

ROBUST SPARSE SPECTRAL FITTING IN ELEMENT AND BEAM SPACES FOR DIRECTIONS-OF-ARRIVAL AND POWER ESTIMATION

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ABSTRACT

In this paper, we propose a robust sparse spectrum fitting method (RSpSF) for Directions-Of-Arrival (DOA) and power estimation in the presence of general form of modeling errors in the array manifold matrix. By exploiting the group sparsity between the power spectrum and the modeling errors, RSpSF formulates the estimator as a convex optimization program. Then, in order to reduce its computational complexity, we apply a beam-space technique to RSpSF and obtain another convex estimator, the beam-space RSpSF (BMRSpSF). Simulation examples are presented to demonstrate the robustness of the proposed methods to off-grid DOAs and to random array calibration errors.

Index Terms— Direction Of Arrival (DOA), sparsity, robust, beam-space

1. INTRODUCTION

By exploiting sparsity in array data or covariance domains, a number of DOA estimation methods have emerged with promising improvements in resolution and accuracy over many well-established methods. The l_1 -SVD algorithm [1] uses the Group-Lasso [2] technique in the data-domain. In [3], a sparse spatial spectrum estimation method, designated here as SpSF, is built on a sparse representation of the covariance matrix of uncorrelated sources, to provide both direction and source power estimates. In the presence of modeling errors in the array manifold matrix, however, the performances of these methods degrade. This issue was discussed in [4], in which a method (designated SpSFMU here) using convex optimization was developed for the special case of off-grid DOAs.

In this paper, we present a new convex DOA estimator, which we call the Robust Sparse Spectral Fitting (RSpSF), which takes the general form of the modeling errors into account. Examples are presented to demonstrate robustness of

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the method, not only to off-grid DOAs, but also to random array calibration errors. The formulation is carried out in both element and beam-space domains, with the latter providing advantages of operating in such a domain, including computational savings.

Consider an antenna array of M elements and assume L narrow-band far-field signals impinge on the array from directions $\theta_1, \theta_2, \dots, \theta_L$ in the presence of additive white Gaussian noise of variance σ^2 .

We define an overcomplete dictionary $\Phi = [a(\phi_1), \dots, a(\phi_K)] \in \mathbb{C}^{M \times K}$, where (ϕ_1, \dots, ϕ_K) are the K potential source directions at the resolution of interest and $K \gg \max(M, L)$, and $a(\phi_k)$ represents the array response at direction ϕ_k . In this framework, Φ is pre-determined and does not depend on the actual source locations. Assume that each of $(\theta_1, \dots, \theta_L)$ belongs to (ϕ_1, \dots, ϕ_K) . The t th snapshot of the array output can be rewritten as:

$$y(t) = \Phi s(t) + n(t), \quad (1)$$

where $s(t) = [s_1(t), \dots, s_K(t)]^T$ is the expanded snapshot of the arriving signals, whose k th entry is t th sample of the l th signal if $\phi_k = \theta_l$, otherwise is 0. Hence $s(t)$ in (1) is sparse. In effect, the problem of source DOA estimation is replaced by the estimation problem of the positions of the non-zero entries in $s(t)$.

In Section 2, the proposed direction finding algorithm is described. In order to reduce the computational complexity of RSpSF, a beam-space RSpSF named BMRSpSF, is proposed in Section 3. In Section 4, the estimation performances of the proposed methods compared to l_1 -SVD, SpSF and SpSFMU by statistical simulations. Finally, Section 5 concludes the paper.

