SCHEDULING AND POWER CONTROL IN STATISTICAL BEAMFORMING NETWORKS USING B BITS OF FEEDBACK

Sagnik Ghosh, Bhaskar D. Rao, and James R. Zeidler

University of California, San Diego
Electrical and Computer Engineering
9500 Gilman Dr., La Jolla, CA 92093
s1ghosh@ucsd.edu, brao@ece.ucsd.edu, zeidler@ece.ucsd.edu

ABSTRACT

In this work, we develop limited feedback techniques that utilize both Channel State Information (CSI) and Channel Distribution Information (CDI) for communication in multiuser MIMO beamforming networks with SINR constraints on the links. We minimize power in the network using a CDI-based algorithm, and then use limited CSI feedback to improve on this scheme by reducing power consumption further and transmit opportunistically. We study three cases of the CSI feedback channel–with one bit of bandwidth, with infinite bits, and with B bits. We develop feedback techniques for the one-bit and infinite-bit cases, and then derive the optimal quantizer for the B-bit case. Our results show that significant power reduction can be achieved using a small number of bits.

Index Terms— covariance feedback, beamformers, multiuser MIMO, outage probability, Channel Distribution Information

I. INTRODUCTION

Multiuser MIMO (MU-MIMO) networks are increasingly popular in communications systems today to increase capacity and combat interference. To fully exploit the advantages of MIMO, full CSI of all nodes in the network is required [1]. This information can be difficult to measure and attain. While receivers are generally able to measure the channel from its corresponding transmitter using training sequences, estimating the channel of the interfering transmitters is more difficult. In addition, feeding back full CSI to the transmitter is expensive.

Due to these difficulties, much work has been done to reduce feedback to the transmitter (for example, [2],[3]). One method to reduce feedback is to utilize statistical information about the channel, or Channel Distribution Information (CDI) [4]-[8]. CDI changes less frequently than CSI. Thus, schemes based on CDI require significantly less feedback. However, schemes based on CDI use more power or achieve less capacity as compared to their CSI counterparts. Thus, systems that use limited CSI in conjunction with CDI are desired to keep feedback low while approaching the performance of perfect CSI of the network.

In this work, we build on our CDI framework from [8] by adding limited, decentralized CSI feedback for scheduling and power reduction. In [8], algorithms for joint beamforming and power control are developed to meet an SINR threshold for each user while minimizing power. Since only statistical information is used, the algorithms are designed to meet some outage requirement for each user. This outage is undesirable, and CSI feedback to the transmitter can be used to combat outage. Since the receiver can measure the instantaneous SINR on its link, it can inform the transmitter to adjust its power accordingly. If the SINR is above the desired threshold, the transmitter can reduce its power to meet the threshold. However, if the SINR is below the desired threshold, then the transmitter should not transmit and wait for channel conditions to be more favorable. If the transmitter increases its power, then interference to other users is increased and those users may no longer be able to meet their SINR requirements. Thus, the transmitter decides whether or not to decrease its power or to not transmit at all.

Feeding back the SINR from the receiver to the transmitter can be expensive and take up significant bandwidth. Taking this into consideration, we develop an optimal thresholding scheme using B bits of feedback. Section II discusses the problem setup. Section III studies two extreme cases followed by a derivation of the set of equations to be solved for using B bits of feedback. Section IV discusses the results, and Section V discusses further improvements that can be made to this scheme.

This work was sponsored in part by the Center for Wireless Communications, UC Discovery grant Com09R-156604 and NSF grant CCF-1115645.

