MAXIMUM LIKELIHOOD ESTIMATION OF TRANSITION PROBABILITIES USING ANALYTICAL CENTER CUTTING PLANE METHOD FOR UNKNOWN MANEUVERING EMITTER TRACKING BY A WIRELESS SENSOR NETWORK

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ABSTRACT

We consider the problem of unknown maneuvering emitter tracking by a wireless sensor network using the interacting multiple models (IMM) with the TDOA and FDOA measurements. Essential to this tracking framework is the Markov transition probability matrix (TPM) governing the jumps between multiple dynamic motion models for the maneuvering target. In practice, the TPM is unknown and has to be estimated. In this paper, we consider the maximum likelihood (ML) estimation of the TPM and propose a recursive algorithm to update the ML TPM estimate using the analytical center cutting plane method (ACCPM). Compared to the general batch ML method, the resulting recursive ML estimation method has a much lower per sample complexity. Simulation results show the efficacy of the proposed method with improved tracking performance.

Index Terms— Maneuvering emitter tracking, ML estimation, EKF-IMM, Markovian jump system, Convex optimization

1. INTRODUCTION

Detection, localization, and tracking (DLT) of an unknown emitter is an important surveillance task with many civilian and military applications. In a wireless sensor network (WSN) with synchronized sensor nodes, the DLT can be performed by either the time difference of arrival (TDOA) or frequency difference of arrival (FDOA) information [1, 2] or both. In this paper, we focus on unknown maneuvering emitter tracking using TDOA and FDOA measurements.

Maneuvering target tracking can be performed by the interacting multiple model (IMM) algorithm [3] whereby multiple models representing the target motion equations transit from one to another according to a Markov chain with known transition probabilities. However, these transition probabilities (or transition probability matrix, TPM in short) are not known in practice and has to be estimated.

In this paper, we consider the problem of maximum likelihood estimation of the TPM and formulate it as a convex optimization problem. Since the ML estimate of TPM corresponds to the analytic center of a polytope defined by the measurements, we propose to use the analytical center cutting plane method (ACCPM) [4] to recursively update the ML TPM estimate. In this way, the ML TPM estimation can be performed with substantially lower per sample complexity than the batch method, which is very important in practice.

To confirm the effectiveness and the efficiency of the proposed estimation method, we simulate the use of joint TDOA and FDOA measurements to track an unknown maneuvering emitter by performing the extended Kalman filter with interacting multiple models (EKF-IMM). Simulation results show that the ACCPM [4] is an efficient recursive TPM estimation method which, together with EKF-IMM, can yield a good overall tracking performance.

2. PROBLEM FORMULATION

2.1. Likelihood function of multiple model transition probability

Consider the discrete Markovian jump system (MJS):

$$\mathbf{x}(k) = \mathbf{F}[m(k), \mathbf{x}(k-1)] + \mathbf{w}[k, m(k)],$$

$$\mathbf{z}(k) = \mathbf{H}[m(k), \mathbf{x}(k)] + \mathbf{v}[k, m(k)],$$
 (1)

where $\mathbf{x}(k)$ is the continuous base state vector, $\mathbf{z}(k)$ is the measurement vector, $\mathbf{w}(k)$ and $\mathbf{v}(k)$ are the state process noise and measurement noise, respectively, \mathbf{F} is the base state transition function, \mathbf{H} is the observation function, and m(k) is the modal state with $m(k) \in \mathbb{M} \triangleq \{1, 2, \dots, r\}$. We here consider m(k) as a Markov chain with initial and transition probabilities denoted by

$$\mu_j(0) = P(m_j(0)), \tag{2}$$

$$\pi_{ij} = P(m_j(k)|m_i(k-1)), \ i, j = 1, \cdots, r,$$
(3)

where $m_i(k)$ stands for the event $\{m(k) = i\}$ for simplicity. For convenience, we define $\mathbf{z}^k \triangleq \{\mathbf{z}(1), \dots, \mathbf{z}(k)\}$.

Define the multiple model transition probability matrix (TPM):

$$\mathbf{P} \triangleq \{\pi_{ij}, i, j = 1, \dots, r\}.$$
(4)

Similar to [3], we assume that \mathbf{P} is an unknown *random* but *time-invariant* matrix.

