ASYNCHRONOUS BIDIRECTIONAL RELAY-ASSISTED COMMUNICATIONS

Reza Vahidnia and Shahram ShahbazPanahi

Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, Oshawa, Ontario, Canada

ABSTRACT

In this paper, an asynchronous bidirectional relay network is considered. We assume that the signal path going through each relay has a propagation delay which is different from those of the other relaying paths. Such an assumption leads to inter-symbol-interference (ISI) at the two transceivers. As such, orthogonal frequency division multiplexing (OFDM) is deployed at the two transceivers to tackle ISI, while the relays, for the sake of simplicity, use amplify-and-forward relaying technique. Using a max-min fair design approach, an SNR balancing technique is presented to jointly obtain the beamformer weights and transceivers' subcarrier power loading under a total transmit power budget. Simulation results are presented to show the performance of this approach.

Index Terms— Relay networks, cooperative communication, SNR balancing, distributed beamforming, power allocation, Asynchronous relaying.

1. INTRODUCTION

Different aspects of collaborative communication schemes have been significantly investigated in the literature. Distributed beamforming belongs to a subclass of collaborative communication schemes where linear processing (i.e., amplify-and-forward protocol) is deployed at the relays [1, 2, 3]. Recently, distributed beamforming has been the center focus of the design and analysis with the aim to develop bandwidth-efficient two-way relaying schemes [4, 5, 6, 7, 8]. The majority of the reported results are based on the assumption that the relay nodes are synchronized at the symbol level and/or ignore the fact the rely paths impose different processing/propagation delays on the signal. In fact, lack of time synchronization and/or different relay path delays result in frequency selectivity of the effective channel between the two transceivers and causes inter-symbol-interference (ISI).

In [9], the filter-and-forward (FF) relaying technique was introduced as a means to combat the ISI caused by the frequency selectivity of the channel links in one-way relay networks. In the FF approach, the channel equalization is implemented in a distributed manner, by deploying finite impulse response (FIR) filters at the relays. Thus, in FF relaying, the burden of compensating the ISI is on shoulders of the relays.

In this paper, we consider simple amplify-and-forward relaying techniques for an asynchronous *two-way* relay network which aims to establish a reliable link between two transceivers. To combat ISI caused by unknown relay path delays in the network, the OFDM technology is used at the transceivers. We then aim to optimally determine the subcarrier power loading at the transceivers and relay beamforming weights. To do so, we present a max-min fair design approach under a total power constraint. We formulate the corresponding optimization problem and present that in a compact form which can be solved using sequential quadrature programming.

Notation: Continuous-time and discrete-time convolutions are represented by $\star_{\rm c}$ and $\star_{\rm d}$, respectively. We use $E\{.\}$ to denote statistical expectation. Complex conjugate, transpose and Hermitian transpose are denoted by $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$, respectively. Also $\mathrm{diag}\{\cdot\}$ stands for a diagonal matrix.

2. SYSTEM MODEL

We consider a two-way relay network which consists of L single-antenna relay nodes establishing a bidirectional connection between two transceivers. Relays are assumed to adopt amplify-and-forward relaying protocol. Signals transmitted by Transceiver p, going through the lth relay and received at Transceiver q, for $p, q \in \{1, 2\}$ are assumed to be subject to different propagation delays denoted by $\tau_{l_{pq}}$. Assuming flat fading and reciprocal channels between relays and transceivers, we can represent the effective linear time-invariant (LTI) channel between Transceivers p and q by a 2×2 *channel impulse response matrix* as $\mathbf{H}(t) = \begin{pmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{pmatrix}$. Considering w_l as the complex weight of the lth relay and representing the amplitude of the flat fading channel coefficient between Transceiver p and the lth relay as g_{lp}, the total gain of the signal path originating from Transceiver p, going through the lth relay, and ending at Transceiver q, can be written as $\alpha_{l_{pq}} \triangleq w_l g_{lp} g_{lq}$. The finite impulse response (FIR) representation of $h_{pq}(t)$ can then be written as

$$h_{pq}(t) = \sum_{l=1}^{L} \alpha_{l_{pq}} \delta(t - \tau_{l_{pq}}), \quad \text{for} \quad p, q \in \{1, 2\}$$