2. ROBUST SPARSE SPECTRAL FITTING

Assuming uncorrelated and zero-mean sources and using the model (1), the array covariance matrix, R , is represented by:

$$R = E(yy^H) = \sum_{k=1}^K p_k a(\phi_k) a^H(\phi_k) + \sigma^2 I, \quad (2)$$

where $p_k = E(|s_k|^2)$, $E(\cdot)$ denotes the expectation operation and $[\cdot]^H$ is Hermitian Transpose. From the definition of $p_k = \varepsilon_l = E(|x_l|^2)$ is the power of the l th source if $\phi_k = \theta_l$, otherwise $p_k = 0$. This means that $\vec{p} = [p_1, p_2, \dots, p_K]^T$ is sparse (since $K \gg L$) and it is just the desired spatial spectrum. Thus, if we can estimate p , we can simultaneously estimate the DOAs and the signal strengths. Vectorization of (2) gives

$$\vec{R} = \text{vec}(R) = \sum_{k=1}^K p_k \vec{a}(\phi_k) + \sigma^2 \vec{I}, \quad (3)$$

where $\vec{a}(\phi) = \text{vec}(a(\phi)a^H(\phi))$.

In the presence of errors in the steering vectors, we denote the true steering vector of the signal coming from the resolution cell centered at ϕ_k by $a_t(\phi_k)$ and define $a_T(\phi_k) = a_t(\phi_k)\sqrt{p_k}$ and $\vec{a}_T(\phi_k) = \text{vec}(a_T(\phi_k)a_T^H(\phi_k))$. Then, we can write:

$$\vec{a}_T(\phi_k) = \vec{a}(\phi_k)p_k + \vec{e}_k, \quad (4)$$

where \vec{e}_k is the "error term" between the true model $\vec{a}_T(\phi_k)$ and the presumed model $\vec{a}(\phi_k)p_k$.

Under the assumption of the sparse spatial spectrum [5], we can formulate an estimator

$$\min_p \|\vec{R} - \sum_{k=1}^K \vec{a}_T(\phi_k)\|_2 + \gamma \|p\|_2 \quad (5)$$

$$s.t. \|\vec{a}_T(\phi_k) - \vec{a}(\phi_k)p_k\|_r \leq \beta_k,$$

where γ and β_k are the regularization parameters used to penalize or account for the noise variance and \vec{e}_k respectively. However, this formulation introduces too many parameters to be tuned, which makes it completely impractical. Thus, by observing that if $p_k = 0$ then $\beta_k = 0$, we can use the following function to promote group sparsity between p and $\beta = [\beta_1, \beta_2, \dots, \beta_K]^T$:

$$\sum_{k=1}^K \delta_k, \text{ with } \delta_k = \|p_k, \beta_k\|_2. \quad (6)$$

Then we formulate RSpSF as:

$$\min_{\vec{a}_T, p, \beta} \|\vec{R} - \sum_{k=1}^K \vec{a}_T(\theta_k)\|_2 + \gamma \sum_{k=1}^K \delta_k \quad (7)$$

$$s.t. \|\vec{a}_T(\phi_k) - \vec{a}(\phi_k)p_k\|_r \leq \beta_k.$$

It is worth emphasizing that (7) is a convex optimization problem, and thus can be efficiently solved. Note that the selection of r depends on the distribution of \vec{e}_k . For example, if e_k follows a Gaussian distribution, r can be 2, but if e_k itself is sparse (e.g. some sensors fail) $r = 1$ would be a better choice.

To minimize the objective function of (7), RSpSF tends to smooth between p_k and β_k if $\delta_k > 0$. However, since $\beta_k = p_k \|\vec{e}_k\|_r$, RSpSF leads to a biased estimate of β_k or p_k . Therefore, in order to get more accurate estimates of the

signal power, we utilize the idea of Adaptive Lasso [6] [7] to improve the estimator (7). Assume that $p_k^{(1)}$ and $\beta_k^{(1)}$ are the initial estimates obtained from (7), respectively. Then we propose the following improved formulation:

$$\min_{\vec{a}_T, p, \beta} \|\vec{R} - \sum_{k=1}^K \vec{a}_T(\theta_k)\|_2 + \gamma \sum_{k=1}^K \left\| \frac{p_k}{p_k^{(1)}}, \frac{\beta_k}{\beta_k^{(1)}} \right\|_2 \quad (8)$$

$$s.t. \|\vec{a}_T(\phi_k) - \vec{a}(\phi_k)p_k\|_2 \leq \beta_k.$$

Formulation (8) is a special case of the adaptive lasso (also see Equation (8) in [6]). As shown in [7], adaptive Lasso can reduce the estimation bias, which means that (8) can achieve more accurate estimates of signal power and DOA than (7).