By using the total probability law, the likelihood function with respect to a deterministic \mathbf{P} can be represented by

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$$p\{\mathbf{z}(k)|\mathbf{P}, \mathbf{z}^{k-1}\} = \sum_{j=1}^{r} p\{\mathbf{z}(k), m_{j}(k)|\mathbf{P}, \mathbf{z}^{k-1}\}$$
$$= \sum_{j=1}^{r} p\{\mathbf{z}(k)|m_{j}(k), \mathbf{P}, \mathbf{z}^{k-1}\} P\{m_{j}(k)|\mathbf{P}, \mathbf{z}^{k-1}\}$$
$$= \sum_{j=1}^{r} p\{\mathbf{z}(k)|m_{j}(k), \mathbf{P}, \mathbf{z}^{k-1}\}$$
$$\times \sum_{i=1}^{r} P\{m_{j}(k)|m_{i}(k-1), \mathbf{P}, \mathbf{z}^{k-1}\}$$
$$\times P\{m_{i}(k-1)|\mathbf{P}, \mathbf{z}^{k-1}\}$$
$$= \sum_{j=1}^{r} \Lambda_{j}(k) \sum_{i=1}^{r} \pi_{ij} \mu_{i}(k-1)$$
$$= \mu^{T}(k-1) \mathbf{P} \mathbf{\Lambda}(k), \qquad (5)$$

where

$$\mu_{i}(k) = P\{m_{i}(k)|\mathbf{P}, \mathbf{z}^{k}\}$$

$$\Lambda_{j}(k) = p\{\mathbf{z}(k)|m_{j}(k), \mathbf{P}, \mathbf{z}^{k-1}\},$$

$$\mu(k) = [\mu_{1}(k), \cdots, \mu_{r}(k)]^{T},$$

$$\mathbf{\Lambda}(k) = [\Lambda_{1}(k), \cdots, \Lambda_{r}(k)]^{T},$$
(6)

 $\mu(k)$ denotes mode probabilities, and $\Lambda(k)$ denotes the mode likelihoods. Since the likelihood function with multiple measurements is denoted by

$$p\{\mathbf{z}^{k}|\mathbf{P}\} = p\{\mathbf{z}(k)|\mathbf{P},\mathbf{z}^{k-1}\}p\{\mathbf{z}^{k-1}|\mathbf{P}\},$$
(7)

it is obvious to see that

$$p\{\mathbf{z}^{K}|\mathbf{P}\} = \prod_{k=1}^{K} \boldsymbol{\mu}^{T}(k-1)\mathbf{P}\boldsymbol{\Lambda}(k).$$
(8)

2.2. Maneuvering emitter tracking by the extended Kalman filter and the interacting multiple model method

Since the TDOA and FDOA measurement models are nonlinear with respect to the location and velocity of the unknown maneuvering emitter, the system model (1) can be represented by

$$\mathbf{e}(k) = \mathbf{F}_j(k-1)\mathbf{e}(k-1) + \mathbf{w}(k-1), \qquad (9)$$

$$\mathbf{z}(k) = \begin{bmatrix} \mathbf{z}^{t}(k) \\ \mathbf{z}^{f}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{h}^{t}(\mathbf{e}(k); \tilde{\mathbf{s}}^{(l)}, \tilde{\mathbf{s}}^{(u)}) + \mathbf{v}^{t}(k) \\ \mathbf{h}^{f}(\mathbf{e}(k); \tilde{\mathbf{s}}^{(l)}, \tilde{\mathbf{s}}^{(u)}) + \mathbf{v}^{f}(k) \end{bmatrix},$$
(10)

where $\mathbf{e}(k) = [x(k), y(k), z(k), \dot{x}(k), \dot{y}(k), \dot{z}(k)]^T$ denotes the base state at time k corresponding to the location and velocity of the moving emitter, j denotes the modal state at time k, $\tilde{\mathbf{s}}^{(l)}$ denotes the location of the l-th sensor, $\mathbf{h}^t(k)$ and $\mathbf{h}^f(k)$ denotes the noise-free TDOA and FDOA measurements at time k, respectively, and $\mathbf{w}(k), \mathbf{v}^t(k)$ and $\mathbf{v}^f(k)$ are assumed to be Gaussian distributed and uncorrelated with each other.

we here use the extended Kalman filter with the interacting multiple model (EKF-IMM) method to estimate the TPM \mathbf{P} .

