OPTIMAL RESOURCE ALLOCATION FOR GAUSSIAN RELAY CHANNEL WITH ENERGY HARVESTING CONSTRAINTS

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ABSTRACT

In this paper, we study the three-node Gaussian relay channel with decode-and-forward (DF) relaying, in which the source and relay nodes transmit with power drawn from energy-harvesting sources. Assuming a deterministic energy-harvesting model under which the energy arrival time and the harvested amount are known prior to transmission, the throughput maximization problem over a finite horizon of N transmission blocks is investigated. We consider the no-delay-constrained (NDC) traffic case, for which the relay can store the decoded information from the source with arbitrary delay before forwarding it to the destination in each N-block transmission. Although the formulated problem is non-convex, we prove the optimality of a separation principle for the source and relay power allocation over time, based upon which a two-stage algorithm is developed to obtain the optimal source and relay power profiles separately.

***Index Terms*—** Throughput, relay channel, cooperative communication, energy harvesting.

1. INTRODUCTION

In conventional energy-constrained wireless communication systems such as wireless sensor networks (WSNs), sensors are equipped with fixed energy supply devices, e.g., batteries, which have limited operation time. When thousands of sensors are deployed in a hostile or toxic environment, recharging or replacing batteries becomes inconvenient and even impossible. Hence, harvesting energy from the environment is a much easier and safer way to provide almost unlimited energy supply for WSNs. In [1], the authors investigated the power management strategies for WSNs with energy-harvesting nodes, for which random energy-harvesting models were assumed. For the point-to-point communication powered by energy-harvesting sources, the power allocation problem was studied in, e.g., [2] with the deterministic energy-harvesting model, and in [3] with the random energy-harvesting model.

In this paper, we study the half-duplex orthogonal Gaussian relay channel with energy-harvesting source and relay nodes. We consider the simple case with deterministic source and relay energy profiles, corresponding to practical scenarios where the energy-harvesting level can be predicted with negligible errors, and leave the more general random cases for future study. We further focus on the scenario with no-delay-constrained (NDC) traffic, in which the relay is allowed to store the decoded information from the source with arbitrary delay before forwarding it to the destination. Note that the relay operation in the NDC traffic case is more flexible than that for the delay-constrained (DC) traffic case previously studied in [4], and is thus expected to achieve a higher throughput. We examine the throughput maximization problem over a finite horizon of N-block transmission, which is non-convex in general, and propose an algorithm to compute the globally optimal solution.

Notation: $\log(\cdot)$ and $\ln(\cdot)$ stand for the base-2 and natural logarithms, respectively; $\mathcal{C}(x) = \frac{1}{2}\log(1+x)$ denotes for the AWGN channel capacity; $(x)^+ = \max(0, x)$.

2. SYSTEM MODEL

We consider the classic three-node relay channel, which consists of one source-destination pair and one relay. We assume that the relay node operates in a half-duplex mode over two orthogonal frequency bands, while the source-relay and source-destination use the same band. For simplicity, we assume that the source-relay and relay-destination links operate with equal bandwidth.

We consider the decode-and-forward (DF) relaying scheme, which requires the relay to successfully decode the source message. Moreover, we adopt an N-block transmission protocol: During each of the N source transmission blocks, say, the i-th block, $1 \le i \le N$, the source transmits with power $P_S(i)$; after decoding the source message, the relay transmits with power $P_R(i+1)$ in the $(i+1)$-th block. Moreover, we assume that each block has B channel uses, where B is assumed large enough such that the channel capacity results in [5] are good approximations to the communication rates in practical systems.

In addition to the block transmission model, we assume that the harvested energy arrives at the beginning of each

∗This work is supported in part by DoD-AFOSR under Grant FA9550-09-1-0107 and DoD-DTRA under Grant HDTRA1-08-1-0010.

block with known amounts $E_S(i)$ in the i-th block and $E_R(i+1)$ in the $(i+1)$-th block, $i = 1, 2, \cdots, N$, at the source and the relay, respectively. In this paper, we assume that the battery capacity to store the harvested energy is infinite. Thus, the amount of energy available for each block transmission is constrained by the following source and relay energy-harvesting constraints:

$$\sum_{i=1}^{k} P_S(i) \leq \frac{1}{B} \sum_{i=1}^{k} E_S(i),\ k = 1, \cdots, N, \quad (1)$$

$$\sum_{i=1}^{k} P_R(i+1) \leq \frac{1}{B} \sum_{i=1}^{k} E_R(i+1),\ k = 1, \cdots, N. \quad (2)$$

