Urban Source Localization Based on Time of Arrival Measurement and Street Information

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***Abstract*— In this work, we study the localization of mobile signal emitters using time of arrival (TOA) measurement and additional urban street information. Two algorithms are proposed to improve the localization performance by integrating street information with the TOA measurement. The first algorithm exhaustively searches of all possible road paths. For each possible path, the source location is estimated based a semidefinite programming (SDP) algorithm by minimizing the maximum error measurement between the observed propagation time and the modeled propagation time. Only location on a street that satisfies the minimum mean square error yields estimation output. To reduce complexity, our second joint optimization algorithm combines the two steps together and jointly optimizes the path selection and source location. Numerical results show that both algorithms can improve the localization performance. Our proposed joint optimization approach is more suitable for practical use because of lower complexity and good performance.**

I. Introduction

Source localization has now attracted widely research attention in wireless networks. It has a lot of applications, such as emergency response, mobile gaming, target tracking, signal routing, interference alignment, and wireless security [1] [2] [3]. These applications necessitate that we develop techniques for estimating the location of mobile users in both outdoor and indoor environments. In outdoor environment, global positioning system (GPS) can be used to provide location service. However, it needs support in indoor and some urban situations where the satellite can not be directly found. Therefore, it is necessary to develop other localization techniques besides GPS.

There exist various established methods for source localization that are based on measurement models of received signal time of arrival (TOA), distance measurement, received signal strength (RSS), signal angle of arrival (AOA), and their combinations. In some radio signal applications, distance information is not directly available and must be estimated based on signal measurement such as strength and time of arrival. On the other hand, received signal strength measurements can also be very sensitive to the channel environment. For example, in an environment with rich scatters, signal strength measurement can be difficult to model and relate to the source location information. For these reasons, other measurement models may be more practical. In this work, we are particularly interested in the simple model based on received signals' time of arrival measurement.

In most radio environments with direct line-of-sight radio link or with scatters close to the source or sensors, the TOA measurement is a direct function of the distance between the source and the sensor as the radio propagation velocity is well known. One practical obstacle is the typical lack of synchronization between the source and the receiver. In other words, the receivers often are not aware of the precise starting time instant of source transmission t_0. Therefore, the source location and t_0 should be estimated jointly. Several existing works assume source sensor coordination such that t_0 is known to the sensors [4] [5]. However, by requiring cooperation between the source and the sensors, such assumption severely limits the practical application of related TOA algorithms.

In addition to the above mentioned measurements directly obtained from the signal transmitted between the source and receiver, there is some additional information that can be used to assist source localization. For example, in cellular networks, Cell-ID can provide a rough location estimation of the mobile user based on the cell size. From the Cell-ID, we can obtain information about which area the mobile user is in. Furthermore, the map of that area is a priori known to us, and there is a limit number of mobile streets (paths) for the mobile unit in that area. The mobile user should be on one of the streets (paths). Therefore, the additional roadmap information can be used to assist source localization.

In this work, we will develop source localization algorithms that utilize the practical TOA measurement with unknown t_0. We integrate into our problem formulation a priori street (path) information in areas where the source may reside. We present two methods. The first one is a straightforward high complexity algorithm that exhaustively test all possible mobile paths. We estimate the source location in each path based on a semidefinite programming (SDP) algorithm by minimizing the maximum error measurement between the observed propagation time and the modeled propagation time. Our second algorithm integrates the location estimate and the most likely path decision into a joint estimation problem. By comparison, the second method is less computationally intensive. Both methods are shown to be effective.

The remainder of the paper is organized as follows. In Section II, we describe the system model we use in this paper. Section III presents two different localization approaches for the model. In Section IV, we illustrate the numerical results, followed by the conclusion in Section V.

II. Problem Statement

Consider the scenario that there are N anchor nodes with location $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$ in a 2-D field, where $\mathbf{x}_i = [x_{i1}\ x_{i2}]^T$. These anchor nodes cooperate by helping a data fusion center (DFC) determine an unknown source location $\mathbf{y}$. Given an LOS propagation path, the time of arrival measurement t_i at anchor node $\mathbf{x}_i$ can be easily modeled as

$$t_i = \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| + t_0 + n_i, \qquad i = 1, 2, \cdots, N, \quad (1)$$

where c is the speed of light, $\|\cdot\|$ denotes the Euclidean norm, t_0 is the unknown time instant at which the source transmits the signal to be measured, and n_i is the additive measurement noise (error) with zero mean.