2.2.1. EKF based on TDOA measurements

We first consider the tracking by the EKF based on TDOA measurements, which includes two steps, *i.e.*, the prediction step and the updating step: Prediction step: $\hat{\mathbf{e}}_{j}^{t}(k|k-1) = \mathbf{F}_{j}(k-1)\hat{\mathbf{e}}_{j}(k-1|k-1),$ $\mathbf{P}_{j}^{t}(k|k-1) = \mathbf{F}_{j}(k-1)\mathbf{P}_{j}(k-1|k-1)\mathbf{F}_{j}^{T}(k-1) + \mathbf{R}_{\mathbf{w}}.$

Updating step:

$$\hat{\mathbf{e}}_{j}^{t}(k|k) = \hat{\mathbf{e}}_{j}^{t}(k|k-1) + \mathbf{K}_{j}^{t}(k)\mathbf{r}_{j}^{t}(k),$$
$$\mathbf{P}_{j}^{t}(k|k) = (\mathbf{I} - \mathbf{K}_{j}^{t}(k)\mathbf{H}_{j}^{t}(k))\mathbf{P}_{j}^{t}(k|k-1), \qquad (12)$$

(11)

where

$$\begin{split} \mathbf{r}_{j}^{t}(k) &= \mathbf{z}^{t}(k) - \mathbf{h}_{j}(\hat{\mathbf{e}}_{j}^{t}(k|k-1)), \\ \mathbf{S}_{j}^{t}(k) &= \mathbf{H}_{j}^{t}(k)\mathbf{P}_{j}^{t}(k|k-1)\mathbf{H}_{j}^{tT}(k) + \mathbf{R}_{v}^{t}, \\ \mathbf{K}_{j}^{t}(k) &= \mathbf{P}_{j}^{t}(k|k-1)\mathbf{H}_{j}^{tT}(k)\mathbf{S}_{j}^{t}(k)^{-1}, \\ \mathbf{h}^{t}(\mathbf{e}(k); \tilde{\mathbf{s}}^{(l)}, \tilde{\mathbf{s}}^{(u)}) &= \frac{1}{c} \begin{bmatrix} ||\mathbf{r}^{(2)}(k)|| - ||\mathbf{r}^{(1)}(k)|| \\ \vdots \\ ||\mathbf{r}^{(N)}(k)|| - ||\mathbf{r}^{(1)}(k)|| \end{bmatrix}, \\ \mathbf{r}^{(l)}(k) &= \mathbf{G}_{1}\mathbf{e}(k) - \tilde{\mathbf{s}}^{(l)}, \ l = 1, \cdots, N, \\ \mathbf{H}_{j}^{t}(k) &= \frac{\partial \mathbf{h}^{tT}}{\partial \mathbf{e}(k)} = \frac{1}{c} \begin{bmatrix} (\tilde{\mathbf{i}}_{j}^{(2)}(k) - \tilde{\mathbf{i}}_{j}^{(1)}(k))^{T} \\ \vdots \\ (\tilde{\mathbf{i}}_{j}^{(N)}(k) - \tilde{\mathbf{i}}_{j}^{(1)}(k))^{T} \end{bmatrix} \mathbf{G}_{1}, \\ \tilde{\mathbf{i}}_{j}^{(l)}(k) &= \frac{\tilde{\mathbf{r}}_{j}^{(l)}(k)}{||\tilde{\mathbf{r}}_{j}^{(l)}(k)||}, \\ \tilde{\mathbf{r}}_{j}^{(l)}(k) &= \mathbf{G}_{1}\hat{\mathbf{e}}_{j}^{t}(k|k-1) - \tilde{\mathbf{s}}^{(l)}, \\ \mathbf{G}_{1} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \end{split}$$

where the superscript "l" denotes sensor number and N denotes the number of sensors.