II. PROBLEM FORMULATION

II-A. System Model

This work considers time-varying MIMO channels for multiple users in a network. Consider a MIMO network with L transmit-receive pairs. At link l, the transmitter sends the symbol $s_l(t)$ to the receiver. The transmitter uses unit-norm beamforming vector $\mathbf{v}_l(t)$ to precode the signal and transmits with power $p_l(t)$. The receiver employs the linear unit-norm beamformer $\mathbf{u}_l(t)$ to combine the signal. The channel from transmitter i to receiver l is given by $\mathbf{H}_{li}(t)$. The noise $N_l(t)$ is distributed as a complex circular Gaussian, and represents the combined noise after applying the receive beamforming vector to the incoming signal. The l^{th} received signal is thus given by

$$r_l(t) = \sqrt{p_l(t)}[\mathbf{u}_l^H(t)\mathbf{H}_{ll}(t)\mathbf{v}_l(t)]s_l(t) + N_l(t) + \sum_{i \neq l}^{L} \sqrt{p_i(t)}[\mathbf{u}_l^H(t)\mathbf{H}_{li}(t)\mathbf{v}_i(t)]s_i(t)$$

In schemes that use perfect CSI, a block-fading model is assumed, so the channel stays constant over each block. Then, for notational convenience, the time variable will be dropped for the channel, power allocations, and beamformers. To further simplify notation, define $G_{li} = |\mathbf{u}_l^H \mathbf{H}_{li} \mathbf{v}_i|^2$ as the beamforming channel gain from the transmitter on link i to the receiver at link l and $\sigma^2_{N_l}$

as the noise power for the l^{th} link. Then, under this model, the SINR Γ_l on each link can be shown to be

$$\Gamma_l = \frac{p_l G_{ll}}{\sum_{i \neq l} p_i G_{li} + \sigma_{N_l}^2} \tag{1}$$

If perfect CSI is available, to ensure a reliable link is available to all nodes in the network, each link has an SINR constraint: Γ_l must be greater than a threshold γ_l. The goal is then to minimize the power consumed by the network while meeting all the SINR constraints. The cost function considered in this work is the *weighted sum power*. In this setup, each link l in the network incurs some cost $w_l > 0$ to transmit across its link. An example of a network with varying power costs on the links are networks with varying battery life at the transmitters. For minimizing non-weighted sum power, $w_l = 1$ for $l = 1, \ldots, L$.

To compact notation, define the weighting and power vectors as $\mathbf{w} = \{w_1, \ldots, w_L\}$ and $\mathbf{p} = \{p_1, \ldots, p_L\}$, respectively. The beamforming matrices are defined as $\mathbf{U} = \{\mathbf{u}_1, \ldots, \mathbf{u}_L\}$ and $\mathbf{V} = \{\mathbf{v}_1, \ldots, \mathbf{v}_L\}$. The optimization problem for having perfect CSI can then be stated as follows:

$$\begin{aligned} &\min_{\mathbf{p} \geq 0, \mathbf{U}, \mathbf{V}} \mathbf{w}^T \mathbf{p} \\ &\text{s.t.} \quad \Gamma_l \geq \gamma_l, l = 1, \ldots, L \end{aligned} \tag{2}$$

For a fixed set of channel matrices $\mathbf{H}_{li}$'s, this problem can be solved and will give a set of power allocations and beamformers for the transmitters and receivers in the network [1]. Then, for every change in the $\mathbf{H}_{li}$'s, all the transmit and receive beamformers must be updated, and the power allocation scheme changes. In many networks, the feedback required for these changes in the channel is unrealistic due to rapidly-varying CSI. Thus, this work considers a network where full CDI is available, but full instantaneous CSI is unavailable. All the links are assumed to undergo correlated Rayleigh fading. Then, when only CDI is known, the exact $\mathbf{H}_{li}$'s are not known–instead, they are assumed to be a random variable drawn from a complex-normal distribution:

$$\text{vec}(\mathbf{H}_{li}) \sim CN(0, \mathbf{\Sigma}_{li})$$

The channel covariance matrices, given by the $\mathbf{\Sigma}'_{li}s$, comprise the CDI of the network. This work will consider the case where the channel varies, but the statistics stay constant. Under this model, the expression for SINR given in (1) becomes a random variable since it depends on the channel. The constraints in (2) can then no longer be written as the SINR on link l always exceeding some threshold γ_l–since the SINR is now random, it will drop below γ_l with some probability. Therefore, these absolute constraints change to outage constraints, and links are allowed to have an SINR below their thresholds for specified probabilities. Mathematically, the constraint on link l in (2) becomes $\Pr(\Gamma_l \leq \gamma_l) \leq \alpha_l$, where α_l is the probability that the link is in outage. The main optimization problem using CDI can then be formulated:

