CHANCE-CONSTRAINED OPTIMIZATION OF UPLINK PARAMETERS FOR OFDMA COGNITIVE RADIOS

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ABSTRACT

This paper deals with the resource allocation task for the uplink of OFDMA-based cognitive radio (CR) systems. A weighted sum-rate maximization problem is formulated to optimize the subcarrier assignment as well as the power loading per CR user, while protecting the primary user (PU) systems. Since the CR-to-PU channels may not be accurately acquired, the PU interference constraint is cast as a chance constraint. Consequently, a convex conservative approximation of the chance constraint is employed for tractability reasons. In particular, to mitigate the combinatorial complexity incurred for optimal subcarrier assignment, a separable structure is pursued, and the dual decomposition method is employed to obtain a near-optimal solver. The resultant algorithm is tested via simulated tests.

Index Terms— cognitive radios, resource allocation, OFDMA, channel uncertainty, optimization.

1. INTRODUCTION

Cognitive radios (CRs) aim to mitigate the scarcity of spectral resources by allowing opportunistic use of the bands licensed to primary user (PU) systems. In an *overlay* scenario, CRs target unused parts of the spectrum (called white space) via spectrum sensing. In the *underlay* scenario, which is the setting of interest in this work, CRs operate on the same band as the PUs by carefully controlling the interference caused to the PU system. In this case, the channel gain estimates between the CR transmitters and the PU receivers are required for optimizing the resource allocation (RA) task.

Since CRs do not receive explicit support from the PUs, acquiring accurate channel estimates is often challenging. Therefore, much research effort has been devoted to ensure that the interference constraint is effected robustly against channel uncertainty [1–4]. Statistical knowledge of the channel was assumed in [1–3], and bounded uncertainty (within an ellipsoid) was considered in [4].

Orthogonal frequency division multiple access (OFDMA) is a natural candidate for CR systems due to its flexibility in controlling spectrum usage. A major RA task for OFDMA radios is to allocate subcarriers to different users, and also load each subcarrier with proper power levels. However, very few works have addressed the RA problem for OFDMA-based CRs under channel uncertainty [2, 5]. Moreover, most existing approaches focus on the downlink scenario, and are not readily extendible to the uplink setup.

Robust interference constraints are often cast as chance constraints, which are typically more difficult to handle than their deterministic counterparts as they may be either nonconvex, or, tough to verify as being convex. Moreover, it is sometimes difficult to express these constraints in closed form. In such cases, convex approximation of chance constraints is of practical merit [6].

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The present paper addresses the RA problem for OFDMA uplink CRs with uncertain CR-to-PU channels. A weighted sum-rate maximization problem is formulated under a probabilistic interference constraint and maximum transmit-power constraints for the CR users. The Bernstein method is adopted to approximate the probabilistic constraint by a convex constraint. Even after the approximation, the overall problem is still nonconvex due to the combinatorial assignment of users to each subcarrier. By employing appropriate bounds, the approximation emerging from the interference constraint can be further made *separable* across subcarriers. This opens the door to the dual decomposition approach, which leads to a near-optimal and computationally efficient solution [7].

The rest of the paper is organized as follows. The problem is formulated in Sec. 2, and Bernstein's approximation technique tailored for chance constraints is outlined in Sec. 3. The RA algorithm is developed in Sec. 4. Numerical results are presented in Sec. 5, and conclusions are drawn in Sec. 6.

2. PROBLEM STATEMENT

Consider the uplink mode of a network comprising K CR users communicating with their base station (BS) using OFDMA over N subcarriers. The instantaneous channel gain $h_k^{(n)}$ between CR user $k \in \mathcal{K} \triangleq \{1, 2, \ldots, K\}$ and the CR BS on subcarrier $n \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$ is assumed to be perfectly known. It is further assumed that during the sensing phase, the presence of an active PU has been detected. In order to limit the interference inflicted to the PU, the channels from the CR users to the PU must be known. Let $g_k^{(n)}$ denote the channel gain from the k-th CR to the PU receiver on subcarrier n. Due to the lack of cooperation from the PU system, it is difficult to estimate $g_k^{(n)}$ precisely. To capture this uncertainty, $g_k^{(n)}$ is modeled as a random variable.

