OPTIMAL JOINT BASE STATION ASSIGNMENT AND DOWNLINK BEAMFORMING FOR HETEROGENEOUS NETWORKS

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ABSTRACT

Consider a MIMO heterogeneous network with multiple transmitters (including macro, pico and femto base stations) and many receivers (mobile users). The users are to be assigned to the base stations which then optimize their linear transmit beamformers accordingly. In this work, we consider the problem of joint base station assignment and linear beamformer design to maximize a system wide utility. We first establish the NP-hardness of the resulting optimization problem for a large family of α-fairness utility functions. Then, we propose an efficient algorithm to approximately solve this problem for the special case of sum rate maximization. The simulation results show that the algorithm improves the sum rate.

Index Terms— Heterogeneous Network, Base Station Assignment, MIMO Beamforming, Complexity Analysis

1. INTRODUCTION

The insatiable demand for high speed mobile communication has put the existing wireless cellular infrastructure under severe stress. One effective means to cope with the explosive data growth is to increase the existing spectral efficiency by reducing the cell size, adding more base stations and increasing frequency reuse. These techniques have led to the deployment of pico base stations (pico BS) or relays within a large macro-cell, and have resulted in the use of femto BS (also known as home BS) which are low power, short range transmitters used to enhance the signal quality in residential houses or crowded business areas. The coordination of various base stations and the possibility of load sharing with Wi-Fi/DSL networks make the management of such heterogeneous networks [1] a challenging task.

With the increase in the number of base stations simultaneously operating within the same frequency band, interference has become a major performance limiting factor for heterogeneous networks. As a consequence, there has been an intensive recent research on physical-layer algorithms [4, 5, 6] for interference mitigation. However, most of these algorithms consider the classical model of an interference channel or an interfering broadcast channel where the transmitter-receiver associations are fixed (pre-assigned), and the design scope is restricted to choosing linear transceivers to improve system throughput and user fairness.

In a heterogeneous network consisting of many overlapping cells and base stations, a user can be assigned to any one of the nearby cells. Traditionally, base station assignment is made on the basis of signal strength (or the distances to base stations). However, if a base station in a heterogeneous network is congested, it may be more beneficial to assign users to a different cell even though they may be closest to the congested base station. In this paper we consider the problem of maximizing a system wide utility by simultaneously optimizing both the assignment of users (receivers) to BSs (transmitters) and the design of their linear beamformers. We prove that for a well-known family of system utility functions (α-fairness utilities), the joint BS assignment and transceiver design problem is NP-hard. In addition, for the special case of sum-rate maximization, we propose an iterative algorithm for this joint design problem. Simulation results show that this algorithm can achieve a higher system throughput at a moderate computational cost.

Throughout this paper, the symbol $(\cdot)^H$ denotes complex conjugate transpose of a matrix and $\mathbb{E}(\cdot)$ is the expected value of a random variable. In addition, $\mathbf{I}$ represents the identity matrix of the appropriate size and $\mathbf{e}_\ell$ represents the standard basis vector consisting of all zero elements except the ℓ-th element which is 1.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a wireless network in downlink direction with K transmitters and N receivers, each of them are equipped with multiple antennas. The transmitters can be macro, pico or femto BSs, and the receivers can be mobile users. We assume that all the transmitters use the same bandwidth for communi-

This research is supported in part by the Army Research Office, Grant No. W911NF-09-1-0279, and in part by the National Science Foundation, Grant No. CMMI-0726336.