$$\begin{aligned} &\min_{\mathbf{p} \geq 0, \mathbf{U}, \mathbf{V}} \mathbf{w}^T \mathbf{p} \\ &\text{s.t.} \quad \Pr(\Gamma_l \leq \gamma_l) \leq \alpha_l, l = 1, \ldots, L \end{aligned} \tag{3}$$

This problem was studied in [8]. There are two main drawbacks of this solution versus the CSI solution. First, the outage α_l on the links is undesirable, and second, the power for a given set of channel matrices at any time may be higher than required to meet the SINR thresholds. Thus, in this work we use limited SINR feedback to address these two issues and improve on the CDI solution. Since the receiver can measure its channel, it is no longer limited to using the receive beamforming vector obtained from the CDI-based algorithm and can measure its SINR to feed back to the transmitter. The beamformer used here at the receiver is the LMMSE beamformer.

III. THREE CASES OF SINR FEEDBACK

First, we will consider two extremes of SINR feedback. Then we will study the general case of B bits of feedback.

III-A. Using 1 Bit of Feedback

With 1 bit of feedback, the receiver can inform the transmitter whether or not to transmit based on the channel conditions. Thus if the channel conditions are not favorable, instead of transmitting and having outage, the transmitter will send its data when the channel conditions get better based on feedback from the receiver. The l^{th} receiver at time t can measure the instantaneous SINR $\Gamma_l(t)$. If the SINR is greater than γ_l (when $\Gamma_l(t) \geq \gamma_l$), then the receiver sends back a 1; otherwise, the receiver sends back a 0 (when $\Gamma_l(t) < \gamma_l$). If the transmitter gets a 1, it transmits data at the original power level given by the CDI-based algorithm. Otherwise, if it receives a 0, it stays silent until channel conditions change again. This scheme of the transmitter staying silent when a 0 is received will also be utilized when feeding back B bits. However, if more than 1 bit is available on the feedback channel, the receiver can inform the transmitter to reduce its power.

III-B. Using Infinite Bits of Feedback

With infinite bits of feedback, the receiver can send the transmitter the exact instantaneous SINR $\Gamma_l(t)$. If $\Gamma_l(t) < \gamma_l$, then the transmitter will stay silent, as with the 1-bit feedback case. If the transmitter tried to boost the power to meet the SINR threshold, then interference would be increased to other users and they may not be able to meet their own SINR requirements. However, if $\Gamma_l(t) \geq \gamma_l$, then the transmitter can rescale the power accordingly to meet the SINR threshold exactly and save power. Since $\Gamma_l(t) = \frac{p_l G_{ll}(t)}{\sum_{i \neq l} p_i G_{li}(t) + \sigma_{N_l}^2}$, $\Gamma_l(t) \propto p_l$. The new SINR should meet meet the target SINR γ_l. Therefore, the new power used at time t by transmitter l, $p_l(t)$, is given by

$$p_l(t) = \frac{\gamma_l}{\Gamma_l(t)} p_l$$

Note that this update equation for the power assumes that all the other interfering powers do not change. While this is not an ideal assumption, due to the restrictions being placed on the system, the interference will not rise, and so the SINR requirement γ_l will be met.

