligned and the system performance decreases drastically. The sum-rate performance is also shown in Fig. 5 for the same channel realization, $\alpha = 0.5$ and different filter orders. As it can be observed, increasing the filters order from $L = 1$ to $L = 2$ provides a noticeable improvement in the sum-rate performance. It can also be noticed that, in this example, filters with $L = 5$ perform slightly better than those with $L = 10$. The intuition behind this is that high order filters are able to reduce the interference not only by the alignment, but also by restricting the transmission bandwidth. Thus, their sum-rate performance may be worse than that of filters with lower order.

5. CONCLUSIONS

A new interference leakage minimization method for designing the space-time filters for the K-user convolutive MIMO channel has been proposed. In order to avoid trivial solutions which multiplex the users in the frequency domain, a spectral mask has been incorporated to the optimization problem. Simulation results show that the proposed method successfully reduces the interference while ensuring that all users transmit simultaneously over the same frequency band. As further work, it would be interesting to consider other cost functions (e.g., maximum SINR or maximum capacity), or a joint minimization of the ISI and the IL.

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