We assume that we have the additional roadmap information near the source, i.e. the source $\mathbf{y} = [y_1\ y_2]^T$ must be on one of the K paths (streets) in that area. We assume that the paths are straight lines with width δ_0, which can be denoted by

$$\begin{array}{c} y_2 = a_{11}y_1 + a_{12} + a_{13}\delta, \\ y_2 = a_{21}y_1 + a_{22} + a_{23}\delta, \\ \cdots \\ y_2 = a_{K1}y_1 + a_{K2} + a_{K3}\delta, \end{array} \quad (2)$$

where $|\delta| \leq \frac{1}{2}\delta_0$, and $a_{i3} = \sqrt{a_{i1}^2 + 1}$ for $i = 1, \cdots, K$. Let $\mathbf{p} = [p_1\ p_2\ \ldots\ p_K]^T$ be the vector to indicate which path the source is in, where $p_i = 0$ or $p_i = 1$ for $i = 1, \ldots, K$, and $\sum_{i=1}^{K} p_i = 1$. Then the relation between y_1 and y_2 can be expressed as:

$$y_2 = \begin{bmatrix} y_1 & 1 & \delta \end{bmatrix} \cdot A \cdot \mathbf{p}, \quad (3)$$

where $A = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{K1} \\ a_{12} & a_{22} & \cdots & a_{K2} \\ a_{13} & a_{23} & \cdots & a_{K3} \end{bmatrix}$.

Without any other prior assumptions on the statistics of the TOA measurements, a least square (LS) estimator can be used for the source localization problem. The estimator is to minimize the following mean square error:

$$F = \sum_{i=1}^{N} \left(t_i - \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| - t_0 \right)^2. \quad (4)$$

Combining the constraints, we have the following LS problem:

$$\begin{array}{c} \min\limits_{\mathbf{y}, t_0, \mathbf{p}, \delta} \sum\limits_{i=1}^{N} \left(t_i - \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| - t_0 \right)^2 \\ \text{s.t. } \mathbf{y} = [y_1\ y_2]^T, \\ y_2 = \begin{bmatrix} y_1 & 1 & \delta \end{bmatrix} \cdot A \cdot \mathbf{p}, \\ |\delta| \leq \frac{1}{2}\delta_0, \\ p_i = 0 \ or \ p_i = 1, \ for \ i = 1, \ldots, K, \\ \sum\limits_{i=1}^{K} p_i = 1. \end{array} \quad (5)$$

This is a non-convex mixed integer nonlinear optimization problem. In the next section, we will present two algorithms to solve this complicated problem.

III. Localization Algorithm

Since (5) is a non-convex mixed integer nonlinear optimization problem, it is difficult to solve it directly. One approach is an exhaustive search method. We estimate the source location for each path based on a SDP algorithm [6], and choose the location with the minimum mean square error as the final output. A computationally less intensive alternative is to relax some constraints in (5) to estimate mobile path and location together. We now describe these two methods.

A. K-min Search

The LS formulation (5) is optimum in the maximum likelihood sense when the TOA measurement noise is assumed to be i.i.d. Gaussian. In practice, however, TOA measurement noise may exhibit different characteristics. Therefore, there is strong incentive for us to develop effective localization algorithms that are less dependent of noise assumptions.

Steering away from the LS objective function, we can rewrite the TOA measurement of (1) into

$$t_i - t_0 = \frac{1}{c}\|\mathbf{x}_i - \mathbf{y}\| + n_i. \quad (6)$$

Squaring in both sides, we get

$$(t_i - t_0)^2 - \frac{1}{c^2}\|\mathbf{x}_i - \mathbf{y}\|^2 = \underbrace{(\frac{2}{c}\|\mathbf{x}_i - \mathbf{y}\| + n_i)n_i}_{\omega_i}, \quad (7)$$

for $i = 1, \cdots, N$. The right side of (7) is a noise term ω_i that is not independent for different indices i. At modest to high SNR, $\frac{2}{c}\|\mathbf{x}_i - \mathbf{y}\|$ dominates n_i to allow $\omega_i \approx \frac{2}{c}\|\mathbf{x}_i - \mathbf{y}\| n_i$.

