COOPERATIVE LOCALIZATION IN PARTIALLY CONNECTED MOBILE WIRELESS SENSOR NETWORKS USING GEOMETRIC LINK RECONSTRUCTION

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ABSTRACT

We extend one of our recently proposed anchorless mobile network localization algorithms (called PEST) to operate in a partially connected network. To this aim, we propose a geometric missing link reconstruction algorithm for noisy scenarios and repeat the proposed algorithm in a local-to-global fashion to reconstruct a complete distance matrix. This reconstructed matrix is then used in the PEST to localize the mobile network. We compare the computational complexity of the new link reconstruction algorithm with existing related algorithms and show that our proposed algorithm has the lowest complexity, and hence, is the best extension of the low complexity PEST. Simulation results further illustrate that the proposed link reconstruction algorithm leads to the lowest reconstruction error as well as the most accurate network localization performance.

Index Terms— Cooperative mobile network localization, partial connectivity, distance matrix reconstruction.

1. INTRODUCTION

Numerous applications of wireless sensor networks (WSNs) cannot rely on a pre-existing and fixed infrastructure. In such scenarios, there are typically no anchor nodes (with known locations) and determining the relative location of the sensor nodes is the ultimate goal. The problem of localization in anchorless networks becomes more challenging when the nodes of the network are mobile. In [1] an anchorless localization scheme for mobile networks is proposed wherein each node requires knowledge about its own movement model as a probability distribution in order to do predictions, which is not so simple to acquire and additionally increases the computational complexity significantly. In [2], a method based on extended Kalman filtering is developed which incorporates the locations of the nodes as well as their velocities in a state-space model. But, this algorithm also has a high complexity. In [3], we proposed two anchorless network localization algorithms using novel subspace tracking ideas to adapt the classical multidimensional scaling (MDS) [4] for mobile WSNs. The proposed model-independent algorithms (PEST and PIST) have a considerably lower complexity than existing algorithms as well as an acceptable accuracy. Surprisingly, the problem of partial connectivity in not well investigated in a mobile WSN.

In this paper, we propose to use a local-to-global missing link reconstruction to end up with a reconstructed network distance measurement matrix which can be fed to the PEST algorithm for localization. To this aim, we modify an existing link reconstruction algorithm [5], modify the Nyström algorithm [4] for link reconstruction, and also propose a novel geometric missing link reconstruction algorithm and modify it by proposing a selection criterion for noisy measurements. The rest of the paper is organized as follows. In Section 2, we present the network model and state the problem under consideration. Section 3 tackles the problem of partial connectivity. Section 4 compares the computational complexity of the missing link reconstruction algorithms under consideration. Section 5 provides simulation results for evaluation of missing link reconstruction as well as mobile localization in a partially connected WSN. Finally, concluding remarks are presented in Section 6.

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2. NETWORK MODEL AND PROBLEM STATEMENT

We consider a network of N mobile wireless sensor nodes, living inside a bounded 2-dimensional space. Our network model is based on pairwise distance measurements and these distance measurements themselves can be calculated by means of time of flight (ToF) measurements. Hence, we assume that the ToF information is already converted into noisy distance measurements as

$$r_{i,j,k} = d_{i,j,k} + v_{i,j,k}, \tag{1}$$

where $d_{i,j,k} = \|\mathbf{x}_{i,k} - \mathbf{x}_{j,k}\|$ is the noise-free Euclidean distance, $v_{i,j,k} \sim \mathcal{N}(0, \sigma^2_{v,i,j,k})$ is the uncorrelated additive noise and $\mathbf{x}_{i,k}$ is the actual coordinate vector of the i-th sensor node, all for the k-th snapshot of a mobile scenario. For a free space propagation model, we consider a constant