Assuming that the pulse shaping filter has a representation as $\varphi(t)$, we can write the transmitted signal from Transceiver p as

$$s_p(t) = \sum_{k=-\infty}^{\infty} s_p[k] \varphi(t - kT_{\rm s}), \quad p \in \{1, 2\} \tag{1}$$

where $s_p[k]$ is the kth symbol transmitted by Transmitter p, and $T_{\rm s}$ is the symbol period. Hence, the received signal at Transceiver q, sampled a rate of $1/T_{\rm s}$, can be written as

$$r_q[nT_{\rm s}] = r_q(t)\Big|_{t=nT_{\rm s}} = \sum_{p=1}^{2} s_p[n] \star_{\rm d} h_{pq}[n] \tag{2}$$

where

$$h_{pq}[n] \triangleq \sum_{l=1}^{L} \alpha_{l_{pq}} \varphi(nT_{\rm s} - \tau_{l_{pq}}). \tag{3}$$

It follows from (3) that the effective channels are frequency selective and inter-symbol interference (ISI) is inevitable at high data rates. Hence, we deploy orthogonal frequency division multiplexing (OFDM) scheme at the two transceivers to combat such an ISI.

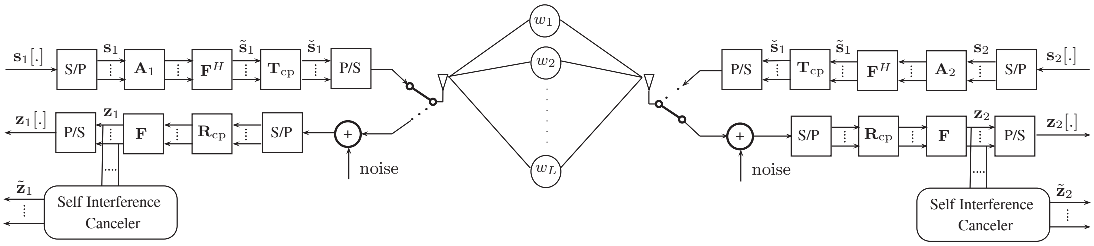

Fig. 1. System model

As depicted in Fig. 1, for the sake of simplicity at the relays, the amplify-and-forward protocol is deployed at the relays. In this figure, the cyclic prefix insertion and deletion operations are represented by matrices $\mathbf{T}_{\rm cp}$ and $\mathbf{R}_{\rm cp}$, respectively, $\mathbf{F}$ is the $N \times N$ DFT matrix and serial-to-parallel and parallel-to-serial operations are presented, respectively, by S/P and P/S. Let N denote the maximum of the lengths of all discrete-time channel impulse responses $\{h_{pq}[\cdot]\}_{p,q=1}^{2}$, for $p, q \in \{1, 2\}$. We also choose the number of subcarriers equal to N. Considering $\varphi(t)$ as a rectangular pulse with a length of $T_{\rm s}$ and introducing an $N \times L$ matrix $\mathbf{B}_{pq}$ whose (n, l)th is given by

$$B_{pq}(n,l) = \begin{cases} g_{lp}g_{lq}, & (n-1)T_{\rm s} \le \tau_{l_{pq}} \le nT_{\rm s} \\ 0, & \text{otherwise}, \end{cases} \tag{4}$$

the contribution of the lth relay to the nth tap of $h_{pq}[\cdot]$ can be described by $B_{pq}(n, l)w_l$. Using (4), the vector of discrete-time channel coefficients, defined as $\mathbf{h}_{pq} \triangleq \Big[h_{pq}[0], \ldots, h_{pq}[N-1]\Big]^T$, can be written as