One way to estimate the optimum $\mathbf{y}$ without assuming any particular characteristics on ω_i is to minimize the ℓ_∞ norm of ω_i. This approach simply tries to minimize the peak error without making any assumption on the noise distribution or on the noise correlation. Therefore its performance is expected to be less sensitive to the noise distribution or correlation [6]. We propose to adopt the min-max criterion for location estimation

$$\hat{\mathbf{y}} = \arg\min_{\mathbf{y}, t_0} \max_{i=1,\cdots,N} \left| (t_i - t_0)^2 - \frac{1}{c^2}\|\mathbf{x}_i - \mathbf{y}\|^2 \right|. \quad (8)$$

.

Note that this min-max formulation (8) is a non-convex problem. We need to do some relaxations below. First, let us introduce two auxiliary variables $y_s = \mathbf{y}^T\mathbf{y}$ and $t_s = t_0^2$. Then we have

$$\begin{aligned} \hat{\mathbf{y}} = \arg \min_{\mathbf{y}, y_s, t_0, t_s} \max_{i=1,\cdots,N} & |t_s - 2t_i t_0 + t_i^2 \\ & - \frac{1}{c^2}(y_s - 2\mathbf{x}_i^T\mathbf{y} + \mathbf{x}_i^T\mathbf{x}_i)|, \end{aligned} \quad (9)$$

which is a convex function in terms of variables $\mathbf{y}$, y_s, t_0, and t_s. However, the two equality constraints $y_s = \mathbf{y}^T\mathbf{y}$ and $t_s = t_0^2$ are not convex and need to be relaxed into approximate convex constraints. In order to transform the problem formulation into a convex optimization problem, we introduce two convex relaxations on the equality constraints. Specifically, we relax the two equalities $y_s = \mathbf{y}^T\mathbf{y}$ and $t_s = t_0^2$ into inequalities $y_s \succeq \mathbf{y}^T\mathbf{y}$ and $t_s \succeq t_0^2$, respectively. Both inequalities can be conveniently expressed in terms of linear matrix inequalities:

$$\begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \succeq 0, \begin{bmatrix} 1 & t_0 \\ t_0 & t_s \end{bmatrix} \succeq 0. \tag{10}$$

For a given $\mathbf{p}$, we have the following SDP convex optimization problem:

$$\begin{aligned} &\min_{\mathbf{y}, y_s, t_0, t_s, \delta} \theta \\ \text{s.t. } & -\theta \le t_s - 2t_i t_0 + t_i^2 - \tfrac{1}{c^2}(y_s - 2\mathbf{x}_i^T \mathbf{y} + \mathbf{x}_i^T \mathbf{x}_i) \le \theta, \\ & i = 1, \cdots, N, \\ & \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \succeq 0, \begin{bmatrix} 1 & t_0 \\ t_0 & t_s \end{bmatrix} \succeq 0, \\ & \mathbf{y} = [y_1 \; y_2]^T, \\ & y_2 = \begin{bmatrix} y_1 & 1 & \delta \end{bmatrix} \cdot A \cdot \mathbf{p}, \\ & |\delta| \le \tfrac{1}{2}\delta_0. \end{aligned} \tag{11}$$

Because there are K choices of $\mathbf{p}$ in total, we propose the K-min algorithm by solving (11) for different $\mathbf{p}$ using interior point methods such as SeDuMi [7]. The final location estimate is selected by identifying the SDP result that generates the minimum objective function value (4).

B. Joint Optimization Solution

We noted that the K-min search approach needs to solve the optimization problem (11) for K times. The complexity may be higher if the suspected area of the mobile unit is large and involves multiple street intersections. Alternatively, we can solve $\mathbf{p}$, $\mathbf{y}$, and t_0 together in (5) by relaxing the constraints. We now present our joint optimization solution (JOS).