A relevant RA problem is to maximize the weighted sum of all CR throughputs under the transmit-power constraints (one per CR), and the PU interference constraint. Let $p^{(n)}$ denote the transmit-power loaded on subcarrier n, where $0 \leq p^{(n)} \leq P_{\max}^{(n)}$. Let $\mathbf{p}$ and $\mathbf{P}_{\max}$ be the vectorized versions of $\{p^{(n)}\}$ and $\{P_{\max}^{(n)}\}$, respectively. Also, let $k(n) \in \mathcal{K}$ represent the index of the user served on subcarrier n, and define $\mathbf{k} \triangleq [k(1), \ldots, k(n)]^T$. With w_k denoting the positive weight for user $k \in \mathcal{K}$, the following chance-constrained optimization problem is of interest

$$\text{(P1)} \quad \max_{0 \preceq \mathbf{p} \preceq \mathbf{P}_{\max}, \mathbf{k} \in \mathcal{K}^N} \sum_{n \in \mathcal{N}} w_{k(n)} \log\left(1 + h_{k(n)}^{(n)} p^{(n)}\right) \quad (1)$$

$$\text{subject to} \sum_{n \in \mathcal{N}: k(n)=k} p^{(n)} \leq P_{k,\max}, \quad k \in \mathcal{K} \quad (2)$$

$$\Pr\left\{\sum_{n \in \mathcal{N}} g_{k(n)}^{(n)} p^{(n)} < I_{\max}\right\} \geq 1 - \epsilon \quad (3)$$

where (3) enforces that the interference power at the PU stays below $I_{\max}$ with probability no less than $1-\epsilon$. The feasible set of (3) can be either convex or nonconvex, depending on the distribution of $g_k^{(n)}$ [8]. For example, $\Pr\left\{\mathbf{a}^T\mathbf{u} < \mathbf{b}\right\} \geq 1-\epsilon$ is convex for $\epsilon < 1/2$, if $[\mathbf{a}^T\ \mathbf{b}^T]^T$ has a symmetric logarithmically concave density [8]. However, even if (3) is convex, it may not be straightforward to express it in closed form, rendering the optimization problem intractable. Moreover, the overall problem would be still nonconvex due to the combinatorial search over $\mathbf{k}$.

In the following, a convex approximation of (3) is advocated, which is also conservative in the sense that the approximate constraint implies the original constraint (3). This will be achieved by using the Bernstein method [8,9]. By judiciously choosing the form of the approximation, one can also ensure that the approximated constraint is separable in n. Then, an efficient near-optimal solution will be obtained via dual decomposition [7].

3. APPROXIMATION OF CHANCE CONSTRAINTS

3.1. Bernstein Approximation

A useful class of approximation techniques for chance constraints known as Bernstein approximations is briefly reviewed in the present context [8,9]. Consider a chance constraint of the form

$$\Pr\left\{f_0(\mathbf{p}) + \sum_{n=1}^{N}\zeta_n f_n(\mathbf{p}) < 0\right\} \geq 1-\epsilon \tag{4}$$

where $\mathbf{p}$ denotes a deterministic parameter, and $\{\zeta_n\}$ are random variables. Suppose one desires to meet this constraint for a given family of $\{\zeta_n\}$ distributions, provided the following assumptions are satisfied.

as1) $\{f_n(\mathbf{p})\}$ are affine in $\mathbf{p}$ for $n=0,1,\ldots,N$.

as2) $\{\zeta_n\}$ are independent of each other, and their marginal distributions $\{\pi_n\}$ belong to *-compact convex sets of probability distributions.

as3) $\{\pi_n\}$ have a common bounded support of $[-1,1]$; that is, $-1 \leq \zeta_n \leq 1$ for all $n=1,\ldots,N$.