$$\mathbf{h}_{pq} = \mathbf{B}_{pq}\mathbf{w} \tag{5}$$

where $\mathbf{w} \triangleq [w_1, \ldots, w_L]^T$ is an $L \times 1$ vector representing the complex relay weights. The noise process is assumed to be spatially and temporally white with a variance of σ^2 and it is denoted, at the lth relay, as $\gamma_l(t)$. This noise is amplified by w_l, goes through the channel g_{lq}, and is delayed by τ'_{lq}, and then, it is received at Transceiver q. Note that $\tau'_{lq} < \tau_{l_{pq}}$. Let us define

$$\mathbf{G}_q \triangleq \mathrm{diag}\{g_{1q}, \ldots, g_{Lq}\}, \quad \text{for } \ q \in \{1, 2\} \tag{6}$$

$$\Gamma_q(m,l) \triangleq \gamma_l(mT_{\rm s} - \tau'_{lq}),\ m = 1, \ldots, M,\ l = 1, \ldots, L \tag{7}$$

where $N_{\rm cp}$ represents the length of cyclic prefix, and $M = N + N_{\rm cp}$ is the total length of an OFDM symbol. Here, $\Gamma_q(m, l)$ is the (m, l)th element of the $M \times L$ matrix $\mathbf{\Gamma}_q$. Representing the receiver noise at Transceiver q by an $M \times 1$ vector $\mathbf{n}'_q$, we can formulate the noise vector at Transceiver q as

$$\mathbf{n}_q \triangleq \mathbf{\Gamma}_q\mathbf{G}_q\mathbf{w} + \mathbf{n}'_q \ \ q \in \{1, 2\} \tag{8}$$

where the first term is the relay received noises, after amplification by the relays and after propagation through the corresponding channels, when they add up at Transceiver q. The vector of information symbols transmitted by Transceiver q can be written as $\mathbf{s}_q \triangleq \Big[s_q[1], \ldots, s_q[N]\Big]^T$ for, $q \in \{1, 2\}$. The received signal after going through cyclic prefix removal and DFT blocks can be represented as

$$\mathbf{z}_1 \triangleq \mathbf{A}_1\mathcal{D}_{11}\mathbf{s}_1 + \mathbf{A}_2\mathcal{D}_{21}\mathbf{s}_2 + \mathbf{F}\mathbf{R}_{\rm cp}\mathbf{n}_1 \tag{9}$$

$$\mathbf{z}_2 \triangleq \mathbf{A}_1\mathcal{D}_{12}\mathbf{s}_1 + \mathbf{A}_2\mathcal{D}_{22}\mathbf{s}_2 + \mathbf{F}\mathbf{R}_{\rm cp}\mathbf{n}_2 \tag{10}$$

where $\mathcal{D}_{pq} \triangleq \mathrm{diag}\{\mathbf{F}\mathbf{h}_{pq}\}$ is a diagonal matrix whose diagonal elements are equal to thr frequency response of $h_{pq}[n]$ at different subcarriers, $\mathbf{A}_q \triangleq \mathrm{diag}\{\sqrt{P_{iq}}\}_{i=1}^{i=N}$ is a diagonal matrix with diagonal elements equal to the amplification factors for each subcarrier, and P_{iq} is the power allocated to the ith subcarrier at Transceiver q (i.e., $\mathrm{E}\left\{|\mathrm{s}_q[\,i\,]|^2\right\} = 1$). Also, the cyclic prefix removal matrix is defined as $\mathbf{R}_{\rm cp} \triangleq [\mathbf{O} \ \ \mathbf{I}_N]$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix and $\mathbf{O}$ is an all-zero $N \times M$ matrix. After canceling the self interference, the received signals are given by

$$\tilde{\mathbf{z}}_1 \triangleq \mathbf{z}_1 - \mathbf{A}_1\mathcal{D}_{11}\mathbf{s}_1 = \mathbf{A}_2\mathcal{D}_{21}\mathbf{s}_2 + \mathbf{F}\mathbf{R}_{\rm cp}\mathbf{n}_1 \tag{11}$$

$$\tilde{\mathbf{z}}_2 \triangleq \mathbf{z}_2 - \mathbf{A}_2\mathcal{D}_{22}\mathbf{s}_2 = \mathbf{A}_1\mathcal{D}_{12}\mathbf{s}_1 + \mathbf{F}\mathbf{R}_{\rm cp}\mathbf{n}_2\,. \tag{12}$$

In the next section, we present our joint power allocation and distributed beamforming algorithm.