3. BEAM-SPACE ROBUST SPARSE SPECTRAL FITTING

Since the Number of Optimization Variable (NOV) of RSpSF is $M \times K$, its computational complexity may be prohibitive for real time applications. Thus, we consider a transformation from element-space to beam-space,

$$y_B(t) = W^H \Phi s(t) + W^H n(t) = \Phi_B s(t) + W^H n(t), \quad (9)$$

where W is the $M \times B$ beamforming matrix. B is the dimension of beamspace and $\Phi_B = W^H [a(\phi_1), \dots, a(\phi_K)]$. Define $a_B(\phi_k) = W^H a(\phi_k)$ and $\vec{a}_B(\phi_k) = \text{vec}(a_B(\phi_k)a_B^H(\phi_k))$. In order to keep white noise in the beam-space output, we select W such that $W^H W = I_B$. Here, we use the beamforming matrix [8]

$$W = \frac{1}{\sqrt{K}} \left[a \left(m \frac{2}{K} \right), \dots, a \left((m+B-1) \frac{2}{K} \right) \right], \quad (10)$$

where m denotes the index of the subband under examination (more details can be found in [8]). Therefore, the covariance matrix in beam-space is

$$R_B = W^H R W + \sigma^2 I_B$$

$$= \sum_{k=1}^K p_k (W^H a(\phi_k)) (a^H(\phi_k) W) + \sigma^2 I_B, \quad (11)$$

and $\vec{R}_B = \text{vec}(R_B)$.

Define $a_{BT}(\phi_k) = W^H a_t(\phi_k)\sqrt{p_k}$ and $\vec{a}_{BT}(\phi_k) = \text{vec}(a_{BT}(\phi_k)a_{BT}^H(\phi_k))$. Then we can get the beam-space version of RSpSF (BMRSpSF) as

$$\min_{\vec{a}_{BT}, p, \beta} \|\vec{R}_B - \sum_{k=1}^K \vec{a}_{BT}(\theta_k)\|_2 + \gamma \sum_{k=1}^K \delta_k \quad (12)$$

$$s.t. \|\vec{a}_{BT}(\phi_k) - \vec{a}_B(\phi_k)p_k\|_r \leq \beta_k.$$

It is apparent that the computational complexity of RSpSF is reduced due to that the NOV of RSpSF decreases from

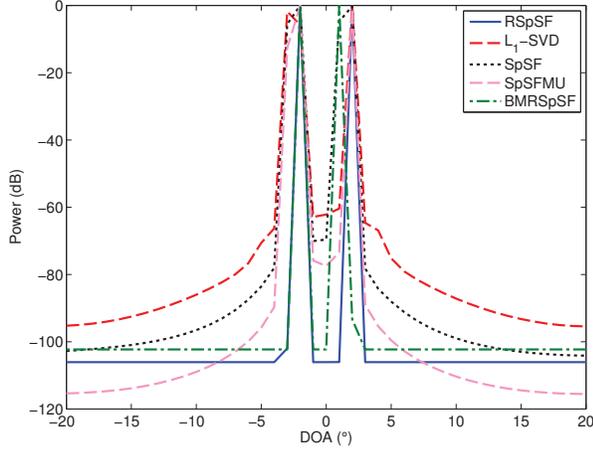


Fig. 1. Normalized spectrum of the proposed methods, l_1 -SVD, SpSF and SpSFMU for two independent sources. SNR=10dB. DOAs: -2.4° and 1.6°

$M^2 \times K$ to $B^2 \times K$. We have also observed in our simulations that BMRSpSF is much less sensitive to the selection of γ , another potential advantage of beam-space. Furthermore, this beam-space technique can be applied in many other sparsity-exploiting DOA estimation methods, e.g. SpSF and l_1 -SVD, to achieve lower computational complexity.