Under these assumptions, the following constraint constitutes a conservative substitute and thus implies (4)

$$\inf_{\rho>0}\left[f_0(\mathbf{p}) + \rho\sum_{n=1}^{N}\Omega_n\left(\rho^{-1}f_n(\mathbf{p})\right) + \rho\log\left(\frac{1}{\epsilon}\right)\right] \leq 0 \tag{5}$$

where $\Omega_n(y) \triangleq \max_{\pi_n}\log\left(\int\exp(xy)d\pi_n(x)\right)$. Moreover, it is guaranteed that (5) is convex [8,9]. The approximation is useful when $\{\Omega_n(y)\}$ can be evaluated efficiently. In general, one can consider an upperbound for $\Omega_n(y)$ given by

$$\Omega_n(y) \leq \max\{\mu_n^- y, \mu_n^+ y\} + \frac{\sigma_n^2}{2}y^2, \quad n=1,\ldots,N \tag{6}$$

where μ_n^-, μ_n^+ with $\mu_n^- \leq \mu_n^+$ and σ_n are constants that depend on the given families of probability distributions. Some examples are given in Table 1 in [9]. Replacing $\Omega_n(\cdot)$ in (5) with this upperbound, and invoking the arithmetic-geometric inequality, yields

$$f_0(\mathbf{p}) + \sum_{n=1}^{N}\max\{\mu_n^- f_n(\mathbf{p}), \mu_n^+ f_n(\mathbf{p})\} + \sqrt{2\log\left(\frac{1}{\epsilon}\right)}\left(\sum_{n=1}^{N}\sigma_n^2 f_n(\mathbf{p})^2\right)^{\frac{1}{2}} \leq 0 \tag{7}$$

as a convex conservative surrogate for (4).

Suppose now that the distributions of $g_k^{(n)}$ have bounded supports $[a_k^{(n)}, b_k^{(n)}]$. The case with unbounded supports will be treated in Sec. 3.2. Introduce constants $\alpha_k^{(n)} \triangleq \frac{1}{2}(b_k^{(n)} - a_k^{(n)})$ and $\beta_k^{(n)} \triangleq \frac{1}{2}(b_k^{(n)} + a_k^{(n)})$ to normalize the supports to $[-1,1]$ per as3); that is, $\alpha_k^{(n)}\zeta_n + \beta_k^{(n)} \in [a_k^{(n)}, b_k^{(n)}]$. Then, letting $f_0(\mathbf{p}) = -I_{\max} + \sum_{n=1}^{N}\beta_{k(n)}^{(n)}p^{(n)}$ and $f_n(\mathbf{p}) = \alpha_{k(n)}^{(n)}p^{(n)}$ for $n \in \mathcal{N}$, it follows that (4) is equivalent to (3). Thus, substituting into (7), and noting that $p^{(n)} \geq 0$, one obtains

$$-I_{\max} + \sum_{n=1}^{N}\beta_{k(n)}^{(n)}p^{(n)} + \sum_{n=1}^{N}\mu_{k(n)}^{(n)+}\alpha_{k(n)}^{(n)}p^{(n)} + \sqrt{2\log\frac{1}{\epsilon}}\left(\sum_{n=1}^{N}(\sigma_{k(n)}^{(n)}\alpha_{k(n)}^{(n)}p^{(n)})^2\right)^{\frac{1}{2}} \leq 0. \tag{8}$$

The overall RA problem corresponding to (P1) with (3) replaced by (8) is still nonconvex due to the combinatorial search in $\mathbf{k}$. In fact, as the variables $p^{(n)}$ are coupled through the last term in (8), the search complexity grows rapidly as the number of subcarriers (N) increases. To mitigate these issues, we further approximate (8) by noting that the last term in (8) involves the ℓ_2-norm of the vector $[\sigma_{k(1)}^{(1)}\alpha_{k(1)}^{(1)}p^{(1)}, \ldots, \sigma_{k(N)}^{(N)}\alpha_{k(N)}^{(N)}p^{(N)}]$, and that $\|\mathbf{x}\|_2 \leq \sqrt{N}\|\mathbf{x}\|_\infty$ for any $\mathbf{x} \in \mathbb{R}^N$. Thus, the constraint becomes

$$\sum_{n=1}^{N}\gamma_{k(n)}^{(n)}p^{(n)} + \sqrt{2N\log\frac{1}{\epsilon}}\max_{n\in\mathcal{N}}\sigma_{k(n)}^{(n)}\alpha_{k(n)}^{(n)}p^{(n)} \leq I_{\max} \tag{9}$$

where $\gamma_{k(n)}^{(n)} \triangleq \mu_{k(n)}^{(n)+}\alpha_{k(n)}^{(n)} + \beta_{k(n)}^{(n)}$.