cation. For the sake of simplicity, we assume that there are M antennas at each transmitter and L antennas at each receiver. We denote the transmitter k by $\mathcal{B}_k$ and the user n by $\mathcal{U}_n$. The channel between $\mathcal{B}_k$ and $\mathcal{U}_n$ is denoted by $\mathbf{H}_{nk} \in \mathbb{C}^{L\times M}$. The users need to be assigned to the transmitters. By assignment of user $\mathcal{U}_n$ to transmitter $\mathcal{B}_k$, we mean that $\mathcal{U}_n$ is served by $\mathcal{B}_k$. Let us define binary variables $a_{nk} \in \{0,1\}$ for all $n = 1,\cdots,N$, and $k = 1,\cdots,K$, to represent the assignments. The binary variable $a_{nk} = 1$ if and only if $\mathcal{U}_n$ is served by $\mathcal{B}_k$. Due to the practical limitations, each user can be served by only one transmitter. Hence, we should have

$$\sum_{k=1}^{K} a_{nk} \leq 1. \tag{1}$$

The transmitter $\mathcal{B}_k$ transmits the signal $\mathbf{x}_k \in \mathbb{C}^M$ which is a linear combination of the data streams intended for its assigned user.

$$\mathbf{x}_k = \sum_{n}^{N} a_{nk}\mathbf{V}_{nk}\mathbf{s}_n, \tag{2}$$

where the vector $\mathbf{s}_n \in \mathbb{C}^{d_n}$ consists of d_n data streams of independent Gaussian random variables with covariance matrix $\mathbf{I}_{d_n\times d_n}$. These data streams are intended for user $\mathcal{U}_n$. The matrix $\mathbf{V}_{nk} \in \mathbb{C}^{M\times d_n}$ is the linear beamformer applied to the data streams of user $\mathcal{U}_n$ by transmitter $\mathcal{B}_k$. The expected power of signal $\mathbf{x}_k$, should be less than the power budget at $\mathcal{B}_k$ denoted by P_k. Assuming independence between the data streams of different users, we have

$$\mathbb{E}(\mathbf{x}_k^H\mathbf{x}_k) = \sum_{n=1}^{N} a_{nk}\mathrm{Tr}(\mathbf{V}_{nk}^H\mathbf{V}_{nk}) \leq P_k \tag{3}$$

The received signal at user $\mathcal{U}_n$ is

$$\mathbf{y}_n = \sum_{k=1}^{K}\mathbf{H}_{nk}\mathbf{x}_k + \mathbf{z}_n, \tag{4}$$

where $\mathbf{z}_n$ is white Gaussian noise with covariance matrix $\sigma_n^2\mathbf{I}_{L\times L}$ and $\mathbf{H}_{kn} \in \mathbb{C}^{L\times M}$ is the channel between $\mathcal{B}_k$ and $\mathcal{U}_n$. We assume that each receiver treats all the signals intended for the other users as noise (which is reasonable assumption in practice). Then, the achievable rate of $\mathcal{U}_n$ is

$$R_n = \sum_{k=1}^{K} a_{nk}\log\det\Bigg(\mathbf{I} + \bigg(\sigma_n^2\mathbf{I} + \sum_{k=1}^{K}\sum_{m\neq n}\mathbf{H}_{nk}\mathbf{V}_{mk}\mathbf{V}_{mk}^H\mathbf{H}_{nk}^H\bigg)^{-1}\mathbf{H}_{nk}\mathbf{V}_{nk}\mathbf{V}_{nk}^H\mathbf{H}_{nk}^H\Bigg).$$

The objective is to maximize a system wide utility function $U(R_1,\cdots,R_N)$ by choosing the assignment variables a_{nk} and the beamformers $\mathbf{V}_{nk}$. Therefore, the problem of joint BS assignment and beamforming can be formulated as follows:

$$\max_{\{a_{nk},\mathbf{V}_{nk}\}} \quad U(R_1,\cdots,R_N) \tag{P}$$

$$\text{s.t. } a_{nk} \in \{0,1\}\ \forall\, n,\ k \text{ and } \sum_k a_{nk} \leq 1,\ \forall\, n \tag{C1}$$

$$\sum_n a_{nk}\mathrm{Tr}(\mathbf{V}_{nk}\mathbf{V}_{nk}^H) \leq P_k,\ \forall k \tag{C2}$$