4. SIMULATION RESULTS

In this section, we compare the estimation performance of the two new methods to those of l_1 -SVD [1], SpSF [3] and SpSFMU [4]. Two types of the modeling error in array manifold matrix are considered. The first one is the off-grid DOAs introduced in [4], and the second one is random calibration errors.

We consider a uniform linear array (ULA) of $M = 8$ sensors separated by half a wavelength of the narrowband sources. Two far-field zero-mean sources ($L = 2$) impinge on this array and the noise is AWGN with unit variance. $T = 200$ snapshots are assumed to be available and the candidate directions are set to be uniformly distributed from -90° to 90° with 1° separation. Assume that e_k follows a Gaussian distribution, then we choose $r = 2$. $B = 3$ and $m = -1$ are used in BMRSpSF. The DOA estimation error is defined as $RMSE = \frac{1}{L} \sum_{l=1}^L \sqrt{E(\hat{\theta}_l - \theta_l)^2}$, where $\hat{\theta}_l$ is the estimate of θ_l . The normalized spectrum is defined as $P = \lg(P/P_{max})$, where P is the spatial spectrum and P_{max} is the maximum of the P . For l_1 -SVD, the power estimation is achieved by least squares using the estimated directions. All regularization parameters are empirically and independently chosen for best performance.

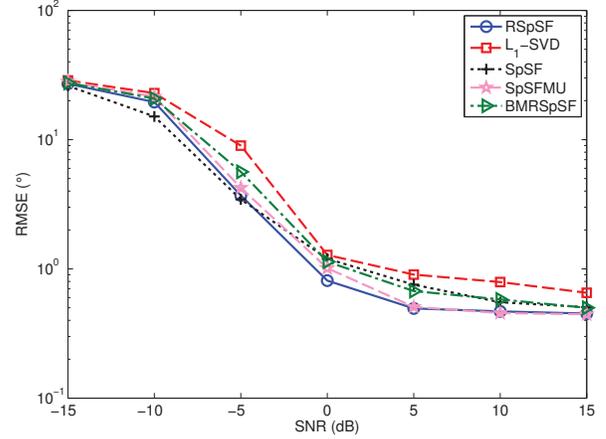


Fig. 2. RMSE of DOA estimation of the proposed methods, l_1 -SVD, SpSF and SpSFMU for two independent sources. DOAs: -2.4° and 1.6°

4.1. Off-Grid DOAs

In this case, two uncorrelated sources come from -2.4° and 1.6° with equal SNR of $10dB$.

Figure 1 gives an example of the normalized spatial spectrum of RSpSF, BMRSpSF, l_1 -SVD, SpSF and SpSFMU. In this scenario, all methods are able to resolve the sources. The estimates of RSpSF and BMRSpSF are $[-2^\circ, 2^\circ]$ and $[-2^\circ, 1^\circ]$, and the estimates of l_1 -SVD, SpSF and SpSFMU are $[-3^\circ, 2^\circ]$, $[-2^\circ, 2^\circ]$ and $[-2^\circ, 2^\circ]$ respectively. The power estimates obtained by RSpSF and BMRSpSF are $[9.74, 9.53]$ and $[9.98, 9.74]$, which are very close to the true value and better than those by l_1 -SVD, SpSF and SpSFMU, which are $[9.03, 11.8]$, $[8.91, 8.47]$ and $[9.01, 9.31]$, respectively.

Figure 2 compares the RMSEs of DOA estimation of each algorithm for varying SNR. We use 200 Monte Carlo tests for each SNR with 200 snapshots for each trial. Through this simulation, it can be seen that RSpSF, BMRSpSF, l_1 -SVD, SpSF and SpSFMU have similar resolution thresholds, and RSpSF can achieve lower RMSE than the other methods. It is worth mentioning that the time consumption of BMRSpSF is only half of RSpSF. With larger B , we can get better estimation performance from BMRSpSF, with the price of larger computational complexity.