Alternatively, one can appeal to the fact that $\|\mathbf{x}\|_2 \leq \|\mathbf{x}\|_1$ to obtain yet another substitute for (3) as

$$\sum_{n=1}^{N}\gamma_{k(n)}^{(n)}p^{(n)} + \sqrt{2\log\frac{1}{\epsilon}}\sum_{n=1}^{N}|\sigma_{k(n)}^{(n)}\alpha_{k(n)}^{(n)}p^{(n)}| \leq I_{\max}. \tag{10}$$

Both (9) and (10) are amenable to dual decomposition, as will be discussed in Sec. 4.

3.2. Extensions to Channels with Unbounded Support

In the preceding discussion, Bernstein approximations were applied to bounded channel gains. While this may be reasonable considering the finite dynamic ranges of the A/D converters in the radios, presuming too large a range for the uncertain parameters inevitably leads to a very loose approximation of the chance constraint. An alternative approach is developed here when the channel distributions are known (as opposed to the previous case where the *family* of possible distributions were known.)

Upon defining $I \triangleq \sum_n g_{k(n)}^{(n)}p^{(n)}$, it is possible to express $\Pr\{I < I_{\max}\}$ in (3) as

$$\Pr\{I < I_{\max}|\mathbf{a} \leq \mathbf{g} \leq \mathbf{b}\}\Pr\{\mathbf{a} \leq \mathbf{g} \leq \mathbf{b}\} + \Pr\{I < I_{\max}|\mathbf{g} < \mathbf{a} \text{ or } \mathbf{g} > \mathbf{b}\}\Pr\{\mathbf{g} < \mathbf{a} \text{ or } \mathbf{g} > \mathbf{b}\} \tag{11}$$

where $\mathbf{g} \triangleq [g_{k(1)}^{(1)}, ..., g_{k(N)}^{(N)}]^T$, and $\mathbf{a} \triangleq [a_{k(1)}^{(1)}, ..., a_{k(N)}^{(N)}]^T$ and $\mathbf{b} \triangleq [b_{k(1)}^{(1)}, ..., b_{k(N)}^{(N)}]^T$ are appropriate constants determined such that $\delta \triangleq \Pr\{\mathbf{a} \le \mathbf{g} \le \mathbf{b}\} \in (1-\epsilon, 1)$. Then, neglecting the second term in (11), inequality (3) can be approximated conservatively as

$$\Pr\{I < I_{\max} \,|\mathbf{a} \le \mathbf{g} \le \mathbf{b}\} \ge \frac{1-\epsilon}{\Pr\{\mathbf{a} \le \mathbf{g} \le \mathbf{b}\}} = \frac{1-\epsilon}{\delta} \triangleq 1-\epsilon' \tag{12}$$

which can now be approximated by (9) or (10) with ϵ replaced by ϵ'.

To make things concrete, consider the case where the *p.d.f.* of $g_k^{(n)}$ is exponential (Rayleigh fading) with mean $\bar{g}_k^{(n)}$ as

$$f_{g_k^{(n)}}(x) = \frac{1}{\bar{g}_k^{(n)}} \exp\left(-\frac{x}{\bar{g}_k^{(n)}}\right) . \tag{13}$$

It then follows from independence that

$$\begin{aligned}\Pr\{\mathbf{a} \le \mathbf{g} \le \mathbf{b}\} &= \prod_{n=1}^{N} \Pr\left\{a_{k(n)}^{(n)} \le g_{k(n)}^{(n)} \le b_{k(n)}^{(n)}\right\} \\ &= \prod_{n=1}^{N}\left[\exp\left(-\frac{a_{k(n)}^{(n)}}{\bar{g}_{k(n)}^{(n)}}\right) - \exp\left(-\frac{b_{k(n)}^{(n)}}{\bar{g}_{k(n)}^{(n)}}\right)\right] = \delta. \end{aligned} \tag{14}$$