$$\gamma = d^2_{i,j,k}/\sigma^2_{v,i,j,k}, \tag{2}$$

which punishes the longer distances with larger measurement errors. Meanwhile, we consider a simple finite-range model where the distances can be measured only if they are below a certain communication range r_0, otherwise they cannot be measured and we call them *missing links*. A wide variety of movement models can be considered for the mobile nodes since in [3] we explain that the proposed algorithms, one of which is also considered here, are blind to the movement model. The problem considered herein can be stated as follows. Having a fully connected network, the squared noisy distance measurements $r^2_{i,j,k}$ between the nodes can be collected in a distance matrix $\mathbf{D}_k$, i.e., $[\mathbf{D}_k]_{i,j} = r^2_{i,j,k}$, after which the double-centered distance matrix can be calculated as $\mathbf{B}_k = -1/2\mathbf{H}_N\mathbf{D}_k\mathbf{H}_N$ using the centering operator $\mathbf{H}_N = \mathbf{I}_N - \mathbf{1}_N\mathbf{1}_N^T/N$, where $\mathbf{I}_N$ denotes an $N \times N$ identity matrix and $\mathbf{1}_N$ represents an $N \times 1$ vector of all ones. Then, $\mathbf{B}_k$ can be used in the PEST to track the locations of the nodes in an iterative manner [3]. However, unlike [3], we here consider a partially connected network. To be able to modify our previously proposed PEST algorithm to operate in partially connected networks, we propose to recover the missing links in a local-to-global fashion and then use the PEST. As we use the PEST, the network localization will be anchorless.

3. TACKLING PARTIAL CONNECTIVITY

We first consider the problem of missing link reconstruction, which is then used in a local-to-global fashion to reconstruct $\mathbf{D}_k$.

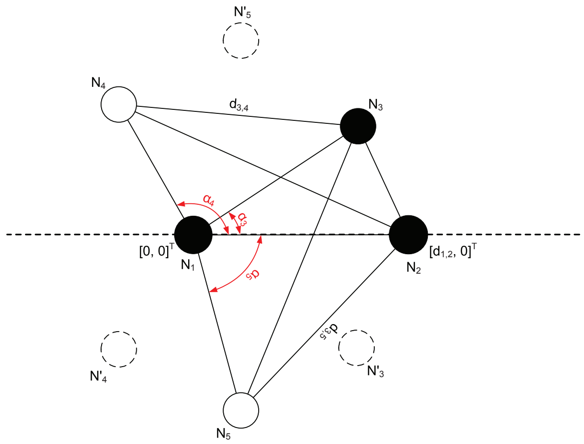

Fig. 1. Geometric link reconstruction (GLR)

3.1. Missing Link Reconstruction

In [6], a distributed algorithm for anchorless localization based on building a relative coordinate system is explained. For every node of the network a relative coordinate system is considered which is used to localize the neighboring nodes. We will here exploit this idea to reconstruct missing links in our mobile network. We propose to build a local coordinate system only around 5 nodes including 3 interconnected nodes (N_1 to N_3) and 2 other nodes (N_4 and N_5) which are both connected to the first three and the link between the last two nodes is missing as shown in Fig. 1. Let us start with the noiseless case. We choose one of the first three nodes as N_1 and place it on the origin of the local coordinate system $[0,\ 0]^T$. Since we know $d_{1,2}$, we can set the coordinates of N_2 to $[d_{1,2},\ 0]^T$. Now, by calculating $\cos(\alpha_3)$ using

$$\cos(\alpha_3) = \frac{d_{1,2}^2 + d_{1,3}^2 - d_{2,3}^2}{2d_{1,2}d_{1,3}}, \tag{3}$$