$$R_n = \sum_{k=1}^{K} a_{nk}\log\det\Bigg(\mathbf{I} + \bigg(\sigma_n^2\mathbf{I} + \sum_{k=1}^{K}\sum_{m\neq n}\mathbf{H}_{nk}\mathbf{V}_{mk}\mathbf{V}_{mk}^H\mathbf{H}_{nk}^H\bigg)^{-1}\mathbf{H}_{nk}\mathbf{V}_{nk}\mathbf{V}_{nk}^H\mathbf{H}_{nk}^H\Bigg), \tag{C3}$$

3. COMPLEXITY ANALYSIS

In this section we consider the problem (P) and analyze the complexity of solving this problem globally under different utility functions.

One important family of system wide utility functions that has been extensively studied in literature is α-fairness utility function [2]. For any constant $\alpha \geq 0$, it is defined as follows

$$U_\alpha(R_1,\cdots,R_N) = \begin{cases} \sum_{n=1}^{N}\frac{R_n^{1-\alpha}}{1-\alpha} & \text{if } \alpha \neq 1; \\ \sum_{n=1}^{N}\log(R_n) & \text{if } \alpha = 1. \end{cases} \tag{5}$$

We denote this family of utility functions with $\mathcal{F}$. Most of the well known utility functions belong to $\mathcal{F}$ [2, 3]. Four of those utility functions are listed in the following table.

α	***Utility***	***Expression***
0	*Sum-Rate*	$\sum_{n=1}^{N} R_n$
1	*Proportional Fairness*	$\sum_{n=1}^{N}\log(R_n)$
2	*Harmonic-Rate*	$(\sum_{n=1}^{N} R_n^{-1})^{-1}$
∞	*Max-Min*	$\min_{1\leq n\leq N} R_n$

We have the following result, regarding the complexity status of problem (P).

Theorem 1 *The problem (P) is NP-hard when the utility function $U(\cdot)$ is the Sum-Rate utility function. Furthermore, for any other utility function $U(\cdot)$ in $\mathcal{F}$, it is NP-hard if* $\min(L,M) \geq 3$.

Proof: Proof of the NP-hardness for the Sum-Rate utility is based on a polynomial time reduction from the MAX 2-SAT problem. MAX 2-SAT is the problem of determining the maximum number of clauses among a set of 2-SAT Boolean clauses that can be satisfied simultaneously using Boolean variable assignments. Since the MAX 2-SAT problem is NP-hard, the problem of optimal base station assignment and beamformer design is NP-hard as well. Due to space limitation we omit the details of the proof here.

To prove the NP-hardness of maximizing any utility function $U(\cdot) \in \mathcal{F}$ for the case where $\min(L,M) \geq 3$, we use a polynomial time reduction from the Graph 3-Colorability problem. Graph 3-Colorability problem is the problem of determining if the vertices of a graph G can be colored with 3 colors such that none of the adjacent nodes are colored the same. It is well known that the Graph 3-Colorability problem is NP-complete. Therefore, the problem of joint optimal assignment and beamformer design is NP-hard. To construct the polynomial time reduction, consider a graph $G = (V, E)$, where $|V| = N$. Furthermore, consider N mobile users in the system, each corresponding to a node in graph G. Assume there are $3N$ BSs, each with a power budget of $P = 1$. Let us also assume that the noise power at any user is $\sigma^2 = 1$. For any user $\mathcal{U}_i$, there are 3 corresponding BSs. We denote them by $\mathcal{B}_{i_1}$, $\mathcal{B}_{i_3}$, and $\mathcal{B}_{i_3}$. The channels are constructed as follows:

1. The channel between $\mathcal{B}_{i_\ell}$ and $\mathcal{U}_i$ is $\mathbf{e}_\ell \mathbf{e}_\ell^H$, for all $i = 1, \cdots, N$, and all $\ell = 1, 2, 3$.
2. If $\{i, j\} \in E$, the channel between $\mathcal{B}_{i_\ell}$ and $\mathcal{U}_j$ is $\frac{1}{2}\mathbf{e}_\ell \mathbf{e}_\ell^H$, for all $\ell = 1, 2, 3$, and otherwise it is zero.