Assuming $\mathbf{z} = [z_1, \cdots, z_K]^T$, where $z_i = p_i y_1$. Let $\mathbf{a}_1 = [a_{11}, \cdots, a_{K1}]^T$, $\mathbf{a}_2 = [a_{12}, \cdots, a_{K2}]^T$, $\mathbf{a}_3 = [a_{13}, \cdots, a_{K3}]^T$, $\mathbf{u} = [1, \cdots, 1]^T$, then we have

$$\begin{aligned} y_1 &= \mathbf{u}^T \mathbf{z}, \\ \mathbf{y} &= [\mathbf{u}^T \mathbf{z}, \mathbf{a}_1^T \mathbf{z} + \mathbf{a}_2^T \mathbf{p} + \mathbf{a}_2^T \delta]^T. \end{aligned}$$

We relax the original LS optimization problem (5) to the following formulation:

$$\begin{aligned} &\min_{\mathbf{y}, \mathbf{z}, \mathbf{p}, y_s, t_0, t_s, \delta} \theta \\ \text{s.t. } & -\theta \le t_s - 2t_i t_0 + t_i^2 - \tfrac{1}{c^2}(y_s - 2\mathbf{x}_i^T \mathbf{y} + \mathbf{x}_i^T \mathbf{x}_i) \le \theta, \\ & i = 1, \cdots, N, \\ & \begin{bmatrix} \mathbf{I} & \mathbf{y} \\ \mathbf{y}^T & y_s \end{bmatrix} \succeq 0, \begin{bmatrix} 1 & t_0 \\ t_0 & t_s \end{bmatrix} \succeq 0, \\ & \mathbf{y} = [\mathbf{u}^T \mathbf{z} \;\; \mathbf{a}_1^T \mathbf{z} + \mathbf{a}_2^T \mathbf{p} + \mathbf{a}_2^T \delta]^T, \\ & |\delta| \le \tfrac{1}{2}\delta_0, \\ & 0 \le p_i \le 1, \; for \; i = 1, \ldots, K \\ & \sum_{i=1}^{K} p_i = 1, \\ & \begin{bmatrix} \mathbf{I} & \mathbf{p} \\ \mathbf{p}^T & 1 \end{bmatrix} \succeq 0. \end{aligned} \tag{12}$$

Once again, the joint optimization formulation (12) can be solved using SeDuMi [7].

Compared with $K-$min, JOS can estimate the source location by solving one SDP instead of K. Therefore, its complexity is much lower than the $K-$min search formulation. However, JOS requires more relaxations and this may induce some performance loss. We will compare the performance of these two approaches in the next section.

IV. Numerical Result

In this section, we compare the performance of proposed algorithms in Section III. We label the two algorithms as $K-$min and JOS in our simulation results. In addition, we also compare the performance of the proposed algorithms with the min-max TOA algorithm in [6] without exploiting the additional path information, labeled as "Simple TOA". We also include the Cramér-Rao lower bound (CRLB) as a reference.

In the following examples, we place eight sensors in a 2-dimensional area at $\mathbf{x}_1 = [400, 400]^T$, $\mathbf{x}_2 = [400, -400]^T$, $\mathbf{x}_3 = [-400, 400]^T$, $\mathbf{x}_4 = [-400, -400]^T$, $\mathbf{x}_5 = [800, 800]^T$, $\mathbf{x}_6 = [800, -800]^T$, $\mathbf{x}_7 = [-800, 800]^T$, $\mathbf{x}_8 = [-800, -800]^T$. We evaluate the mean squared error (MSE) of the source location as the performance metric against different strengths of the noise standard deviation. For simplicity, we convert the noise into the distance domain.

Example 1: In this example, the source is possibly in one of the 3 paths, where

$$A = \begin{bmatrix} 100 & -1 & 0 \\ 0 & -100 & -50 \\ 100 & 1.414 & 1 \end{bmatrix}.$$

The source is located at $[0, -100]^T$ on the second path, and $\delta_0 = 4$. In Fig. 1, we show the performance of different approaches as well as the CRLB. $K-$min clearly gives best performance in this example. The JOS algorithm delivers slightly worse results than $K-$min, but performs far better than the pure TOA algorithm without the path information. Therefore, we find that additional roadmap information can substantially improve the localization performance.

Example 2: In this example, we assume there are 6 paths, where

$$A = \begin{bmatrix} 100 & 1.5 & -1 & 0 & -20 & 12 \\ 0 & -50 & -100 & -50 & 0 & 30 \\ 100 & 1.803 & 1.414 & 1 & 20.025 & 12.042 \end{bmatrix},$$

and $\delta_0 = 4$. The source is located at $[0, -100]^T$ on the third path. In Fig. 2, we illustrate the performance of different algorithms. We can see the gap between $K-$min and JOS becomes less evident. Both are better than the simple TOA algorithm. Since there are 6 possible traveling paths, the objective function value (4) can be very close to one other. Thus, sometimes even $K-$min may fail to find the correct path. For this reason, the results of K-min and JOS are similar.