the location of N_3 will then be $[d_{1,3}\cos(\alpha_3),\ d_{1,3}\sqrt{1-\cos(\alpha_3)^2}]^T$ or $[d_{1,3}\cos(\alpha_3),\ -d_{1,3}\sqrt{1-\cos(\alpha_3)^2}]^T$ but we set it to the former. In order to acquire a rigid configuration (up to a translation and orthogonal transformation) we calculate the two possible locations for N_4 (also N_5) similar to N_3 and decide between the two possible locations by comparing the distances $d(N_4, N_3)$ and $d(N_4', N_3)$ with the available measured $d_{3,4}$ and choose the one which is equal to it. For a noisy scenario, however, we will have to choose the location which yields a closer distance compared to the noisy measured $r_{3,4}$. The same explanations hold for N_5. Now, having the relative location of N_4 and N_5 in the considered coordinate system we can calculate their missing distance. We call this algorithm geometric link reconstruction (GLR). Note that considering the above explanations, this 5-node setup is the simplest configuration of nodes with unknown locations (fits in anchorless network localization) by means of which we can recover one missing link.

For the case of noisy measurements, however, we expect that the accuracy of our relative location estimates for N_4 and N_5 will depend on the choice of the base-line nodes N_1 and N_2. For the sake of simplicity, let us assume that N_2 is already perfectly located using the available information. Further, the location estimation error in both N_4 and N_4' is similar with respect to the base-line since N_3 is only used to choose N_4 or N_4'. Therefore, the Cramér-Rao bound (CRB) of our location estimate will depend on the measurement vector $\mathbf{r} = [r_{1,4},\ r_{2,4}]^T$, where $r_{i,4} = \sqrt{(x_4-x_i)^2+(y_4-y_i)^2}$. Under the above assumptions, the CRB of the N_4 location estimate for general Gaussian noise can be derived using the Fisher information matrix (FIM) as explained in [7]

$$\mathbf{I}(N_4) = \begin{bmatrix} (\frac{\partial \mathbf{r}}{\partial x_4})^T\mathbf{C}^{-1}(\frac{\partial \mathbf{r}}{\partial x_4}) & (\frac{\partial \mathbf{r}}{\partial x_4})^T\mathbf{C}^{-1}(\frac{\partial \mathbf{r}}{\partial y_4}) \\ (\frac{\partial \mathbf{r}}{\partial y_4})^T\mathbf{C}^{-1}(\frac{\partial \mathbf{r}}{\partial x_4}) & (\frac{\partial \mathbf{r}}{\partial y_4})^T\mathbf{C}^{-1}(\frac{\partial \mathbf{r}}{\partial y_4}) \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \mathrm{tr}[\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial x_4}\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial x_4}] & \mathrm{tr}[\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial x_4}\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial y_4}] \\ \mathrm{tr}[\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial y_4}\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial x_4}] & \mathrm{tr}[\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial y_4}\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial y_4}] \end{bmatrix}. \tag{4}$$

For distance-dependent measurement noise (as defined by (1) and (2)), the covariance matrix of the measurements $\mathbf{C}$ will be

$$\mathbf{C} = \mathbb{E}\{(\mathbf{r}-\mathbb{E}\{\mathbf{r}\})(\mathbf{r}-\mathbb{E}\{\mathbf{r}\})^T\} = \begin{bmatrix} \frac{d_{1,4}^2}{\gamma} & 0 \\ 0 & \frac{d_{2,4}^2}{\gamma} \end{bmatrix}. \tag{5}$$

Our derivations show that the second term of (4) is independent of γ and is negligible compared to the first term for large values of γ. Thus, the FIM can be approximated by the first term of (4) as

$$\mathbf{I}(N_4) \approx \gamma \begin{bmatrix} (\frac{x_4^2}{d_{1,4}^4}) + \frac{(x_4-d_{1,2})^2}{d_{2,4}^4} & y_4(\frac{x_4}{d_{1,4}^4} + \frac{x_4-d_{1,2}}{d_{2,4}^4}) \\ y_4(\frac{x_4}{d_{1,4}^4} + \frac{x_4-d_{1,2}}{d_{2,4}^4}) & y_4^2(\frac{1}{d_{1,4}^4} + \frac{1}{d_{2,4}^4}) \end{bmatrix}. \tag{6}$$