2.2.2. EKF based on FDOA measurements

Since both the TDOA and FDOA observation model share the same state transition equation, the prediction step for the FDOA case can be omitted. We here give the updating step for the FDOA EKF tracking case:

$$\hat{\mathbf{e}}_{j}^{f}(k|k) = \hat{\mathbf{e}}_{j}^{t}(k|k) + \mathbf{K}_{j}^{f}(k)\mathbf{r}_{j}^{f}(k),$$
$$\mathbf{P}_{j}^{f}(k|k) = (\mathbf{I} - \mathbf{K}_{j}^{f}(k)\mathbf{H}_{j}^{f}(k))\mathbf{P}_{j}^{t}(k|k),$$
(13)

where the FDOA Kalman gain is given by

$$\begin{split} \mathbf{r}_{j}^{f}(k) &= \mathbf{z}^{f}(k) - \mathbf{h}_{j}^{f}(\hat{\mathbf{e}}_{j}^{t}(k|k)), \\ \mathbf{K}_{j}^{f}(k) &= \mathbf{P}_{j}^{t}(k|k) \mathbf{H}_{j}^{fT}(k) \mathbf{S}_{j}^{f}(k)^{-1}, \\ \mathbf{S}_{j}^{f}(k) &= \mathbf{H}_{j}^{f}(k) \mathbf{P}_{j}^{t}(k|k) \mathbf{H}_{j}^{fT}(k) + \mathbf{R}_{\mathbf{v}}^{f}, \\ \mathbf{h}^{f}(\mathbf{e}(k); \tilde{\mathbf{s}}^{(l)}, \tilde{\mathbf{s}}^{(u)}) &= \frac{f_{0}}{c} \begin{bmatrix} (\mathbf{i}_{j}^{(2)}(k) - \mathbf{i}_{j}^{(1)}(k))^{T} \\ \vdots \\ (\mathbf{i}_{j}^{(N)}(k) - \mathbf{i}_{j}^{(1)}(k))^{T} \end{bmatrix} \mathbf{G}_{2}\mathbf{e}(k), \\ \mathbf{H}_{j}^{f}(k) &= \frac{\partial \mathbf{h}^{fT}}{\partial \mathbf{e}(k)} = -\frac{f_{0}}{c} \begin{bmatrix} \left[\hat{\mathbf{A}}_{j}^{(1)}(k) - \hat{\mathbf{A}}_{j}^{(2)}(k) \\ \vdots \\ \left[\hat{\mathbf{A}}_{j}^{(1)}(k) - \hat{\mathbf{A}}_{j}^{(2)}(k) \end{bmatrix}^{T} \\ \vdots \\ \begin{bmatrix} \hat{\mathbf{A}}_{j}^{(1)}(k) - \hat{\mathbf{A}}_{j}^{(N)}(k) \\ \mathbf{i}_{j}^{(1)}(k) - \mathbf{i}_{j}^{(N)}(k) \end{bmatrix}^{T} \end{bmatrix}, \end{split}$$

$$\begin{split} \hat{\mathbf{i}}_{j}^{(l)}(k) &= \frac{\hat{\mathbf{r}}_{j}^{(l)}(k)}{||\hat{\mathbf{r}}_{j}^{(l)}(k)||}, \\ \hat{\mathbf{r}}_{j}^{(l)}(k) &= \mathbf{G}_{1}\hat{\mathbf{e}}_{j}^{t}(k|k) - \tilde{\mathbf{s}}^{(l)}, \\ \hat{\mathbf{A}}_{j}^{(l)}(k) &= (\mathbf{I} - \hat{\mathbf{i}}_{j}^{(l)}(k)\hat{\mathbf{i}}_{j}^{(l)T}(k)) \frac{\mathbf{G}_{2}\mathbf{e}_{j}^{t}(k|k)}{||\hat{\mathbf{r}}_{j}^{(l)}(k)||}, \\ \mathbf{G}_{2} &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \end{split}$$

and f_0 denotes the carrier frequency.

2.2.3. $\mu_j(k)$ and $\Lambda_j(k)$ of IMM

In the tracking process of EKF-IMM, model likelihood function $\Lambda_j(k)$ is obtained by

$$\Lambda_j(k) = \mathcal{N}(\mathbf{r}_j(k); \mathbf{0}; \mathbf{S}_j(k)), \qquad (14)$$

where $\mathbf{r}_{j}(k) = \begin{bmatrix} \mathbf{r}_{j}^{tT}(k) & \mathbf{r}_{j}^{fT}(k) \end{bmatrix}^{T}$, $\mathbf{S}_{j}(k) = \begin{bmatrix} \mathbf{S}_{j}^{t}(k) & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{j}^{f}(k) \end{bmatrix}$. The mode probability $\mu_{j}(k)$ is given by