III-C. Using B Bits of Feedback

In using B bits of feedback, ideas from the two extreme cases will be combined. When the SINR at user l, $\Gamma_l(t)$, is below the threshold γ_l, 0 will be fed back and the transmitter will not transmit. However, when $\Gamma_l(t)$ is above the threshold γ_l, the transmitter will reduce its power depending on the region that $\Gamma_l(t)$ falls in. Take for example $B = 2$ bits of feedback, and a set of thresholds $\gamma_l = \gamma_l^{(0)} < \gamma_l^{(1)} < \gamma_l^{(2)}$. If $\Gamma_l(t) < \gamma_l^{(0)}$, 0 is sent back. If $\gamma_l \leq \Gamma_l(t) < \gamma_l^{(1)}$, 1 is sent back, and the power is not reduced. If $\gamma_l^{(1)} \leq \Gamma_l(t) < \gamma_l^{(2)}$, 2 is sent back, and the power at the transmitter is updated as $p_l(t) = \frac{\gamma_l}{\gamma_l^{(1)}} p_l$. This power reduction is guaranteed to meet the SINR threshold γ_l, since $\Gamma_l(t) \geq \gamma_l^{(1)}$. If $\gamma_l^{(2)} \leq \Gamma_l(t)$, 3 is sent back, and the power at the transmitter is updated as $p_l(t) = \frac{\gamma_l}{\gamma_l^{(2)}} p_l$, with similar reasoning as before. This scheme can be extended to an arbitrary number of bits–the SINR is scaled according to the lower bound on the region.

For the previously described scheme, the thresholds $\gamma_l^{(i)}$ must be determined. For notational convenience, the user subscript l will be dropped for this discussion, as this process must be applied for each user. Also, define $p^{(i)}$ and $\gamma^{(i)}$ to be the power transmitted and the SINR value assumed, respectively, when the value $i+1$ is fed back. In this notation, note that $p^{(0)} = p$, the original power transmitted. Over time, the average power used by the transmitter will be minimized, so the optimization problem is

$$\min_{\gamma^{(i)}, i=1\ldots 2^B-2} E[p(t)] \tag{4}$$

The objective function $E[p(t)]$ is given by

$$E[p(t)] = \sum_{i=0}^{2^B-2} \Pr(p^{(i)}) p^{(i)}$$

As indicated in the above example for $B = 2$, $p^{(i)} = \frac{\gamma}{\gamma^{(i)}} p^{(0)}$. Also, recall that the statistical information of the channels is known, so $\Pr(p^{(i)})$ can be calculated from the CDF of the SINR. This expression is given by $\Pr(p^{(i)}) = F(\gamma^{(i+1)}) - F(\gamma^{(i)})$, where $F(\gamma) = \Pr(\Gamma \leq \gamma)$. Also, define $\gamma^{(2^B-1)} = \infty$ and therefore $F(\gamma^{(2^B-1)}) = 1$ for notational convenience. Thus, the optimization problem in (4) can now be written as

$$\begin{aligned}
&\min_{\gamma^{(i)}, i=1\ldots 2^B-2} \sum_{i=0}^{2^B-2} \left(F(\gamma^{(i+1)}) - F(\gamma^{(i)})\right) \frac{\gamma}{\gamma^{(i)}} p^{(0)} = \\
&\min_{\gamma^{(i)}, i=1\ldots 2^B-2} \gamma p^{(0)} \left(\sum_{i=0}^{2^B-2} \frac{F(\gamma^{(i+1)}) - F(\gamma^{(i)})}{\gamma^{(i)}} \right)
\end{aligned} \tag{5}$$

To solve the problem in (5), the following theorem is useful:

Theorem 1: If $F(\gamma)$ is a continuous differentiable function, then the problem in (5) has a unique global minimum solution.

Proof: This will be proved using partial derivatives and setting them equal to 0. Note that an underlying condition to this optimization problem is $\gamma^{(1)} < \gamma^{(2)} < \ldots < \gamma^{(2^B-2)} < \gamma^{(2^B-1)} = \infty$. Therefore, from the assumptions on $F(\gamma)$, $F(\gamma^{(1)}) < F(\gamma^{(2)}) < \ldots < F(\gamma^{(2^B-2)}) < F(\gamma^{(2^B-1)}) = 1$. The goal is to check if these conditions are inherently satisfied when the partial derivatives are taken. If they are, then the set of $2^B - 2$ variables and $2^B - 2$ equations can be solved to yield the optimal solution.