Now, by considering the configuration shown in Fig.1, the CRB after elaborate simplifications can be given by

$$\mathrm{CRB}_{N_4} \approx \frac{(d_{1,4}^2 + d_{2,4}^2)(d_{1,4}^2 d_{2,4}^2)}{4\gamma A_{(N_1,N_2,N_4)}^2}, \tag{7}$$

where $A_{(N_1,N_2,N_4)}$ indicates the area of the triangle with vertices N_1, N_2 and N_4. The same calculations can be carried out for the case of distance-independent measurement noise with $v_{i,j} \sim \mathcal{N}(0,\sigma_v^2)$. For that case, $\mathbf{C} = \sigma_v^2\mathbf{I}_2$ and the second term of (4) will be zero, and therefore, the CRB expression boils down to

$$\mathrm{CRB}_{N_4} = \frac{\sigma_v^2 d_{1,4}^2 d_{2,4}^2}{2A_{(N_1,N_2,N_4)}^2}. \tag{8}$$

These CRB expressions provide a selection criterion (SC) for choosing the base-line nodes N_1 and N_2. Considering the aforementioned assumption that N_2 is perfectly located, the location estimates of N_4 and N_5 can be considered statistically independent which results in

$$\mathrm{SC} = \mathrm{CRB}_{\text{total}} = \mathrm{CRB}_{N_4} + \mathrm{CRB}_{N_5}. \tag{9}$$

The pair of nodes that provides the minimum SC in (9) will be chosen as N_1 and N_2. We call this modified algorithm for noisy scenarios, the modified GLR (MGLR).

One interesting solution proposed in [5] called linear algebraic reconstruction (LAR) proves that if we have a similar 5-node setup as explained for the GLR, the missing distance can be recovered by considering the singularity of the Schur complement of $\mathbf{D}^{(5)}$ (noisy distance matrix for N_1 to N_5 with missing link set to zero) with respect to $\mathbf{D}^{(3)}$ (noisy distance matrix for N_1 to N_3) as defined by

$$\mathbf{D}^{(5)} = \begin{bmatrix} \mathbf{D}^{(3)} & \mathbf{E} \\ \mathbf{E}^T & \mathbf{0}_{2\times 2} \end{bmatrix}. \tag{10}$$

This will give us a second-order polynomial with two roots corresponding to the missing distance. The root which constructs a rank-2 $\mathbf{B}^{(5)}$ matrix corresponding to the reconstructed complete $\hat{\mathbf{D}}^{(5)}$, will be chosen. Although the algorithm is exact for noiseless scenarios, in a noisy scenario, none of the two roots will construct a $\mathbf{B}^{(5)}$ matrix with rank two. A simple modification that comes to mind is to construct both $\mathbf{B}^{(5)}$ matrices and choose the one which is closer to a rank-2 matrix. To this aim, we can define a rank selection metric $\rho = \sum_{i=1}^{2}|\lambda_i| / \sum_{i=3}^{5}|\lambda_i|$ (where $\{\lambda_i\}$ denote the eigenvalues of $\mathbf{B}^{(5)}$) and choose the root which yields a larger ρ. We call this algorithm the modified LAR (MLAR). The other possible solution is to simplify the Nyström algorithm (on behalf of all Nyström-based algorithms explained in [4]) for the case of the explained 5-node setup with one missing link. To do this, we first calculate the relative coordinates of N_1 to N_3, denoted by a 2×3 matrix $\mathbf{Y}$, by doing a double-centering on $\mathbf{D}^{(3)}$ and then computing an EVD on $\mathbf{B}^{(3)}$ as

$$\mathbf{B}^{(3)} = -\tfrac{1}{2}\mathbf{H}_3\mathbf{D}^{(3)}\mathbf{H}_3, \;\; \mathbf{B}^{(3)} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^T, \;\; \mathbf{Y} = \boldsymbol{\Sigma}_s^{\frac{1}{2}}\mathbf{U}_s^T,$$