3. SNR BALANCING APPROACH

The main goal of this section is to maximize the quality of transceivers' signals (which are expressed in terms of transceivers' received SNRs) for both transceivers across all subcarriers. Our proposed approach is to jointly design the optimal relay beamforming coefficients and subcarrier transmitted powers at the two transceivers such that the smallest received SNR, across all subcarriers of the two transceivers, is maximized subject to a total power constraint. Denoting the received SNR of Transceiver q on the ith subcarrier as SNR_{iq}, this maximization can be formulated as

$$\max_{\mathbf{p}_1,\,\mathbf{p}_2 \ge \mathbf{0}} \ \max_{\mathbf{w}} \ \min_{i \in \{1,\cdots,N\}} \ \min_{q \in \{1,2\}} \mathrm{SNR}_{iq}(\mathbf{w}) \tag{13}$$

$$\text{subject to } \frac{\mathbf{1}^T\mathbf{p}_1}{N} + \frac{\mathbf{1}^T\mathbf{p}_2}{N} + \sum_{l=1}^{L}\tilde{P}_l \le P_{\max}\,.$$

where $P_{\max}$ is the maximum available total power, $\tilde{P}_l$ is the transmited power of the lth relay, and $\mathbf{p}_q \triangleq [P_{1q}, \ldots, P_{Nq}]^T$, for $q \in \{1, 2\}$. In the remainder of the paper, we assume that $p, q \in \{1, 2\}$ and $p \ne q$, i.e, $p = 1$ if $q = 2$ and $p = 2$ if $q = 1$. Let us define the ith Vandermonde column of $\mathbf{F}^H$ as

$$\mathbf{f}_i = \frac{1}{\sqrt{N}}\Big[\ 1 \quad e^{\left(j\frac{2\pi(i-1)}{N}\right)} \quad \ldots \quad e^{\left(j\frac{2(N-1)(i-1)\pi}{N}\right)}\ \Big]^T.$$

Using (11) and (12) along with the fact that $\mathrm{E}\left\{|\mathrm{s}_p[\,i\,]|^2\right\} = 1$ for $p \in \{1, 2\}$ and $i \in \{1, 2, \ldots, N\}$, the power of the ith subcarrier signal, received by Transceiver q is denoted by $P^{\rm s}_{iq}$ and is written as

$$P^{\rm s}_{iq} = P_{ip}\mathrm{E}\left\{\Big|\mathrm{s}_p[\,i\,]\Big|^2\right\}\mathbf{h}_{pq}^H\mathbf{f}_i\mathbf{f}_i^H\mathbf{h}_{pq} = P_{ip}|\mathbf{f}_i^H\mathbf{B}_{pq}\mathbf{w}|^2\,. \tag{14}$$

Also, it follows from (8) that the power of the noise on the ith subcarrier received at Transceiver q is given by

$$P^{\rm n}_{iq} \triangleq {\rm E}\{\mathbf{w}^H\mathbf{G}_q^H\mathbf{\Gamma}_q^H\mathbf{R}_{\rm cp}^T\mathbf{f}_i\mathbf{f}_i^H\mathbf{R}_{\rm cp}\mathbf{\Gamma}_q\mathbf{G}_q\mathbf{w}\} + {\rm E}\{\mathbf{n'}_q^H\mathbf{R}_{\rm cp}^T\mathbf{f}_i\mathbf{f}_i^H\mathbf{R}_{\rm cp}\mathbf{n'}_q\} = \mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2 \quad (15)$$