$$\mu_j(k) = \frac{1}{c(k)} \Lambda_j(k) \sum_{i=1}^r \pi_{ij} \mu_i(k-1),$$
(15)

where $c(k) = \sum_{j=1}^{r} \Lambda_j(k) \sum_{i=1}^{r} \pi_{ij} \mu_i(k-1).$

3. ML ESTIMATION OF THE TPM USING THE ANALYTIC CENTER CUTTING-PLANE METHOD

The analytic center of a set of convex inequalities and linear equalities,

$$f_i(\mathbf{x}) \le 0, \ i = 1, \cdots, n, \ \mathbf{F}\mathbf{x} = \mathbf{c},$$
 (16)

is defined as an optimal point for the (convex) problem

$$\min \quad -\sum_{i=1}^{n} \log(-f_i(\mathbf{x})),$$
s. t. $\mathbf{F}\mathbf{x} = \mathbf{c}.$

$$(17)$$

with variables $\mathbf{x} \in \mathbf{R}^m$ and implicit constraints $f_i(\mathbf{x}) < 0, i = 1, \dots, n$ [5].

Fortunately, the maximum likelihood (ML) estimation of \mathbf{P} by maximizing the likelihood function $p\{\mathbf{z}^{K}|\mathbf{P}\}$ of (8) can be performed by solving the following analytical center problem:

$$\min_{\mathbf{P}} -\sum_{k=1}^{K} \log \mu'(k-1) \mathbf{P} \mathbf{\Lambda}(k),
s. t. \sum_{\substack{j=1\\ \pi_{ij} \ge 0, \ i = 1, \dots, r.}}^{r} \pi_{ij} = 1,$$
(18)

For simplicity and without loss of generality, we consider r = 3 in the following. Let $\mathbf{x} = [\pi_{11}, \pi_{12}, \pi_{21}, \pi_{22}, \pi_{31}, \pi_{32}]^T$, (18) can be equivalently written as

$$\min_{\mathbf{x}} \quad -\sum_{k=1}^{K} \log(c_k + \mathbf{a}_k^T \mathbf{x}) - \epsilon_1 \sum_{i=1}^{3} \log(-\mathbf{b}_i^T \mathbf{x} + d_i) \\
-\epsilon_2 \sum_{j=1}^{6} \log(x_j),$$
(19)

where ϵ_1 and ϵ_2 denote constants for penalization and are usually small,

$$c_{k} = \sum_{i=1}^{3} \mu_{i}(k-1)\mathbf{\Lambda}^{T}(k)\mathbf{h}_{1},$$

$$\mathbf{a}_{k}^{T} = \sum_{i=1}^{3} \mu_{i}(k-1)\mathbf{\Lambda}^{T}(k)\mathbf{H}_{i},$$

$$\mathbf{b}_{1} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^{T},$$

$$\mathbf{b}_{2} = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}^{T},$$

$$\mathbf{b}_{3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}^{T},$$

$$d_{1} = d_{2} = d_{3} = 1,$$

$$\mathbf{h}_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{H}_{2} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{H}_{3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

The optimization problem (19) can be solved using the iterative procedure of the weighted ACCPM [4]:

- Step 1: Let $\mathbf{A}^0 = \{\mathbf{B} eye(r \times (r-1))\}$ with $\mathbf{B} = [\mathbf{b}_1 \cdots \mathbf{b}_r], \mathbf{x}^0$ be a point that satisfies the constraints, $\mathbf{w}^0 = ones(r^2, 1)/\epsilon, \ \epsilon = \epsilon_1 = \epsilon_2 = 10^{-3}, \ \mathbf{c}^0 = [ones(r, 1) \ zeros(r(r-1), 1)], \text{ where ones, eye, and zeros are Matlab notations. } \mathbf{s}^0 = \mathbf{c}^0 - (\mathbf{A}^0)^T \mathbf{x}, \ \mathbf{H}^0 = \mathbf{A}^0 diag\{\mathbf{w}^0\} diag\{\mathbf{s}^0\}^{-2} \mathbf{A}^{0T}, \ \mathbf{g}^0 = -\mathbf{A}^0 diag\{\mathbf{s}^0\}^{-1} \mathbf{w}^0, K \ \text{and } k = 1 \ (diag \ \text{is also a Matlab notation).}$
- Step 2: Compute the approximate analytic center \mathbf{x}^k of the set $\Omega^k = \{\mathbf{x} \in \mathbb{R}^m : \mathbf{c}^k (\mathbf{A}^k)^T \mathbf{x} \ge 0\}$ by the updating scheme from the previous set Ω^{k-1} ; assume that $\beta = 10^{-5}$ and compute $(\mathbf{x}^k, \mathbf{s}^k)$ with \mathbf{x}^k approximate analytic center of $\Omega^k = \{\mathbf{x} \in \mathbb{R}^m : \mathbf{c} (\mathbf{A}^k)^T \mathbf{x} \ge 0\}$ using the updating scheme; generate a hyperplane $\{\mathbf{a}_{k+1}^T \mathbf{x} \le \mathbf{a}_{k+1}^T \mathbf{x}^k\}$ and a new set $\Omega^{k+1} = \{\mathbf{x} \in \mathbb{R}^m : \mathbf{c}^{k+1} (\mathbf{A}^{k+1})^T \mathbf{x} \ge 0\}$, where $\mathbf{A}^{k+1} = (\mathbf{A}^k, \mathbf{a}_{k+1}), \mathbf{c}^{k+1} = \begin{pmatrix} \mathbf{c}^k \\ \mathbf{a}_{k+1}^T \mathbf{x}^k \end{pmatrix}$; apply variable stepsize dual Newton iteration method to recent until a new approximate analytic center \mathbf{x}^{k+1} is obtained with $\delta(\mathbf{x}) = \sqrt{\mathbf{g'}\mathbf{H}^{-1}\mathbf{g}} < 0.06$.

Step 3: Set k := k + 1, if k > K, stop; otherwise, return to Step 2.

With the above iterative procedure, the unknown maneuvering emitter tracking can be performed by the ACCPM, whose steps are listed in table 1.

Complexity analysis of the method: The complexity of the AC-CPM is $O^*(\frac{m^2}{\epsilon^2})$ [4], where O^* denotes that lower order terms are ignored; ϵ denotes the radius of the ball which is contained in the solution set; *m* denotes the dimension of the optimization variables. The computational complexity of the EKF is $O(b^3)$, where *b* is the size of the state vector. Assuming that *M* is the number of states in the state space, the computational complexity of the IMM theoretically includes *M* times the one of the EKF, *M* times the one of

Table 1. Steps for tracking by the ACCPM

Step 2: Compute the means and covariances of the base states.

Step 3: Compute the outputs of the extended Kalman filter which include mode likelihood functions, means and covariances of the base states.

Step 4: Solve the optimization problem (19) by ACCPM. *Step* 5: Update mode probabilities.

the states estimation at the interacting stage, and M times the one of the states estimation at the mode probability updating stage. In these stages, Mb(b+2) multiplications and (2M-1)(b+3)/2 sums are required for each mixing, respectively, and M(b+2) multiplications and M(2b-2) sums are required for each mode probabilities updating, respectively. These yield the IMM computational complexity approximately denoted by $O^*(Mb^3)$. Assume that K is the required number of the cycles for accurate TPM estimation, the total computational complexity of the proposed method is $O^*(KMb^3 + \frac{m^2}{c^2})$.

4. COMPUTER SIMULATION

In the simulation, the unknown maneuvering emitter is considered to move according to one of the 3 dynamic models: (1) constant velocity (CV) motion model; (2) clockwise coordinated turn (CT) model; (3) anticlockwise CT model. Five static sensors are used to estimate the TPM. The MJS used in the simulation is denoted by (9) and (10). The state transition matrix [6] for CV, clockwise CT, and anticlockwise CT models are respectively denoted by

$$\mathbf{F}_1(k) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \otimes \mathbf{I}_2, \tag{20}$$

$$\mathbf{F}_{j}(k) = \begin{bmatrix} 1 & 0 & \frac{\sin(\Omega_{j}(k)T)}{\Omega_{j}(k)} & -\frac{(1-\cos(\Omega_{j}(k)T))}{\Omega_{j}(k)} \\ 0 & 1 & \frac{(1-\cos(\Omega_{j}(k)T))}{\Omega_{j}(k)} & \frac{\sin(\Omega_{j}(k)T)}{\Omega_{j}(k)} \\ 0 & 0 & \cos(\Omega_{j}(k)T) & -\sin(\Omega_{j}(k)T) \\ 0 & 0 & \sin(\Omega_{j}(k)T) & \cos(\Omega_{j}(k)T) \end{bmatrix}, \quad (21)$$