where $\mathbf{H}_n$ stands for an $n \times n$ centering operator and subscript s indicates the submatrices corresponding to the eigenvectors with the 2 largest positive eigenvalues. Next, we also bring the center of gravity of the group containing N_4 and N_5 to the origin and exploit the known distances between N_4 and N_5 and the other nodes to recover the coordinates of N_4 and N_5, denoted by a 2×2 matrix $\mathbf{Z}$, as in [4]

$$\mathbf{F} = -\tfrac{1}{2}\mathbf{H}_3\mathbf{E}\mathbf{H}_2, \;\; \mathbf{Z} = \mathbf{Y}^{-T}\mathbf{F}.$$

Finally, the missing distance can be recovered from the dummy locations we calculated in $\mathbf{Z}$ for the nodes 4 and 5.

3.2. Distance Matrix Reconstruction and Network Localization

To be able to reconstruct the network distance matrix completely, and subsequently use it in the PEST, we propose to repeat the missing link reconstruction for all the missing links in a local-to-global fashion. Therefore, in every snapshot of the mobile network, we first discover the missing links, then for every pair of nodes with a missing link we try to find three other nodes meeting the requirements explained in Subsection 3.1. Obviously, in sparsely connected networks, there may be two nodes for which we cannot find the three neighboring nodes as explained earlier (irrecoverable missing links). To alleviate this problem, we should always recover the missing links which are recoverable in a first round and in the next round there is a good chance that some of the irrecoverable missing links can be recovered due to previously recovered missing links. We repeat this procedure as long as we can recover some missing links. Notably, as we recover the missing links the probability that we can find more than one group of three nodes meeting the required conditions increases. In those cases, we choose one of these groups which meets the following criterion

$$\arg\min_{g,l} \; \mathrm{SC}_{g,l} \quad g = 1, 2, \cdots, G; \; l = 1, 2, 3, \tag{11}$$

where G denotes the number of possible 3-node neighboring groups and l indicates the index of the chosen edge determined by N_1 and N_2. At the end, if there are still a few missing links not recovered, for mobile networks with slow dynamics, we can always exploit the previously recorded distance measurements (or recovered distance estimates) and use them instead of the shortest path estimate, which hopefully can give us better estimates. This can be further refined by filling the missing distances with r_0 if the previously recorded distance measurement (or recovered distance estimate) for that link is less than r_0 as

$$[\hat{\mathbf{D}}_k]_{i,j} = \begin{cases} [\hat{\mathbf{D}}_{k-1}]_{i,j} & \text{if } (i,j) \text{ is irrecoverable } \& \ [\hat{\mathbf{D}}_{k-1}]_{i,j} > r_0^2, \\ r_0^2 & \text{if } (i,j) \text{ is irrecoverable } \& \ [\hat{\mathbf{D}}_{k-1}]_{i,j} \leq r_0^2. \end{cases} \tag{12}$$

By exploiting this property of mobile networks, we depart from the existing literature that may leave some nodes not localized [6]. The reconstructed distance matrix at the k-th snapshot ($\hat{\mathbf{D}}_k$) will be fed to the PEST to recover the locations of the mobile nodes. The whole process of localization in a partially connected mobile network is shown in Algorithm 1.

4. RECONSTRUCTION COMPUTATIONAL COMPLEXITY

We define the reconstruction computational complexity as the number of operations required to reconstruct one missing link. For the sake of simplicity, we do not count the number of additions and subtractions due to the negligible complexity in comparison with the other operations. Also, we consider the same complexity for multiplications and divisions, and hence, we present the sum of them as the number of floating point operations (FLOPS). The results of the computational complexity for the MLAR, the Nyström, the GLR and the MGLR algorithms are summarized in Table 1. To calculate the total number of FLOPS required, we assume the same methods and complexities as explained in [3] for matrix inverse, scalar square root (SQRT) and EVD computation. As can be seen from the last column of the table, the GLR and the MGLR algorithms have the lowest complexities among all the algorithms under consideration and this makes them preferable for practical implementations, especially for sparsely connected networks with a lot of missing links. Note that this amount of complexity times the number of missing links in a given network yields the total complexity overload imposed by the network distance matrix reconstruction process. It is noteworthy that in the GLR (and MGLR), after fixing the locations of N_1 to N_3 in the relative coordinate system, we could also use them to find the locations of N_4 and N_5 using classical trilateration; however, it requires much higher complexity and thus we prefer the proposed MGLR.