It is natural to choose the lowerbound as $\mathbf{a} = \mathbf{0}$. To determine $\mathbf{b}$, $\Pr\left\{0 \le g_{k(n)}^{(n)} \le b_{k(n)}^{(n)}\right\}$ is enforced to be constant across subcarriers. Then, $\mathbf{b}$ is obtained as

$$b_{k(n)}^{(n)} = \bar{g}_{k(n)}^{(n)} \log \frac{1}{1-\delta^{\frac{1}{N}}}, \quad n \in \mathcal{N} . \tag{15}$$

It is worth noting that even if channel *p.d.f.*'s are exactly known to be exponential, the *p.d.f.* of I is not easily expressed in closed form [10].

4. RESOURCE ALLOCATION ALGORITHM

The OFDMA RA problems with separable structure can be tackled efficiently in the dual domain. In this approach, the overall problem is divided into multiple smaller per-subcarrier subproblems, which can be solved independently, coordinated by the dual variables. Moreover, it can be shown that the duality gap vanishes as the number of subcarriers increases. The approach has been widely applied to the RA problems for multi-carrier systems [7].

Problem (P1) with (3) substituted with (10) is clearly separable in n. Upon using (9) and introducing auxiliary variables $\mathbf{u} \triangleq [u_1, \ldots, u_N]^T$, the following separable constraints are seen to be equivalent.

$$\sum_{n=1}^{N} \gamma_{k(n)}^{(n)} p^{(n)} + \sqrt{2\log\frac{1}{\epsilon}} \sum_{n=1}^{N} u_n \le I_{\max} \tag{16}$$

$$\sqrt{N}\sigma_{k(n)}^{(n)}\alpha_{k(n)}^{(n)} p^{(n)} \le \sum_{n'=1}^{N} u_{n'}, \quad n = 1, \ldots, N \tag{17}$$

We continue the derivation of the RA algorithm using the above constraints. The case with (10) can be handled similarly.

Introducing dual variables $\boldsymbol{\lambda} \triangleq [\lambda_1, \lambda_2, \ldots, \lambda_N]^T \succeq \mathbf{0}$, $\boldsymbol{\mu} \triangleq [\mu_1, \mu_2, \ldots, \mu_K]^T \succeq \mathbf{0}$ and $\nu \ge 0$ to relax (17), (2), and (16),

Initialize $\boldsymbol{\Sigma}$ and $\boldsymbol{\theta} \triangleq [\boldsymbol{\mu}^T \boldsymbol{\lambda}^T]^T$. Set tolerance τ
Repeat
  If $\boldsymbol{\theta} < 0$, for some indices $i \in \mathcal{I}$,
    set $\mathbf{d} = \sum_{i \in \mathcal{I}} \mathbf{e}_i$ ($\mathbf{e}_i$ is the i-th canonical basis)
  Otherwise:
    Find $\mathbf{k}^*$ and $\mathbf{p}^*$ from (25)–(26)
    Set $\mathbf{d}$ as the subgradient of $D(\cdot)$ w.r.t. $[\boldsymbol{\mu}^T \boldsymbol{\lambda}^T]^T$
  If $\sqrt{\mathbf{d}^T \boldsymbol{\Sigma} \mathbf{d}} < \tau$, stop
  Perform the ellipsoid update:
    $\mathbf{d} \leftarrow \mathbf{d} / \sqrt{\mathbf{d}^T \boldsymbol{\Sigma} \mathbf{d}}$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \boldsymbol{\Sigma}\mathbf{d}/(N+K+1)$
    $\boldsymbol{\Sigma} \leftarrow \frac{(N+K)^2}{(N+K)^2-1}\left(\boldsymbol{\Sigma} - \frac{2}{N+K+1}\boldsymbol{\Sigma}\mathbf{d}\mathbf{d}^T\boldsymbol{\Sigma}\right)$

Table 1. Overall RA algorithm.