where $\mathbf{D}_q \triangleq \mathbf{G}_q^H\mathbf{\Gamma}_q^H\mathbf{R}_{\rm cp}^T\mathbf{f}_i\mathbf{f}_i^H\mathbf{R}_{\rm cp}\mathbf{\Gamma}_q\mathbf{G}_q$ can be proven to be an $L \times L$ diagonal matrix whose its lth diagonal element is $D_q(l,l) = \sigma^2 g_{lq}^2$, for $l = 1, \ldots, L$, (see [10] for details). Now using (14) and (15), the received SNR on the ith subcarrier at Transceiver q can be formulated as

$${\rm SNR}_{iq}(\mathbf{w}) \triangleq \frac{P^{\rm s}_{iq}}{P^{\rm n}_{iq}} = \frac{P_{ip}|\mathbf{f}_i^H\mathbf{B}_{pq}\mathbf{w}|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2} \quad , i = 1, \ldots, N\,. \quad (16)$$

Let us define $\boldsymbol{\gamma}_l \triangleq [\gamma_l(T_{\rm s}) \quad \gamma_l(2T_{\rm s}) \quad \cdots \quad \gamma_l(MT_{\rm s})]^T$ as the noise vector at the relays. In order to obtain the transmit power of the lth relay, and according to Fig. 1, the signal relayed by the lth relay can be expressed as

$$\mathbf{x}_l \triangleq w_l\left(g_{l1}\mathbf{T}_{\rm cp}\mathbf{F}^H\mathbf{A}_1\mathbf{s}_1 + g_{l2}\mathbf{T}_{\rm cp}\mathbf{F}^H\mathbf{A}_2\mathbf{s}_2 + \boldsymbol{\gamma}_l\right) \quad (17)$$

where $\mathbf{g}_1 \triangleq [g_{11} \quad g_{21} \quad \cdots \quad g_{L1}]^T$ and $\mathbf{g}_2 \triangleq [g_{12} \quad g_{22} \quad \cdots \quad g_{L2}]^T$ are the channel coefficient vectors. Let $\tilde{s}_{iq}$ be the ith entry of $\tilde{\mathbf{s}}_q \triangleq \mathbf{F}^H\mathbf{A}_q\mathbf{s}_q$, for $q = 1, 2$. Then, we can write the power of this element as

$${\rm E}\{|\tilde{s}_{iq}|^2\} = \mathbf{f}_i^T\mathbf{A}_q E\{\mathbf{s}_q\mathbf{s}_q^H\}\mathbf{A}_q^H\mathbf{f}_i^* = \frac{1}{N}\mathbf{1}^T\mathbf{p}_q\,. \quad (18)$$

As it is shown in (18), the power of different entries of $\tilde{\mathbf{s}}_q$ are all equal to $\frac{1}{N}\mathbf{1}^T\mathbf{p}_q$ for different subcarriers. On the other hand, the cyclic prefix insertion matrix has no effect on the average power of signals going through $\mathbf{T}_{\rm cp}$ block, which are defined by $\check{\mathbf{s}}_q \triangleq \mathbf{T}_{\rm cp}\mathbf{F}^H\mathbf{A}_q\mathbf{s}_q$. Hence,

$${\rm E}\{\check{\mathbf{s}}_q^H\check{\mathbf{s}}_q\} = \frac{M}{N}\mathbf{1}^T\mathbf{p}_q\,. \quad (19)$$

Therefore, it follows from (17) and (19) that the transmit power of the lth relay can be obtained as

$$\tilde{P}_l = \frac{1}{M}{\rm E}\{\mathbf{x}_l^H\mathbf{x}_l\} = \frac{|w_l|^2}{N}\left(g_{l1}^2\mathbf{1}^T\mathbf{p}_1 + g_{l2}^2\mathbf{1}^T\mathbf{p}_2 + N\sigma^2\right) \quad (20)$$