Algorithm 1 Localization in partially connected networks

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Start with an initial location guess
for k = 1 to K (movement steps) do
    Step I: {Reconstruction}
    D̂_k ← D_k
    while no. of recoverable missing links > 0 do
        Look for groups of three appropriate nodes in D̂_k
        Choose one appropriate group and N_1 and N_2 using (11)
        Recover the missing using MGLR and fill D̂_k
    end while
    Complete irrecoverable missing links using (12)
    Step II: {Localization}
    Use D̂_k in PEST to recover the locations
end for
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Table 1. Reconstruction computational complexity

Algorithm	Mult.	SQRT	Matrix inverse	EVD	Tot. FLOPS
MLAR	37	3	1 (3×3)	2 (5×5)	404
Nyström	122	2	-	1 (3×3)	173
GLR	37	5	-	-	97
MGLR	67	5	-	-	127

5. SIMULATION RESULTS

We start by illustrating the effect of the proposed MGLR algorithm on a 5-node link reconstruction setup. The nodes are randomly deployed in an area of 100×100 square meters and the link between N_4 and N_5 is always missing. The result is shown in Fig. 2 where we plot the root mean squared error (RMSE) of missing link reconstruction versus γ. The results are averaged over 50000 Monte Carlo (MC) trials for 50 random configurations of nodes and 1000 realizations of the noise. The results reveal that GLR performs better than the MLAR and the Nystöm. Moreover, the MGLR which exploits the proposed SC outperforms all the other algorithms. Remember that the MGLR has a much lower complexity compared to the MLAR and the Nystöm, as well. In the next simulations, we present the results of exploiting the MLAR, the Nyström and the MGLR in distance matrix reconstruction for anchorless localization of a mobile network as explained in Subsection 3.2 and briefly illustrated in Algorithm 1. To this aim, we consider a network of $N = 10$ mobile