respectively, one can write the Lagrangian as

$$\begin{aligned} L(\mathbf{p}, \mathbf{u}; \boldsymbol{\lambda}, \boldsymbol{\mu}, \nu) = \sum_{n \in \mathcal{N}} w_{k(n)} \Bigg[\log\left(1 + h_{k(n)}^{(n)} p^{(n)}\right) \\ - \left(\nu\gamma_{k(n)}^{(n)} + \mu_{k(n)} + \lambda_n \sqrt{N}\sigma_{k(n)}^{(n)}\alpha_{k(n)}^{(n)}\right) p^{(n)} \\ + \left(\sum_{n' \in \mathcal{N}} \lambda_{n'} - \nu\sqrt{2\log\frac{1}{\epsilon}}\right) u_n\Bigg] + \nu I_{\max} + \sum_{k \in \mathcal{K}} \mu_k P_{k,\max}. \end{aligned} \tag{18}$$

Therefore, the dual function is

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \sup_{\mathbf{0} \preceq \mathbf{p} \preceq \mathbf{P}_{\max}, \mathbf{u}, \mathbf{k} \in \mathcal{K}^N} L(\mathbf{p}, \mathbf{u}; \boldsymbol{\lambda}, \boldsymbol{\mu}, \nu) \tag{19}$$

$$= \sup_{\substack{\mathbf{0} \preceq \mathbf{p} \preceq \mathbf{P}_{\max} \\ \mathbf{k} \in \mathcal{K}^N}} \sum_{n \in \mathcal{N}} L_n(p^{(n)}, k(n)) + \nu I_{\max} + \sum_{k \in \mathcal{K}} \mu_k P_{k,\max} \tag{20}$$

where

$$L_n(p^{(n)}, k) \triangleq w_k \log\left(1 + h_k^{(n)} p^{(n)}\right) - t_k^{(n)} p^{(n)} \tag{21}$$

$$t_k^{(n)} \triangleq \nu\gamma_k^{(n)} + \mu_k + \lambda_n \sqrt{N}\sigma_k^{(n)}\alpha_k^{(n)} \tag{22}$$

and $\nu = \left(2\log\frac{1}{\epsilon}\right)^{-\frac{1}{2}} \sum_{n' \in \mathcal{N}} \lambda_{n'}$. The dual problem is thus

$$\inf_{\boldsymbol{\lambda} \succeq \mathbf{0}, \boldsymbol{\mu} \succeq \mathbf{0}} D(\boldsymbol{\lambda}, \boldsymbol{\mu}) . \tag{23}$$

It can be seen from (20) that the optimization can be decoupled to per-tone problems given by

$$\max_{0 \le p^{(n)} \le P_{\max}^{(n)}, k(n) \in \mathcal{K}} L_n(p^{(n)}, k(n)) \tag{24}$$

If $k(n) = k$, the optimal power loading $p^{*(n)}[k]$ can be shown to be

$$p^{*(n)}[k] = \left[\frac{w_k}{t_k^{(n)}} - \frac{1}{h_k^{(n)}}\right]_0^{P_{\max}^{(n)}}, \quad n \in \mathcal{N} \tag{25}$$

where $[\cdot]_a^b \triangleq \min\{\max\{0, a\}, b\}$. The optimal user allocation $\mathbf{k}^*$ is then given by

$$k^*(n) \in \arg\max_{k \in \mathcal{K}} L_n(p^{*(n)}[k], k), \quad n \in \mathcal{N} \tag{26}$$

and the optimal power loading by

$$p^{*(n)} = p^{*(n)}[k^*(n)], \quad n \in \mathcal{N} . \tag{27}$$

The dual problem (23) can be solved using, e.g., the subgradient method, or the ellipsoid method. The overall RA algorithm based on the ellipsoid method is given in Table 1.

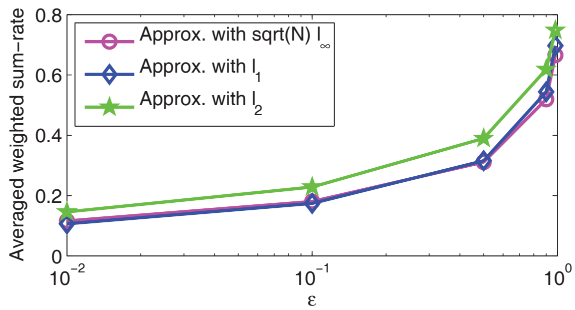

Fig. 1. Weighted sum-rates vs. ϵ.