Now using (16) and defining $\mathbf{a}_i \triangleq \mathbf{B}_{pq}^H\mathbf{f}_i$, for $i \in \{1, \cdots, N\}$ we rewrite the optimization problem (13) as

$$\max_{\mathbf{p}_1,\mathbf{p}_2\geq\mathbf{0}} \ \max_{\mathbf{w}} \ \min_{i\in\{1,\cdots,N\}} \ \min_{q\in\{1,2\}} \frac{P_{ip}|\mathbf{a}_i^H\mathbf{w}|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2} \quad (21)$$
$$\text{subject to } \frac{\mathbf{1}^T\mathbf{p}_1}{N} + \frac{\mathbf{1}^T\mathbf{p}_2}{N} + \sum_{l=1}^{L}\tilde{P}_l \leq P_{\max}\,.$$

Without loss of optimality, we can assume that all SNRs in (13) are balanced, i.e., ${\rm SNR}_{ip} = {\rm SNR}_{jq}$ for $p, q \in \{1,2\}$, $i, j \in \{1, 2, \ldots, N\}$ .To show this, assume for any values of i, j, p and q, ${\rm SNR}_{iq}$ is greater than ${\rm SNR}_{jp}$. Then, we can balance the SNRs by decreasing P_{ip} (which does not violate the constraint nor does it change the optimal value of objective function) ensuring that ${\rm SNR}_{iq} = {\rm SNR}_{jp}$. This property leads us to the following relationships between the subcarrier transmit powers:

$$P_{iq} = \frac{P_{1q}|\mathbf{a}_1^H\mathbf{w}|^2}{|\mathbf{a}_i^H\mathbf{w}|^2} \quad (22)$$

$$\frac{P_{11}}{\mathbf{w}^H\mathbf{D}_2\mathbf{w}+\sigma^2} = \frac{P_{12}}{\mathbf{w}^H\mathbf{D}_1\mathbf{w}+\sigma^2}\,. \quad (23)$$

Defining $\mathbf{J}(\mathbf{w}) \triangleq \left[\frac{1}{|\mathbf{a}_1^H\mathbf{w}|^2}, \cdots, \frac{1}{|\mathbf{a}_N^H\mathbf{w}|^2}\right]^T$ and using (22), we can write $\mathbf{p}_q$ as

$$\mathbf{p}_q = P_{1q}|\mathbf{a}_1^H\mathbf{w}|^2\mathbf{J}(\mathbf{w}) \quad (24)$$

Using (20) and (24), we rewrite the optimization problem (21) as

$$\max_{\mathbf{p}_1,\mathbf{p}_2\geq\mathbf{0}} \ \max_{\mathbf{w}} \ \frac{P_{11}|\mathbf{a}_1^H\mathbf{w}|^2}{\mathbf{w}^H\mathbf{D}_2\mathbf{w}+\sigma^2} \quad (25)$$
$$\text{subject to } P_{ip} = \frac{P_{1p}|\mathbf{a}_1^H\mathbf{w}|^2}{|\mathbf{a}_i^H\mathbf{w}|^2}, p \in \{1,2\}, \quad i \in \{1,2,\ldots,N\}$$
$$\frac{P_{11}}{\mathbf{w}^H\mathbf{D}_2\mathbf{w}+\sigma^2} = \frac{P_{12}}{\mathbf{w}^H\mathbf{D}_1\mathbf{w}+\sigma^2}$$
$$\frac{P_{11}|\mathbf{a}_1^H\mathbf{w}|^2\mathbf{1}^T\mathbf{J}(\mathbf{w})}{N} + \frac{P_{12}|\mathbf{a}_1^H\mathbf{w}|^2\mathbf{1}^T\mathbf{J}(\mathbf{w})}{N} + \sum_{l=1}^{L}\frac{|w_l|^2}{N}\left(g_{l1}^2\mathbf{1}^T\mathbf{p}_1 + |g_{l2}|^2\mathbf{1}^T\mathbf{p}_2 + N\sigma^2\right) \leq P_{\max}\,.$$