Then, in order to show the NP-hardness of (P), it suffices to prove the following claim.

Claim 1 *In the above constructed network, the optimization problem* (P) *has optimal value greater than or equal to* $U(\log(2), \cdots, \log(2))$ *if and only if the graph* G *is 3-colorable.*

Due to space limitation we have omitted the proof of Claim 1.

Remark 1 *Although the scenario considered here is MIMO, the proofs can be generalized to the OFDM setup. In the OFDM scenario, the multiple dimensions of the input and output correspond to the orthogonal channels. Hence, the channels are diagonal in OFDM. In addition, in OFDM setup the beamformers are also restricted to be diagonal (see [3]).*

Remark 2 *It should be noted that the results here are not direct consequences of the results in [3]. The proofs in [3] are mainly based on the scenarios with strong cross links and weak direct links (high interference). But in our case there are no preassigned direct links.*

4. SUM-RATE MAXIMIZATION USING MATRIX-WEIGHTED-SUM-MSE MINIMIZATION

In this section, we develop an algorithm to approximately solve the problem (P) with sum rate utility as its objective. Our goal is to devise a transceiver design scheme based on the optimization problem (P). Using the technique in [6], problem (P) can be equivalently written as:

$$\begin{aligned}
\min_{\{a, \mathbf{V}, \mathbf{U}, \mathbf{W}\}} \quad & \sum_{k=1}^{K} \sum_{n=1}^{N} a_{nk} \left(\mathrm{Tr}(\mathbf{W}_{nk}\mathbf{E}_{nk}) - \log\det(\mathbf{W}_{nk}) \right) \\
\text{s.t.} \quad & \text{(C1), (C2) and (C3)}, \\
& \mathbf{W}_{nk} \succeq \mathbf{0}, \ \forall\, n,\ k,
\end{aligned} \tag{6}$$

where $\mathbf{E}_{nk} \triangleq (\mathbf{I} - \mathbf{U}_{nk}^H \mathbf{H}_{nk} \mathbf{V}_{nk})(\mathbf{I} - \mathbf{U}_{nk}^H \mathbf{H}_{nk} \mathbf{V}_{nk})^H + \sum_{\ell=1}^{K} \sum_{m \neq n} \mathbf{U}_{nk}^H \mathbf{H}_{n\ell} \mathbf{V}_{m\ell} \mathbf{V}_{m\ell}^H \mathbf{H}_{n\ell}^H \mathbf{U}_{nk} + \sigma_n^2 \mathbf{U}_{nk}^H \mathbf{U}_{nk}$ is the MSE value of user n when it is served by $\mathcal{B}_k$ (see [6]) and $\mathbf{U}_{nk}$ is the receive beamformer of user n for decoding the signals from transmitter k.

Our approach is to apply coordinate descent method to problem (6). However, the major difficulty lies in dealing with the discrete variables $\{a_{nk}\}_{n,k}$. Next, we will reformulate problem (6) in an equivalent form with no discrete variables. For any constant $c \geq 0$, let us add an auxiliary term to the MSE value and define $\mathbf{E}_{nk}(c) \triangleq \mathbf{E}_{nk} + c \sum_{\ell \neq k} \|\mathbf{V}_{n\ell}\|^2 \mathbf{U}_{nk}^H \mathbf{U}_{nk}$. Notice that if user n is served by only one base station, we have $\|\mathbf{V}_{n\ell}\|^2 \cdot \|\mathbf{U}_{nk}\|^2 = 0, \forall k, \ell, k \neq \ell$. Therefore, $\mathbf{E}_{nk}(c) = \mathbf{E}_{nk},\ \forall c$. On the other hand, if $\|\mathbf{V}_{n\ell}\|^2 \cdot \|\mathbf{U}_{nk}\|^2 > 0$, for some $\ell, k, \ell \neq k$, then $\mathrm{Tr}(\mathbf{W}_{nk}\mathbf{E}_{nk}(c))$ is an increasing function of c for every positive definite matrix $\mathbf{W}_{nk}$. Using this observation, we can prove the following claim:

Claim 2 *Assume the channel matrices are full column rank and* $\sigma_n^2 > 0, \forall n$. *Then, problem* (6) *is equivalent to the following optimization problem*

$$\begin{aligned}
\min_{\{\mathbf{V}_{nk}, \mathbf{U}_{nk}, \mathbf{W}_{nk}\}} \quad & \sum_{k=1}^{K} \sum_{n=1}^{N} f_{nk} \\
\text{s.t.} \quad & \sum_{n=1}^{N} \mathrm{Tr}(\mathbf{V}_{nk}\mathbf{V}_{nk}^H) \leq P_k,\ \forall k
\end{aligned} \tag{7}$$

where f_{nk} *is extended real valued function defined by*

$$f_{nk} \triangleq \lim_{c \to +\infty} \mathrm{Tr}(\mathbf{W}_{nk}\mathbf{E}_{nk}(c)) - \log\det(\mathbf{W}_{nk}).$$

The equivalence is in the sense that if $\{\mathbf{U}_{nk}^*, \mathbf{V}_{nk}^*, \mathbf{W}_{nk}^*\}_{n,k}$ *is the optimal solution of* (7), *then there exists* $\{a_{nk}^*\}_{n,k}$ *so that* $\{a_{nk}^*, \mathbf{U}_{nk}^*, \mathbf{V}_{nk}^*, \mathbf{W}_{nk}^*\}_{n,k}$ *is the optimal solution of* (6).

Now we use Claim 2 to propose a heuristic algorithm for joint BS assignment and beamformer design problem. In the optimization problem (7), we need to minimize the utility function of (7) which is the limit of the function $f^c \triangleq \sum_{n=1}^{N} \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{W}_{nk}\mathbf{E}_{nk}(c)) - \log\det(\mathbf{W}_{nk})$ when $c \to \infty$. In our approach, we consider an iterative approach where at each iteration we try to minimize f^c for a fixed value of c and then we increase the value of c iteratively. More specifically, we consider the following optimization problem at each iteration.

$$\begin{aligned}
\min_{\{\mathbf{V}_{nk}, \mathbf{U}_{nk}, \mathbf{W}_{nk}\}} \quad & \sum_{k=1}^{K} \sum_{n=1}^{N} \mathrm{Tr}(\mathbf{W}_{nk}\mathbf{E}_{nk}(c)) - \log\det(\mathbf{W}_{nk}) \\
\text{s.t.} \quad & \sum_{n=1}^{N} \mathrm{Tr}(\mathbf{V}_{nk}\mathbf{V}_{nk}^H) \leq P_k,\ \forall k \\
& \mathbf{W}_{nk} \succeq \mathbf{0},\ \forall n,\ \forall k.
\end{aligned} \tag{8}$$

Although this optimization is non convex, it can be solved to a KKT point using coordinate descent approach [6]. If we fix all the variables $\mathbf{W}, \mathbf{V}, \mathbf{U}$ except only one set of variables, there is a closed form solution for the non-fixed variable. Using this observation, we can devise a coordinate descent based approach for solving problem (8) to a KKT point [6]. The overall proposed algorithm is summarized in Figure 1.