Since $\gamma p^{(0)}$ is a constant with respect to the objective function, it will be ignored as it has no impact on the solution. Let $F'(\gamma)$ be the derivative of $F(\gamma)$. The partial derivative with respect to $\gamma^{(i)}$ is given by

$$\begin{aligned}
\frac{\partial}{\partial \gamma^{(i)}} E[p(t)] &= \frac{\partial}{\partial \gamma^{(i)}} \left(\sum_{i=0}^{2^B-2} \frac{F(\gamma^{(i+1)}) - F(\gamma^{(i)})}{\gamma^{(i)}} \right) \\
&= \frac{\partial}{\partial \gamma^{(i)}} \left(\frac{F(\gamma^{(i+1)})}{\gamma^{(i)}} - \frac{F(\gamma^{(i)})}{\gamma^{(i)}} + \frac{F(\gamma^{(i)})}{\gamma^{(i-1)}} \right) \\
&= -\frac{F(\gamma^{(i+1)})}{(\gamma^{(i)})^2} - \frac{F'(\gamma^{(i)})}{\gamma^{(i)}} + \frac{F(\gamma^{(i)})}{(\gamma^{(i)})^2} + \frac{F'(\gamma^{(i)})}{\gamma^{(i-1)}}
\end{aligned}$$

Consider $i = 2^B - 2$. Taking the partial and setting it equal to 0 gives

$$\begin{aligned}
&-\frac{F(\gamma^{(2^B-1)})}{(\gamma^{(2^B-2)})^2} - \frac{F'(\gamma^{(2^B-2)})}{\gamma^{(2^B-2)}} + \frac{F(\gamma^{(2^B-2)})}{(\gamma^{(2^B-2)})^2} + \frac{F'(\gamma^{(2^B-2)})}{\gamma^{(2^B-3)}} = \\
&-\frac{1}{(\gamma^{(2^B-2)})^2} - \frac{F'(\gamma^{(2^B-2)})}{\gamma^{(2^B-2)}} + \frac{F(\gamma^{(2^B-2)})}{(\gamma^{(2^B-2)})^2} + \frac{F'(\gamma^{(2^B-2)})}{\gamma^{(2^B-3)}} = 0
\end{aligned}$$

Solving for $\gamma^{(2^B-3)}$ gives

$$\gamma^{(2^B-3)} = \frac{F'(\gamma^{(2^B-2)})\gamma^{(2^B-2)}}{F'(\gamma^{(2^B-2)})\gamma^{(2^B-2)} + 1 - F(\gamma^{(2^B-2)})} \gamma^{(2^B-2)}$$

Now show that $\gamma^{(2^B-3)} < \gamma^{(2^B-2)}$, which is equivalent to showing

$$\frac{F'(\gamma^{(2^B-2)})\gamma^{(2^B-2)}}{F'(\gamma^{(2^B-2)})\gamma^{(2^B-2)} + 1 - F(\gamma^{(2^B-2)})} < 1 \tag{6}$$

Since the term $F'(\gamma^{(2^B-2)})\gamma^{(2^B-2)}$ appears in the numerator and denominator of the fraction in (6), it will suffice to show $1 - F(\gamma^{(2^B-2)}) > 0$. Since $F(\gamma^{(2^B-2)}) < 1$ (otherwise $\gamma^{(2^B-2)} = \infty$), this statement is true. Thus, the solution to the equation for the partial derivative when $i = 2^B - 2$ is satisfied only when $\gamma^{(2^B-3)} < \gamma^{(2^B-2)}$.