For the i-th source and the $(i+1)$-th relay transmission blocks[1], $i = 1, \cdots, N$, the channel input-output relationships are given as:

$$y_{sr}(i) = \sqrt{h_{sr}} x_s(i) + n_r(i), \quad (3)$$

$$y_{sd}(i) = \sqrt{h_{sd}} x_s(i) + n_d(i), \quad (4)$$

$$y_{rd}(i+1) = \sqrt{h_{rd}} x_r(i+1) + w_d(i+1), \quad (5)$$

where $x_s(i)$ and $x_r(i+1)$ are the transmitted signals in the i-th source and the $(i+1)$-th relay transmission blocks with power $P_S(i)$ and $P_R(i+1)$, respectively; $y_{sr}(i)$ is the received signal at the relay; $y_{sd}(i)$ and $y_{rd}(i+1)$ are the received signals at the destination from the source and the relay, respectively; h_{sr}, h_{rd}, and h_{sd} are the constant channel power gains for the source-relay, relay-destination, and source-destination links, respectively; $n_r(i)$, $n_d(i)$, and $w_d(i+1)$ are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) noises each with zero mean and unit variance. By scaling the source and relay energy profiles with $\frac{1}{h_{sr}}$ and $\frac{1}{h_{rd}}$, respectively, and setting $h_{sr} = h_{rd} = 1$, the received SNR for each link is not changed [4], and thus we can without loss of generality assume that $h_{sr} = h_{rd} = 1$ and $h_{sd} = h_0$ in this paper. Moreover, we only consider the case that $0 \leq h_0 < 1$, which means that the relay can always help with increasing the achievable rate from the source to the destination by using the DF scheme.

3. PROBLEM FORMULATION

In the i-th source transmission block, the source transmits message w_i with power $P_S(i)$ and rate $R(i)$, and the relay decodes w_i reliably only if

$$R(i) \leq \mathcal{C}(P_S(i)). \quad (6)$$

Then, the relay partitions w_i into bins with an equivalent rate $R_B(i+1)$ [5], and transmits the binning index for message w_i in part of messages $v_{i+1}, \cdots, v_{N+1}$. In the $(i+1)$-th block, the relay transmits message v_{i+1} with power $P_R(i+1)$. At the destination, the binning indices for all source messages can be successfully decoded if

$$\sum_{i=1}^{N} R_B(i+1) = \sum_{i=1}^{N} \mathcal{C}(P_R(i+1)), \quad (7)$$

$$\sum_{i=k}^{N} R_B(i+1) \leq \sum_{i=k}^{N} \mathcal{C}(P_R(i+1)),\ 2 \leq k \leq N, \quad (8)$$

which is equivalent to

$$\sum_{i=1}^{k} R_B(i+1) \geq \sum_{i=1}^{k} \mathcal{C}(P_R(i+1)),\ 1 \leq k \leq N-1, \quad (9)$$

$$\sum_{i=1}^{N} R_B(i+1) = \sum_{i=1}^{N} \mathcal{C}(P_R(i+1)). \quad (10)$$

With the decoded binning index, the i-th source message can be decoded successfully at the destination if $R(i) \leq \mathcal{C}(h_0 P_S(i)) + R_B(i+1),\ i = 1, \cdots, N$. Combining this with (6), the achievable rate of the i-th source message is given as

$$\begin{aligned} R(i) &= \min\{\mathcal{C}(P_S(i)), \mathcal{C}(h_0 P_S(i)) + R_B(i+1)\} \\ &= \mathcal{C}(h_0 P_S(i)) + R_B(i+1),\ i = 1, \cdots, N, \end{aligned} \quad (11)$$

where the second equality is due to the fact that we can always decrease $R_B(i+1)$ to make it true. In addition, (11) implies that $\mathcal{C}(h_0 P_S(i)) + R_B(i+1) \leq \mathcal{C}(P_S(i)),\ i = 1, \cdots, N$, which leads to