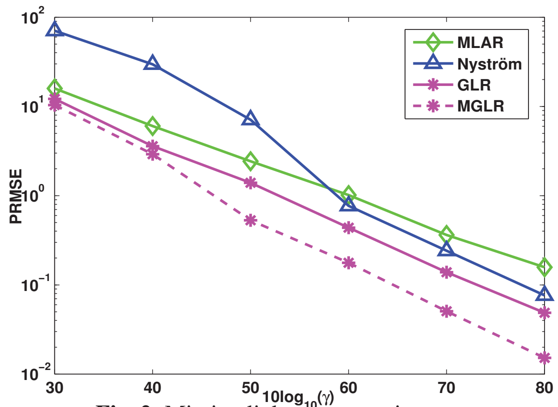

Fig. 2. Missing link reconstruction error

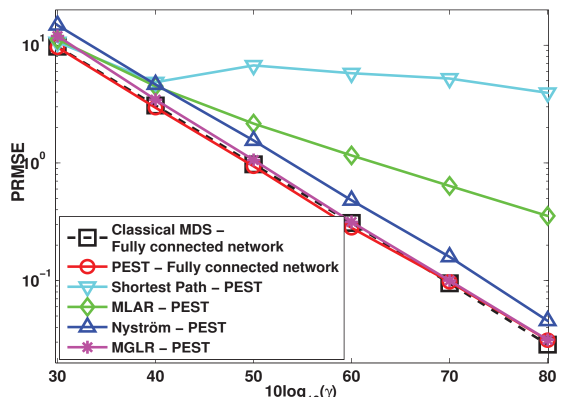

Fig. 3. Results for a partially connected network with $r_0 = 110$m

sensors living inside a 2-dimensional bounded area of 100×100 square meters. Further, to be able to evaluate and plot the results based on the absolute locations, we resolve the unknown translation and orthogonal transformation of our obtained location estimates for all the algorithms by considering 3 anchor nodes. As explained earlier, the distance measurements are impaired by additive distance dependent noise. Note that, for instance, according to (2) at $\gamma = 30$dB we can have a maximum $\sigma_{v,i,j,k} = 100\sqrt{2}/\sqrt{1000} \approx 4.5$m of error on distance measurements. The detail of the movement model is perfectly similar to the explanations in [2, 3] with process noise standard deviation $\sigma_w = 0.1$ and measurement time interval $T_s = 0.1$s. For a quantitative comparison, we define the positioning root mean squared error (PRMSE) of the algorithms at the k-th snapshot as

$$\text{PRMSE} = \sqrt{\frac{\sum_{m=1}^{M}\sum_{n=1}^{N} e_{n,m,k}^2}{M}}, \quad (13)$$

where $e_{n,m,k}$ represents the distance between the real location of the n-th node and its estimated location at the m-th MC trial of the k-th snapshot. All simulations are averaged over $M = 100$ independent MC runs where in each run the nodes move toward random directions starting from random initial locations. Fig. 3 depicts the performance of Algorithm 1 using the MLAR, the Nyström and the MGLR for a partially connected WSN with $r_0 = 110$m (approximately up to 10 missing links). We plot the performance of the classical MDS over the same fully connected network as the lower bound of PRMSE and the PEST over the fully connected network as a base-line algorithm for the sake of comparison [3, 4]. Besides, we also plot the results of using the shortest path algorithm to estimate the missing links in combination with the PEST. The results illustrate that the PEST attains the achievable bound determined by

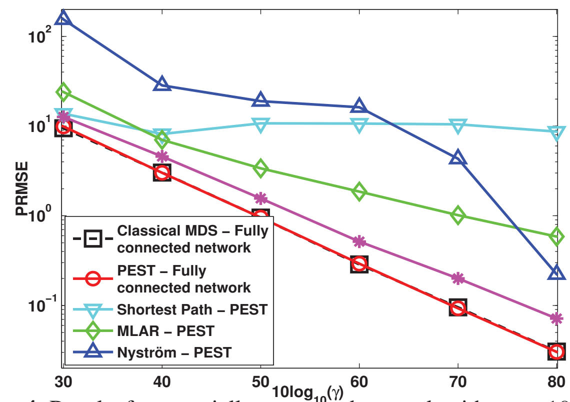

Fig. 4. Results for a partially connected network with $r_0 = 100$m

the classical MDS for the fully connected network. The proposed MGLR algorithm performs the best and is very close to the performance of a fully connected network, which means it is capable of reconstructing up to 10 missing links. Note that the shortest path fails to recover the missing links as it does not show any improvement by increasing γ and also the MLAR performs much worse than the MGLR and the Nyström. Remember that considering the lowest complexity of the MGLR as well as its best accuracy, it is the preferable choice for a partially connected network. Fig. 4 shows the same scenario as in Fig. 3 except for $r_0 = 100$m (approximately up to 14 missing links). It is interesting that while the Nyström shows signs of instability and the MLAR still does not perform well, the MGLR gives the best performance even for a network with $14/\binom{10}{2} > 30\%$ missconnectivity.

6. CONCLUSIONS

We have proposed a geometric link reconstruction algorithm for noisy scenarios. The proposed algorithm is then used in a local-to-global fashion to reconstruct the complete network distance matrix and localize the mobile network. It has been shown that the proposed algorithm has a low computational complexity and outperforms comparable existing approaches in terms of link reconstruction and network localization accuracy in noisy scenarios.

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