Now, consider again the partial with respect to $\gamma^{(i)}$ and set it equal to 0. Solving for $\gamma^{(i-1)}$ gives

$$\gamma^{(i-1)} = \frac{F'(\gamma^{(i)})\gamma^{(i)}}{F'(\gamma^{(i)})\gamma^{(i)} + F(\gamma^{(i+1)}) - F(\gamma^{(i)})} \gamma^{(i)} \tag{7}$$

Assume $\gamma^{(i+1)} > \gamma^{(i)}$. Then, $F(\gamma^{(i+1)}) - F(\gamma^{(i)})$ is positive since F is a monotonically increasing function. Then, by the same arguments given for (6), $\gamma^{(i-1)} < \gamma^{(i)}$. The assumption that $\gamma^{(i+1)} > \gamma^{(i)}$ is satisfied since $\gamma^{(2^B-2)} > \gamma^{(2^B-3)}$, and therefore by the previous argument this assumption will always hold true. Therefore, $\gamma^{(i+1)} > \gamma^{(i)} \, \forall i$. Thus, by solving the partial derivative equations, the underlying condition for the problem is satisfied. These equations are linearly independent since the terms $(\gamma^{(i)})^2$ and $F'(\gamma^{(i)})$ only appear in the i^{th} equation. Since there are $2^B - 2$ equations and $2^B - 2$ variables, the globally optimal solution to (5) will be given by solving these equations. Furthermore, this solution is a minimum since the boundary cases of all $\gamma^{(i)} = \gamma^{(0)}$ or all $\gamma^{(i)} = \infty$ would lead to no power reduction since none of the thresholds would be used to scale the original power. Any solution not at these boundaries would include some power reduction, and so solving the set of partial derivative equations will yield a global minimum.

□

Using Theorem 1, a set of equations is given to satisfy the optimal thresholds $\gamma^{(i)}$ to minimize the average power usage of the system. Currently, these equations are being solved using numerical optimization since the functions are complex. Efficient solving of these equations is still an open problem.

IV. RESULTS

To test this feedback scheme, we study the performance of a single-user MIMO system as well as a 3-user MU-MIMO system. The covariance matrices are generated from the angular spread model [9] and the system is run over 10000 channel matrices. First, the CDI algorithm from [8] is run on the covariance matrices, and

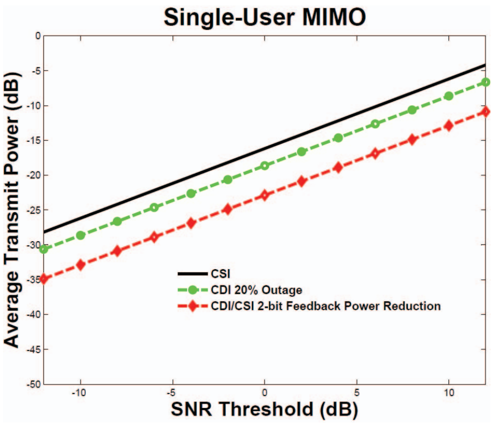

Fig. 1. Single-User Feedback at 20% Outage

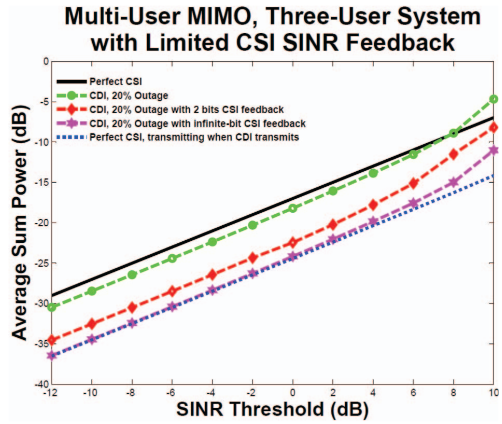

Fig. 2. Multiuser Feedback for 3-User System at 20% Outage

then the CSI feedback scheme discussed in Section III is run when the channel changes.

The results for single-user MIMO are shown in Fig. 1. The CDI algorithm here gives lower power consumption than the CSI case, but is in outage 20% of the time. Using 2 bits of CSI feedback saves significant power as compared to the other two schemes. There is about a 7dB savings over the CSI case and 5dB savings over the CDI case with no CSI feedback. The reason for these savings is that the transmitter does not transmit when channel conditions are bad (which raises the CSI curve), but reduces power when the channel conditions are good. While the power consumption is lower than the CSI case on average, the throughput is also 20% lower for the CDI schemes since the transmitter is not sending data 20% of the time.