Using (23), we rewrite (25) as

$$\max_{P_{11}\geq\mathbf{0}} \ \max_{\mathbf{w}} \ \frac{P_{11}|\mathbf{a}_1^H\mathbf{w}|^2}{\mathbf{w}^H\mathbf{D}_2\mathbf{w}+\sigma^2} \quad (26)$$
$$\text{s.t. } \left(\sigma^2 + \mathbf{w}^H\mathbf{D}_1\mathbf{w}\right)\frac{2P_{11}|\mathbf{a}_1^H\mathbf{w}|^2\mathbf{1}^T\mathbf{J}(\mathbf{w})}{\sigma^2 N} + \sigma^2\mathbf{w}^H\mathbf{w} \leq P_{\max}\,.$$

At the optimum, the constraint in (26) can be shown to be satisfied with equality [10]. Hence, we can rewrite the optimization problem (26) in the following compact form:

$$\max_{\mathbf{w}} \ \frac{N\sigma^2\left(P_{\max} - \sigma^2\mathbf{w}^H\mathbf{w}\right)}{2\left[\left(\mathbf{w}^H\mathbf{D}_2\mathbf{w}+\sigma^2\right)\left(\mathbf{w}^H\mathbf{D}_1\mathbf{w}+\sigma^2\right)\right]\mathbf{1}^T\mathbf{J}(\mathbf{w})} \quad (27)$$
$$\text{subject to } \mathbf{w}^H\mathbf{w} \leq \frac{P_{\max}}{\sigma^2}$$

This constrained nonlinear optimization problem can be solved using a sequential quadratic programming (SQP) solver package. Note that no global optimality can be claimed at this time and there is a possibility for obtaining a local solution. Interestingly, the optimization problem (27) can be rewritten as

$$\max_{\mathbf{w}} \frac{1}{\sum_{i=1}^{N}\frac{1}{\phi_i(\mathbf{w})}} \quad \text{subject to} \quad \|\mathbf{w}\|^2 \leq P_{\max/\sigma^2} \quad (28)$$

where $\phi_i(\mathbf{w}) \triangleq \frac{\left(P_{\max} - \sigma^2\mathbf{w}^H\mathbf{w}\right)|\mathbf{w}^H\mathbf{a}_i|^2}{\left(\mathbf{w}^H\mathbf{D}_2\mathbf{w}+\sigma^2\right)\left(\mathbf{w}^H\mathbf{D}_1\mathbf{w}+\sigma^2\right)}$ is, in light of the results of [11], *is the maximum SNR, for a given beamforming weight vector* $\mathbf{w}$*, in a virtual one-way relay network from Transceiver 1 to Transceiver 2 or (vice versa) over the ith subcarrier that can be achieved under a total power budget* $P_{\max}$. Thus solving the optimization problem (28) means that we aim to maximize the harmonic mean of such maximum SNRs.

4. NUMERICAL RESULTS

We assume 8 relays are to cooperate to establish a bidirectional relay network. The signal paths going through the relays are subject to random propagation delays that are uniformly distributed in the interval $[0, 8T_s]$. The number of subcarriers is assumed to be equal to 128 and the flat fading channel coefficients are considered to be