$$\sum_{i=1}^{k} \mathcal{C}(P_S(i)) - \mathcal{C}(h_0 P_S(i)) \geq \sum_{i=1}^{k} R_B(i+1), \quad (12)$$

for $k = 1, \cdots, N$. From (9), (10), and (12), we obtain

$$\sum_{i=1}^{k} \mathcal{C}\left(h_0 P_S(i)\right) + \mathcal{C}\left(P_R(i+1)\right) \leq \sum_{i=1}^{k} \mathcal{C}\left(P_S(i)\right), \quad (13)$$

for $k = 1, \cdots, N$. Thus, the average throughput for the NDC case is maximized by solving the following problem:

$$\text{(P1)}\ \max\ \frac{\sum_{i=1}^{N} \mathcal{C}\left(h_0 P_S(i)\right) + \mathcal{C}\left(P_R(i+1)\right)}{2(N+1)} \quad (14)$$

$$\text{s. t.} \sum_{i=1}^{k} \mathcal{C}\left(h_0 P_S(i)\right) + \mathcal{C}\left(P_R(i+1)\right) \leq \sum_{i=1}^{k} \mathcal{C}\left(P_S(i)\right), \quad k = 1, \cdots, N,\ (1), \text{and } (2), \quad (15)$$

$$P_S(i) \geq 0,\ P_R(i+1) \geq 0,\ i = 1, \cdots, N, \quad (16)$$

where the factor $\frac{1}{2}$ in (14) is due to half-duplex relaying, and $\frac{1}{N+1}$ is due to the fact that each N-block transmission requires $(N+1)$-block duration. Problem (P1) is non-convex due to the first constraint in (15) [6], and thus difficult to solve at a first glance. However, we will derive the globally optimal solution in the next section.

[1]Note that the $(i+1)$-th relay transmission block in fact corresponds to the i-th source message.

4. OPTIMAL POWER AND RATE ALLOCATION

We first prove that a separation principle for the source and relay power allocation problem holds, upon which Problem (P1) can be solved by a two-stage strategy: First obtain the optimal source power allocation by ignoring the relay, and then optimize the relay power allocation with the obtained source power solution.

4.1. Optimal Source Power Allocation

First, we consider the following source power allocation problem by ignoring the relay:

$$\text{(P2)} \quad \max \sum_{i=1}^{N} \mathcal{C}\left(hP_S(i)\right) \tag{17}$$

$$\text{s. t.} \ (1), \ P_S(i) \geq 0, \ i = 1, \cdots, N, \tag{18}$$

where h is a constant with $0 < h \leq 1$. Problem (P2) has been solved in [2], for which the algorithm to compute the optimal solution is summarized in [4]. Note that the optimal source power profile $P_S^*(i)$'s of Problem (P2) are non-decreasing over i [2].

Since for the NDC case, the relay can store the binning indices of the decoded source messages with arbitrary delay before forwarding them to the destination with best effort transmissions, the relay power profile intuitively should have no effect on the optimal source power profile. This conjecture is affirmed by the following proposition.

Proposition 4.1 For the NDC case, the optimal source power solution for Problem (P2) is also globally optimal for Problem (P1).

The proof is given in [4] and omitted here due to the space limitation. This proposition implies that the separation principle for the source and relay power allocation problems is optimal for Problem (P1).

4.2. Optimal Relay Power Allocation

With the optimal source power profile $P_S^*(i)$'s obtained in Problem (P2), the relay power allocation problem is still nonconvex due to the constraints in (15) [6]. However, by letting $r(i+1) = \mathcal{C}\left(P_R(i+1)\right)$, the relay power allocation problem can be rewritten as

$$\text{(P3)} \quad \max_{r(i+1)\geq 0} \sum_{i=1}^{N} r(i+1) \tag{19}$$

$$\text{s. t.} \ \sum_{i=1}^{k} r(i+1) \leq \sum_{i=1}^{k} \mathcal{C}\left(P_S^*(i)\right) - \mathcal{C}\left(h_0 P_S^*(i)\right), \tag{20}$$

$$\sum_{i=1}^{k} (2^{2r(i+1)} - 1) \leq \frac{1}{B} \sum_{i=1}^{k} E_R(i+1), \ k = 1, \cdots, N. \tag{21}$$

Table 1. Algorithm 1: Compute the optimal solution for Problem (P3).