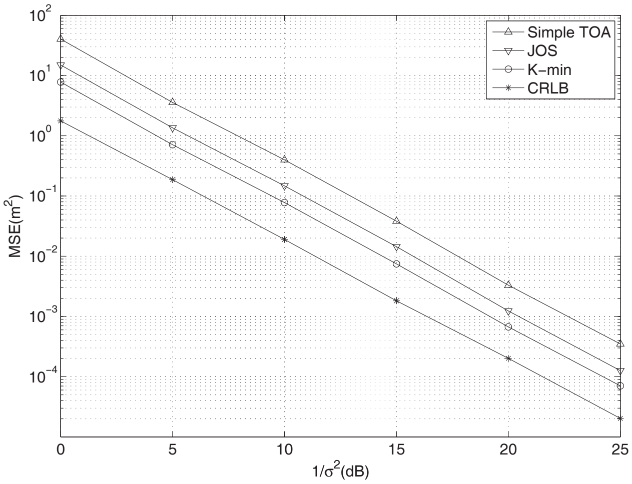

Fig. 1. Comparison of different localization schemes, 3 paths case.

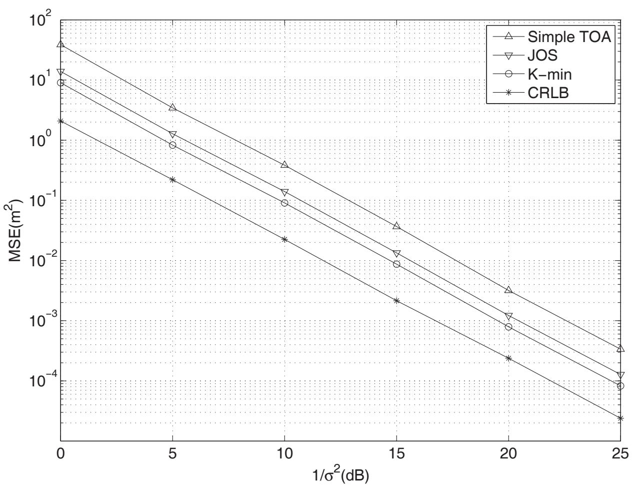

Fig. 2. Comparison of different localization schemes, 6 paths case.

Example 3: In this example, there are still 6 paths as in Example 2 but the source is at $[0, -50]^T$, which is the intersection of the second and the fourth path. The performance of different algorithms is shown in Fig. 3. We can see in this case, the performance of the two new methods are very close. In addition, we observed that the objective function value (4) for $K-$min is very close for the second and the fourth linear paths. And the associate p_i for JOS is close 0.5. Therefore, our two algorithm have very similar performance.

V. Conclusion

We study the problem of source localization based on time of arrival model with additional roadmap information. We present two algorithms to improve the localization performance by exploiting the TOA measurement and additional street path information jointly. Our first algorithm $K-$min exhaustively searches of all the possible paths, and selects path and the resulting location that admits the output error. Our

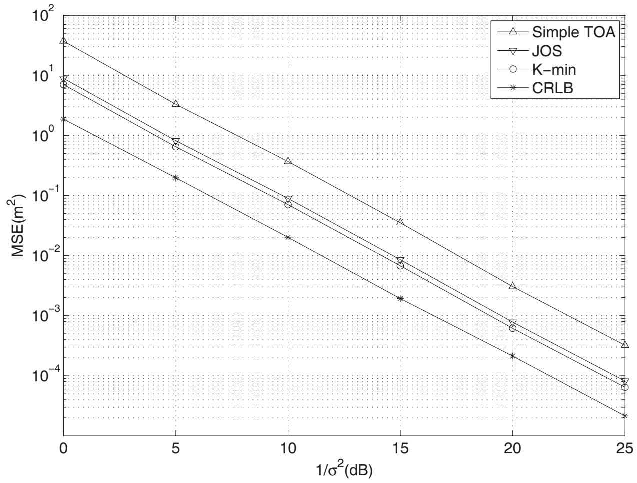

Fig. 3. Comparison of different localization schemes, 6 paths case.

JOS algorithm reduces the algorithm complexity by jointly optimizing the path selection and source location estimate. We present numerical results to illustrate the effective performance by both proposed algorithm in improving the localization performance.

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