4.2. Array Calibration error

We consider that there is random array calibration error which means that the "error term" in equation (4) is

$$\vec{e}_k(\phi) = p_{power} * (Re_p + j * Im_p), \quad (13)$$

where Re_p and Im_p are generated by standard normal distribution, $j = \sqrt{-1}$ and $p_{power} = 0.1$ denotes the strength

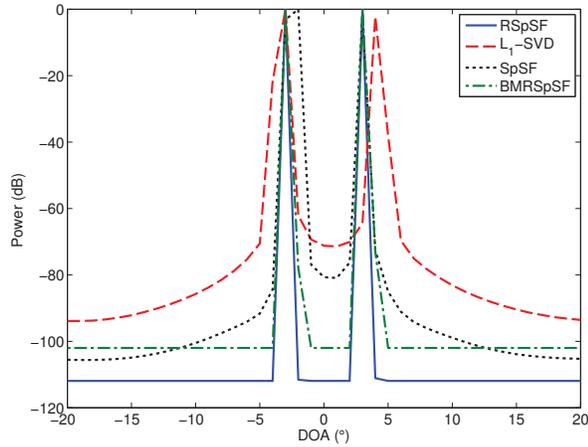


Fig. 3. Normalized spectrum of the proposed methods, SpSF and l_1 -SVD for two independent sources. SNR=10dB. DOAs: -3° and 3°

of the calibration error. In this scenario, two uncorrelated sources are placed at -3° and 3° with $SNR = 10dB$.

An example of the normalized spatial spectrums of all the five methods are presented in figure 3 except that SpSF fails in this case. The DOA estimates of RSpSF, BMRSpSF, l_1 -SVD and SpSF are $[-3^\circ, 3^\circ]$, $[-3^\circ, 3^\circ]$, $[-3^\circ, 4^\circ]$ and $[-2^\circ, 3^\circ]$, respectively. The power estimates obtained by RSpSF, BMRSpSF, l_1 -SVD and SpSF are $[10.69, 10.43]$, $[9.55, 10.55]$, $[12.09, 9.72]$ and $[11.69, 8.64]$, respectively.

The RMSEs of DOA estimation shown in figure 4 are obtained by over 200 independent trails. As illustrated, RSpSF, BMRSpSF and l_1 -SVD have similar large error thresholds which are 5dB above SpSF. Furthermore, RSpSF has better RMSE than l_1 -SVD. As in the Off-Grid DOAs setting, the computational complexity of BMRSpSF is lower than that of RSpSF.

5. CONCLUSION

In this paper, we proposed a new spatial spectrum estimator, the robust sparse spectrum fitting (RSpSF) method, which operates with general forms of the modeling errors in the array manifold matrix. Then, we used the beam-space technique to reduce the computational complexity of RSpSF. This approach can also be applied to many other sparsity-exploiting DOA estimation methods. Through simulations, we demonstrated the effectiveness of the beam-space technique and compared the DOA estimation performance of the proposed methods with l_1 -SVD, SpSF and SpSFMU. The comparison showed that the proposed methods can achieve better DOA estimation performance under the presence of, not only the off-grid DOAs, but also random array calibration errors.

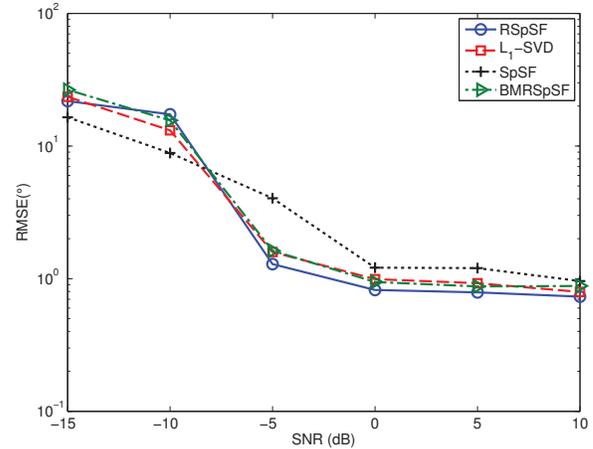


Fig. 4. RMSE of DOA estimation of the proposed methods, SpSF and l_1 -SVD for two independent sources. DOAs: -3° and 3°

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