1. Initialize $i = 1$; while $i \leq N$, repeat
2. Compute

$$i_1 = \arg \min_{i \leq j \leq N} \left\{ \frac{\widetilde{C}_i + \sum_{k=i}^{j} \mathcal{C}\left(P_S^*(k)\right) - \mathcal{C}\left(h_0 P_S^*(k)\right)}{(j-i+1)\,B} \right\},$$

$$i_2 = \arg \min_{i \leq j \leq N} \left\{ \frac{\widetilde{E}_{i+1} + \sum_{k=i}^{j} E_S(k+1)}{(j-i+1)\,B} \right\},$$

$$\widetilde{r}_1 = \frac{\widetilde{C}_i + \sum_{k=i}^{i_1} \mathcal{C}\left(P_S^*(k)\right) - \mathcal{C}\left(h_0 P_S^*(k)\right)}{(i_1-i+1)\,B},$$

$$\widetilde{r}_2 = \mathcal{C}\left(\frac{\widetilde{E}_{i+1} + \sum_{k=i}^{i_2} E_S(k+1)}{(i_2-i+1)\,B} \right),$$

where $\widetilde{C}_1 = \widetilde{E}_2 = 0$, $\widetilde{C}_i = \sum_{k=1}^{i-1} \mathcal{C}\left(P_S^*(k)\right) - \mathcal{C}\left(h_0 P_S^*(k)\right) - r^*(k)$, and $\widetilde{E}_{i+1} = \sum_{k=1}^{i-1} \left(E_S(k+1) - 2^{2r^*(k+1)} - 1\right)$, $i = 2, \cdots, N$. Let $j_0 = \arg\min_{j=1,2} \{\widetilde{r}_j\}$. Set $r^*(j+1) = \widetilde{r}_{j_0}$, $j = i, \cdots, i_{j_0}$, and $i = i_{j_0} + 1$.

3. Algorithm ends.

It can be shown that Problem (P3) is convex over $r(i+1)$'s [6]. By the Karush-Kuhn-Tucker (KKT) optimality conditions, we obtain the optimal solution for Problem (P3) as

$$r^*(i+1) = \left(\frac{1}{2} \log \frac{1 - \sum_{k=i}^{N} \lambda_k}{2 \ln 2 \cdot \sum_{k=i}^{N} \gamma_k} \right)^+, \tag{22}$$

where λ_k and γ_k are the non-negative Lagrangian multipliers corresponding to the k-th constraint in (20) and (21), respectively. It is worth noting that from (22), we observe that the optimal relay transmission rate $r^*(i+1)$ is non-decreasing over i, and strictly increases when any one of the constraints (20) and (21) is satisfied with equality. As such, Problem (P3) can be solved by a forward search algorithm, denoted by Algorithm 1 in Table 1, for which the optimality proof is similar to that of Algorithm II in [4], and thus is omitted here.

Since $r^*(i+1)$'s are non-decreasing, the optimal relay power profile $P_R^*(i+1)$'s of Problem (P3) with $P_R^*(i+1) = 2^{2r^*(i+1)} - 1$, $i = 1, \cdots, N$, are also non-decreasing over i.

4.3. Optimal Rate Scheduling

With the obtained optimal source and relay power profiles $P_S^*(i)$'s and $P_R^*(i+1)$'s in the previous two subsections, the binning rate $R_B(i+1)$ at the relay can be first determined (as will be shown next), and then the source transmission rates $R(i)$'s can be determined from (11). This completes the coding scheme for the NDC case in Section III.

To compute $R_B(i+1)$'s, the following observations are first drawn. If $\mathcal{C}(P_R^*(i+1)) > \mathcal{C}(P_S^*(i)) - \mathcal{C}(h_0 P_S^*(i))$, $\forall i \in$

$\{1, \cdots, N\}$, the relay should transmit not only the binning index of the i-th source message at the $(i+1)$-th block, but also those of source messages $1 \leq j < i$. Moreover, due to the constraint in (15), it follows that if $\mathcal{C}(P_R^*(i+1)) > \mathcal{C}(P_S^*(i)) - \mathcal{C}(h_0 P_S^*(i)), \forall i$, there must exist j with $1 \leq j < i$, such that $\mathcal{C}(P_R^*(j+1)) < \mathcal{C}(P_S^*(j)) - \mathcal{C}(h_0 P_S^*(j))$. The above observations imply that to obtain $R_B(i+1)$'s, we need to find all i's with $\mathcal{C}(P_R^*(i+1)) > \mathcal{C}(P_S^*(i)) - \mathcal{C}(h_0 P_S^*(i))$, and then use their surplus rates to transmit the binning indices of source messages $j \leq i$.