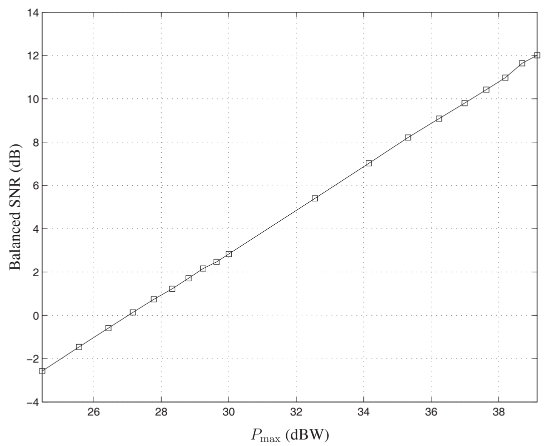

Fig. 2. Maximum balanced SNR versus total transmit power budget $P_{\max}$

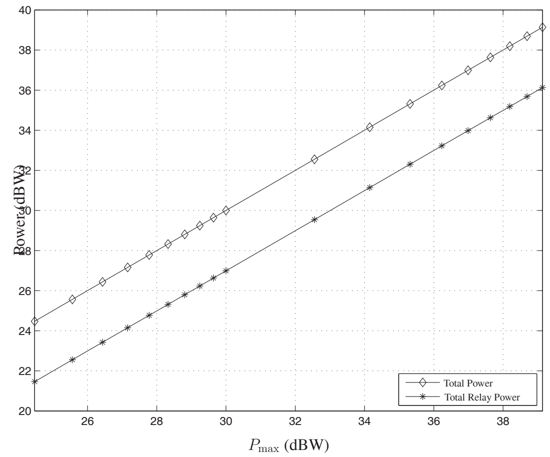

Fig. 3. Total transmit power and relay transmit power versus $P_{\max}$

independent and identically distributed (i.i.d) complex Gaussian random variables with zero-mean and unit variance. Also, the variances of all noises are assumed to be equal to one. The performance of the proposed algorithm, in terms of the maximum balanced SNR versus total transmit power, is depicted in Fig. 2 In Fig. 3, total *relay transmit power* is compared with the total available transmit power. Interestingly, this figure shows that the relays collectively consume half of the available transmit power and the two transceivers spend the remaining half. We have observed that this power allocation scheme holds not only in average but also per channel realization. We have also observed that the SQP approach to solve the SNR balancing problem always yields the same solution regardless of the initial point of the method.

5. CONCLUSION

In this work, we investigated an asynchronous two-way relay network where different relay paths are subject to different processing/propagation delays. Orthogonal frequency division multiplexing (OFDM) is adopted at the transceivers, while relays are required to use simple amplify-and-forward protocol. We proposed an algorithm in order to maximize the smallest SNR over all subcarriers subject to a total transmit power constraint. This approach results in SNR balancing. Simulation results show that using this method, the obtained transmitted power of the relays is equal to the transmitted power of transceivers.

6. REFERENCES

[1] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z. Q. Luo, "Distributed beamforming for relay networks based on second-order statistics of the channel state information," *IEEE Trans. Signal Process.*, vol. 56, pp. 4306–4316, Sep. 2008.

[2] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *IEEE Trans. Inf. Theory*, vol. 55, pp. 2499–2517, Jun. 2009.

[3] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Optimal distributed beamforming for two-way relay networks," *IEEE Trans. Signal Process.*, vol. 58, pp. 1238–1250, Mar. 2010.

[4] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, pp. 699–712, Jun. 2009.

[5] M. Chen and A. Yener, "Interference management for multiuser two-way relaying," in *Proc. CISS'08*, Princeton, NJ, Mar. 19–21, 2008, pp. 246–251.

[6] R. Vaze and R. W. Heath, "Optimal amplify and forward strategy for two-way relay channel with multiple relays," in *Proc. IEEE ITW 2009*, Volos, Greece, Jun. 12–10, 2009, pp. 181–185.

[7] J. Joung and A. Sayed, "Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems," *IEEE Trans. Signal Process.*, vol. 58, pp. 1833–1846, Mar. 2010.

[8] F. Roemer and M. Haardt, "Tensor-based channel estimation and iterative refinements for two-way relaying with multiple antennas and spatial reuse," *IEEE Trans. Signal Process.*, vol. 58, pp. 5720–5735, Nov. 2010.

[9] H. Chen, A. Gershman, and S. Shahbazpanahi, "Filter-and-forward distributed beamforming in relay networks with frequency selective fading," *IEEE Trans. Signal Process.*, vol. 58, pp. 1251–1262, Mar. 2010.

[10] R. Vahidnia and S. Shahbazpanahi, "Joint sub-carrier power loading and distributed beamforming in OFDM-based bidirectional asynchronous relay networks," *submitted to IEEE Trans. Signal Process.*, 2011.

[11] S. Shahbazpanahi and M. Dong, "Achievable rate region and sum-rate maximization for network beamforming for bidirectional relay networks," in *Proc. ICASSP'10*, Mar. 2010, pp. 2510–2513.