We also test the multiuser case with 3 transmit-receive pairs in Fig. 2. For the multiuser case, we see similar trends, but the savings are even more significant for 2 bits of feedback–about 6dB lower than the CSI curve and about 5dB lower compared to the CDI algorithm with no feedback. If we have infinite bits of feedback for the SINR so the SINR is perfectly known at the transmitter, the power savings are about 7.5dB as compared to the CSI curve. Once again, there is about a 20% throughput loss as compared to the CSI case. To make the comparison more fair, the dotted curve represents the CSI-based scheme only when the transmitter transmits using the CDI scheme with limited CSI feedback. The throughput for this scheme is the same as the CDI cases. The CSI curve is lower, as expected, but at low SINR thresholds the difference is small. At higher SINR thresholds, the difference becomes more apparent. These plots show that this type CSI of power reduction can lead to significant savings using very limited feedback in the system.

V. CONCLUSION

In this work, we utilize limited CSI feedback to improve on a CDI-based algorithm for MU-MIMO beamforming networks with SINR requirements. By using low-rate SINR feedback from the receiver to its corresponding transmitter, we show that lower power consumption can be achieved in the network while not wasting power when the SINR requirement is not met. The scheme presented here is also decentralized. While this CSI-based scheme has many advantages to improve on the CDI solution, further improvements can be made with multiple-stage feedback and more aggressive power control. The power reduction invoked by this scheme lowers the interference profile to other users. This means that other users can potentially raise their power slightly while still meeting the SINR requirements for the other users. This is the subject of future research.

VI. REFERENCES

[1] B. Song, R.L. Cruz, and B.D. Rao, "Network Duality for Multiuser MIMO Beamforming Networks and Applications," *IEEE Trans. Commun.*, vol. 55, no. 3, pp. 618–630, Mar. 2007.

[2] M. Sharif and B. Hassibi, "On the Capacity of MIMO Broadcast Channels with Partial Side Information," *IEEE Trans. Inf. Theory*, vo. 41, no. 2, pp. 506–522, Feb. 2005.

[3] B. Mondal and R.W. Heath, "Performance Analysis of Quantized Beamforming MIMO Systems," *IEEE Trans. Signal Process.*, vol. 54, no. 12, pp. 4753–4766, Dec. 2006.

[4] E. Visotski and U. Madhow, "Space-Time Transmit Precoding with Imperfect Feedback," *IEEE Trans. Inf. Theory*, vol. 47, no. 6, pp. 2632–2639, Sept. 2001.

[5] A.M. Tulino, A. Lozano, and S. Verdu, "Capacity-Achieving Input Covariance for Single-User Multi-Antenna Channels," *IEEE Trans. on Wireless Commun.*, vol. 5, no. 3, pp. 662–671, Mar. 2006.

[6] A. Soysal and S. Ulukus, "Optimum Power Allocation for Single-User MIMO and Multi-User MIMO-MAC with Partial CSI," *IEEE J. on Sel. Areas Commun.*, vol. 25, no. 7, pp.1402–1412, Sept. 2007.

[7] A.L. Anderson, J.R. Zeidler, and M.A. Jensen, "Reduced-Feedback Linear Precoding with Stable Performance for the Time-Varying MIMO Broadcast Channel," *IEEE J. on Sel. Areas Commun.*, vol. 26, no. 8, pp. 1483–1493, Oct. 2008.

[8] S. Ghosh, B.D. Rao, and J.R. Zeidler, "Outage-Efficient Strategies for Multiuser MIMO Networks with Channel Distribution Information," *IEEE Trans. Signal Process.*, vol. 58, no. 12, pp. 6312–6324, Dec. 2010.

[9] A.F. Molisch, "A Generic Model for MIMO Wireless Propagation Channels in Macro- and Microcells," *IEEE Trans. Signal Process.*, vol. 52, no. 1, pp. 61-71, Jan. 2004.