Set $c = c_0$, initialize $\mathbf{V}_{nk}$'s randomly
repeat
$\mathbf{W}_{nk} \leftarrow \mathbf{E}_{nk}^{-1}(c), \ \forall n, \ \forall k$
$\mathbf{U}_{nk} \leftarrow \left(\sum_{\ell,m} \mathbf{H}_{n\ell}\mathbf{V}_{m\ell}\mathbf{V}_{m\ell}^H\mathbf{H}_{n\ell}^H + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}_{nk}\mathbf{V}_{nk}$, $\forall n, k$
$\mathbf{V}_{nk} \leftarrow \left(\sum_{\ell,m} \mathbf{H}_{mk}^H\mathbf{U}_{m\ell}\mathbf{W}_{m\ell}\mathbf{U}_{m\ell}^H\mathbf{H}_{mk} + \mu_{nk}^*\mathbf{I}\right)^{-1}\mathbf{H}_{nk}^H\mathbf{U}_{nk}\mathbf{W}_{nk}, \forall\, n, \ \forall\, k$
$c \leftarrow 2c$
until (C1) is satisfied and the iterations converges

Fig. 1. Pseudo code of the proposed algorithm

5. SIMULATION RESULTS

In this section we present simulation results to evaluate the performance of the proposed algorithm. The simulation setup is as follows. We consider a macro-cell as a circle with radius equal to 1 kilometer. The macro BS is assumed to be in the center of the macro cell. $N = 30$ users are placed with uniform distribution within the cell. There are 4 pico BSs which are located at fixed points inside the cell. There are also 5 femto BSs randomly located within the cell. Hence, in total there are $K = 10$ BSs (transmitters) inside the cell. The power of the noise is assumed to be $\sigma^2 = 10^{-12}$. All the transmitters have $M = 4$ antennas, and the receivers are equipped with $L = 2$ antennas. The number of data streams intended for each user is 1 ($d_n = 1, \forall n$). The power budget of the macro, pico and femto BSs are assumed to be P, $0.1P$ and $0.001P$, respectively.

The channels are generated with respect to distances and with path loss exponent 2, plus a random independent Rayleigh fading over all the channel entries. As the results are dependent on the relative positions of the nodes in the network, we first generate the topology of network randomly and fix it through the simulations. The topology of the network is depicted in Figure 2. Then in Figure 3, the achievable sum rate is plotted versus the power. The results are averaged over 10 different random Rayleigh fading realizations. We have compared the proposed algorithm with the WMMSE algorithm [6] with users pre-assigned to the BS on the basis of the strongest direct channel. The preliminary results show an 5-10% improvement in the system throughput. Further evaluation of the algorithm for the congested cases is on-going.

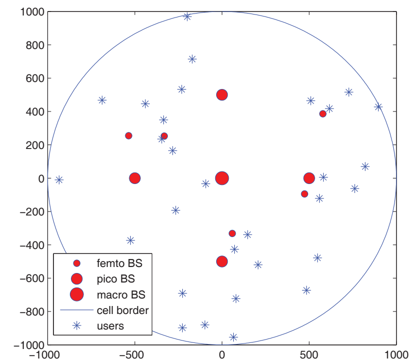

Fig. 2. Relative positions of the transmitters and receivers.

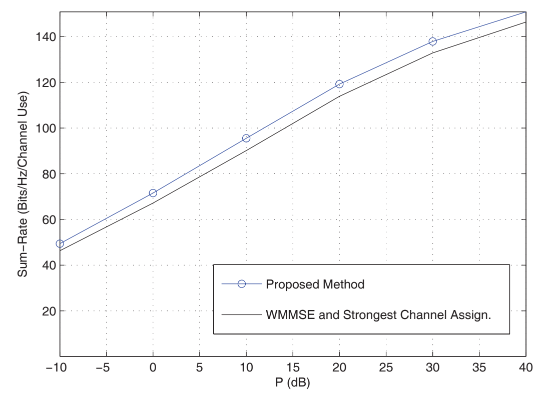

Fig. 3. The achievable sum rate versus the power budget at macro BS

6. REFERENCES

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