where $j = 2, 3, \otimes$ denotes the Kronecker product, \mathbf{I}_2 is the 2×2 identity matrix, $\Omega_j(k) = (-1)^j \frac{a_m}{\sqrt{\dot{x}_k^2 + \dot{y}_k^2}}, \ j = 2, 3, \text{ and } a_m(>0)$ denotes a typical acceleration rate. Assume that TDOA and FDOA measurement noises respectively follows $\mathbf{v}^t(k) \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$ and $\mathbf{v}^f(k) \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_f)$, where $\mathbf{Q}_t = \sigma_t^2 \mathbf{Q}/2$ and $\mathbf{Q}_f = \sigma_f^2 \mathbf{Q}/2$, $\mathbf{Q} = [eye(4, 4) + ones(4, 4)]$. The process noises $\mathbf{w}(k)$ follows $\mathcal{N}(\mathbf{0}, \mathbf{R}_i), \ i = 2, 3$, where $\mathbf{R}_i = diag\{\sigma_{x_i}^2, \sigma_{y_i}^2, \sigma_{x_i}^2, \sigma_{y_i}^2\}$.

 $\mathcal{N}(\mathbf{0}, \mathbf{R}_i), \ i = 2, 3, \text{ where } \mathbf{R}_i = diag\{\sigma_{x_i}^2, \sigma_{y_i}^2, \sigma_{x_i}^2, \sigma_{y_i}^2\}.$ We set $T = 1s, \sigma_{x_1}^2 = \sigma_{y_1}^2 = 100m, \sigma_{x_1}^2 = \sigma_{y_1}^2 = 10m/s, \sigma_{x_1}^2 = \sigma_{y_1}^2 = 100m \text{ and } \sigma_{x_j}^2 = \sigma_{y_j}^2 = 20m/s, \ j = 2, 3.$ $a_m = 100 \text{km/h}^2$. The true initial emitter state is assumed as $\mathbf{e}(0) = \begin{bmatrix} 0 & 0 & 40 & 40 \end{bmatrix}$. The *a priori* mode probability is assumed as $\mu_{0,1} = 0.8, \mu_{0,2} = 0.1$ and $\mu_{0,3} = 0.1$. We also assume that TPM **P** is given by

$$\mathbf{P} = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}.$$
 (22)

The initial value of the TPM is set by $\frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$, where all the entries of 1 are 1. We estimate the TPM P using the ACCPM. The simulation is implemented using Matlab 7.10.0 (R2010a) on an HP Z800 workstation with 3.47GHz and 3.46GHz Intel (R) Xeon (R)

dual core CPU. 500 Monte Carlo (MC) runs are performed. To compare the performance of TPM estimation, we also perform 500 MC runs for the numerical-integration (NI) method [3], where the decoupled version of the method is used and the number of grid TPMs is set as 200. The computation time for the NI method is 932.67seconds under 500MC runs.

In Fig. 1, we plot the root mean square error (RMSE) of the TPM estimate versus the length of the measurements for both the NI [3] and ACCPM, where the ACCPM performed with three different minimum step sizes, *i.e.*, $\gamma = 0.05, 0.005, 0.001$ yield almost the same RMSE. It is seen that the RMSE of the TPM estimate decreases as the length of the measurements grows and goes below 0.1 eventually. In Table 2, we list the computation time of our proposed method with the three step sizes, *i.e.*, $\gamma = 0.05, 0.005, 0.001$. From Fig.1 and Table 2, it can be seen that the ACCPM outperforms the NI [3] with much lower complexity by choosing $\gamma = 0.05$ or 0.005.

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 $\gamma = 0.005$

 $\gamma = 0.001$

Table 2. Computation Time (s) of the ACCPM

 $\gamma = 0.05$

ACCPM



Fig. 1. RMSE of the ML TPM estimate using the ACCPM and the NI [we use black solid line for the NI, and for ACCPM we use red dashed line for step size $\gamma = 0.05$, green point line for $\gamma = 0.005$, and blue dash-dotted line for $\gamma = 0.001$]

Step 1: Compute the mixing probability.