Thus, we develop a backward search algorithm, which is summarized in [4], to obtain one of the feasible solutions for $R_B(i+1)$'s. The main procedure of this algorithm is described as follows. First, $R_B(i+1)$'s are initialized as the minimum values between $\mathcal{C}(P_R^*(i+1))$ and $\mathcal{C}(P_S^*(i)) - \mathcal{C}(h_0 P_S^*(i))$ for all i's, and a parameter t (sum of the positive surplus rates for $\mathcal{C}(P_R^*(i+1)) - (\mathcal{C}(P_S^*(i)) - \mathcal{C}(h_0 P_S^*(i))))$ is set to be 0. The algorithm then searches the values for $R_B(i+1)$'s in a backward way from $i = N$ to 1. For any i-th block, the algorithm computes $temp = \mathcal{C}(P_R^*(i+1)) - (\mathcal{C}(P_S^*(i)) - \mathcal{C}(h_0 P_S^*(i)))$. Then, if $temp > 0$, $temp$ is added to t; if $temp < 0$, the binning rate for the current source message is raised, i.e., $R_B(i+1)$ is increased by $\min(-temp, t)$, and this amount is then subtracted from t.

5. NUMERICAL RESULTS

In this section, we present some numerical results to validate our theoretical results. The source and relay energy profiles are given as $E_S(i) = A_S \sin\left(\frac{i-1}{N} 2\pi + \frac{\pi}{2}\right) + A_S$, $E_R(i+1) = A_R \sin\left(\frac{i-1}{N} 2\pi + \theta\right) + A_R$, $1 \leq i \leq N$, respectively, where $A_S, A_R > 0$ are the amplitudes of the sinusoidal energy profiles at the source and relay, respectively, and θ is the phase shift between these two energy profiles. Here, we choose $B = 100$, $N = 40$, $\theta = \frac{5}{4}\pi$, and $A_S = A_R = 200$. We compare our proposed algorithm for the NDC case with a greedy power allocation strategy, whereby both the source and relay consume as much available power as possible to maximize the instantaneous throughput at each block of the N-block transmission, as well as the optimal power allocation algorithm for the DC traffic case give in [4].

In Fig. 1, we show the average throughputs versus the direct link channel gain h_0 for the proposed power allocation algorithms and the greedy algorithm. It is observed that as the direct link becomes stronger, i.e., h_0 increases, there is a throughput limit of 0.387 bps/Hz. For the NDC case, this throughput limit is achieved even for very small h_0 around 0.05. In contrast, for the DC case, the throughput increases almost linearly and achieves the throughput limit when h_0 exceeds 0.75. For the greedy algorithm, it is observed that the throughput loss can be large, especially when h_0 is small, as compared to the proposed algorithm for the NDC case.

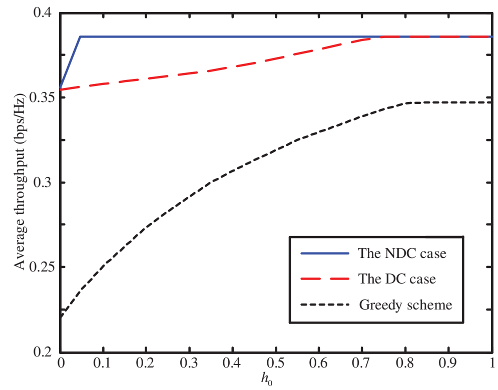

Fig. 1. Throughput comparison of various power allocation schemes for the orthogonal relay channel with energy harvesting constraints.

6. CONCLUTION

In this paper, we studied the throughput maximization problem for the orthogonal relay channel with energy-harvesting source and relay nodes, assuming a deterministic energy-harvesting model. For the case without decoding delay constraint at the destination, we examined the structures of the optimal source and relay power profiles over time, and developed algorithms to compute these optimal power profiles.

7. REFERENCES

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