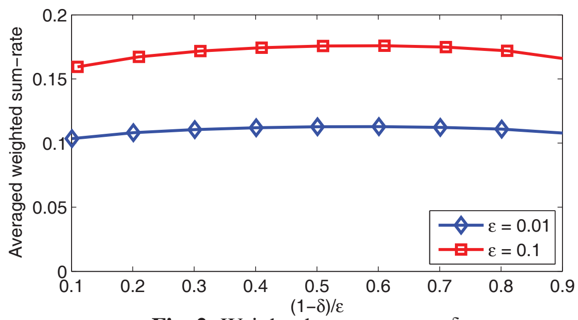

Fig. 2. Weighted sum-rates vs. δ.

5. NUMERICAL TESTS

The proposed RA algorithms were tested via numerical experiments. A 2-user system with $N = 8$ OFDM subcarriers was considered. The wideband links between the CR-BS and the CR users were simulated as 4-path Rayleigh fading channels. The channel gains between the CR users and the PU are modeled as *i.i.d.* and exponentially distributed with $\bar{g}_k^{(n)} = 2$ for all k and n, corresponding to the case where the instantaneous small-scale fading is not known. The truncation thresholds $\mathbf{b}$ were computed via (15). The parameters $\mu_k^{(n)+} = \mu_k^{(n)-}$ and $\sigma_k^{(n)}$ for the Bernstein approximations were chosen from Table 1 in [9] using the known first- and second-order moments of the truncated channel gains.

In Fig. 1, the weighted sum-rates averaged over 80 channel realizations of $\{h_k^{(n)}\}$ are depicted for different values of ϵ when $w_2 = 4w_1 = 0.8$. The value of δ was set to $1 - 0.5\epsilon$. The curve with star markers represents the optimal objective obtained from solving (P1) with (3) replaced by (8) ("ℓ_2-norm" approximation). Since dual decomposition cannot be applied in this case, an exhaustive search of $\mathbf{k} \in \mathcal{K}^N$ was performed. For a given assignment $\mathbf{k}$, the power loading problem is convex. The curves with the diamond and the circle markers correspond to the solutions of (P1) with (10) ("ℓ_1-norm") and (9) ("ℓ_∞-norm") as the approximate interference constraints, respectively. Since smaller ϵ tightens the constraint, the weighted-sum rates increase monotonically in ϵ. Also, since the "ℓ_1-" and the "ℓ_∞-norm" cases are more conservative than the "ℓ_2" case, the rates for the former are lower than those for the latter. However, it is seen from the figure that the differences are small. On the other hand, when the channels $\{g_k^{(n)}\}$ are perfectly known, the average weighted sum-rate was around 1.6, exhibiting a large gap compared to the case when only the statistical knowledge of the channels is available. However, the gap can be reduced when (imperfect) channel estimation is modeled, and will be explored in the journal version.

The sensitivity of the weighted sum-rate performance to the choice of δ is examined in Fig. 2 for $\epsilon = 0.01$ and 0.1. It is seen that the performance is quite robust to the choice of δ.

6. CONCLUSIONS

Maximization of the weighted sum-rate of an OFDMA-based CR uplink was accomplished by optimizing the power loading and the user assignment over the individual subcarriers. The RA problem must ensure that the interference power experienced at each PU's location is less than a pre-specified threshold. Since the channel gains between CR transmitters and PU receivers often cannot be estimated accurately, the PU interference constraint was cast as a chance constraint. As the resulting optimization problem is intractable, a convex conservative surrogate of the chance constraint was employed using Bernstein approximations. On the other hand, due to the combinatorial complexity of searching for the optimal user assignment, OFDMA RA problems are often tackled in the dual domain. To apply this technique, the chance constraint was further approximated so that the overall problem possesses a separable structure. An algorithm based on the dual decomposition method was presented. The numerical tests showed that performance degradation due to the approximation introduced for enforcing the separability is rather insignificant.

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