

# STOCHASTIC LOCALIZATION OF CBRN RELEASES

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## ABSTRACT

We present a novel methodology for chemical, biological, radiological, or nuclear (CBRN) source localization in urban environments. Our approach uses only information reported by CBRN sensors monitoring the environment and thus does not rely on solving any challenging inverse plume dispersion problems. We illustrate and evaluate our technique using three-dimensional CBRN release simulations.

**Index Terms**— CBRN detection, source localization, Markov chains, optimization, sensor placement

## 1. INTRODUCTION

Concern regarding chemical, biological, radiological, or nuclear (CBRN) terrorism is steadily increasing. This is due to gains in the technological capabilities of producing existing and potentially new, more lethal CBRN agents and delivery mechanisms. Urban areas are of particular concern since these areas tend to have large population densities and are centers for large-scale commerce and politics [1].

Detecting and responding to CBRN terrorism within urban areas has its own unique challenges. A typical city tends to be characterized by irregular geometry, many different types of surfaces, and highly dynamic population fluxes. All of these properties make analytic approaches to air flow modeling and agent particle transport difficult. In addition, as cities contain many centers for commerce and dense gatherings of people, accurate release localization and quick response times are of grand importance.

Existing CBRN source localization approaches [2], [3], [4] observe CBRN agent concentrations and solve the inverse problem of tracing dispersion backward in time and space to the source of the release. Limitations of this methodology stem from the irregular and dynamic phenomena typically

found in urban areas. Lack of homogeneity makes modeling of dispersion, a critical component of the approach, difficult.

In the following, we propose a methodology to CBRN source localization. This problem is approached solely on discrete time sequences of observations made by CBRN sensors monitoring the environment through a process of hypothesis testing. Our methodology is model independent, robust to low fidelity modeling, and potentially more accurate in scenarios with highly variable weather conditions. Further, we are able to characterize error probabilities which provide both performance guarantees as well as a conduit to solving the optimal sensor placement problem.

## 2. PROBLEM FORMULATION

Consider a chemical, biological, radiological, or nuclear (CBRN) sensor network deployed in an urban setting. Due to the nature of plume dispersion and existing CBRN attack response techniques, extreme precision on the locations of releases and sensors is not needed. Rather, general locations (e.g., corner of X St. and Y Ave.) suffice for disaster avoidance and response measures. Using this concept, release and potential sensor locations can be discretized to conform to potentially irregular grids. In the following, we assume  $N$  possible CBRN release scenarios represented by the set  $\mathcal{L} = \{L_1, \dots, L_N\}$  and  $M$  possible CBRN sensor locations represented by the set  $\mathcal{B} = \{B_1, \dots, B_M\}$ . The elements of  $\mathcal{L}$  represent at the least a release location but in the following each scenario is assumed to have wind conditions (e.g., wind originating from the N with mean velocity of 3 m/s) associated with it. Wind is a critical component of a release scenario as it is the primary mode of particulate transport.

Let  $\mathbf{y}^i$  denote the vector of sensor observations made by a sensor at location  $B_i$ . Each sensor outputs a CBRN agent concentration estimation and potentially local wind observations. Since different concentrations result in different levels of casualties, counter measures, and responses, observed concentrations can naturally be discretized to concentration levels (e.g.,  $[0, x]$ ,  $[x, y_{LCT50}]$ , etc., where  $y_{LCT50}$  represents a concentration that would lead to an accumulated dosage that produces a 50% chance of survival.) Each  $\mathbf{y}^i$  is mapped to

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a symbol in the finite alphabet  $\Sigma = \{\sigma_1, \dots, \sigma_{|\Sigma|}\}$ . A sequence of  $n$  sensor observations at location  $B_i$  is denoted  $\mathbf{y}^{i,n} = (\mathbf{y}_1^i, \dots, \mathbf{y}_n^i)$ . It is assumed throughout the sequel that an initial state of  $\mathbf{y}_0$  is known a priori.

A CBRN release scenario  $L_j$  is associated with a family of parameterizations  $\Omega_j$  which correspond to variability in weather conditions, amount of agent released, time of day, and time since attack. While not under an attack scenario, a sensor's agent concentration approximation fluctuations are attributed to measurement noise. Accordingly, when there is no CBRN attack, the sensor's concentration readings can be assumed to be independent and identically distributed (iid).

Under a release scenario, however, the agent disperses gradually, resulting in concentration observations that change much slower than a sensor's sampling rate. That is, if a certain concentration level is observed by a sensor at a certain time step, one can expect to see a similar concentration observation in the next time step. Accordingly we model concentration level at a sensor when there is an attack as a first-order Markov chain.

Allow  $M_2(\Sigma \times \Sigma)$  to denote the set of all discrete time Markov transition probability matrices on the state alphabet  $\Sigma = \{\sigma_1, \dots, \sigma_{|\Sigma|}\}$ . For each sensor location, release scenario pair  $(B_i, L_j)$ , associate a series of first-order Markov transition probability matrices  $\mathbf{\Pi}_{\theta_j}, \forall \theta_j \in \Omega_j$ , defined as

$$\mathbf{\Pi}_{\theta_j}^i = \left\{ \pi_{\theta_j}^i(\sigma_v, \sigma_u) \right\}_{u,v=1}^{|\Sigma|}, \quad (1)$$

where  $\pi_{\theta_j}^i(\sigma_v, \sigma_u) = \mathbf{P}[\mathbf{y}_{r+1} = \sigma_u | \mathbf{y}_r = \sigma_v]$ .

Under the assumption of a Markov source, the empirical distribution of a sequence  $\mathbf{y}^n = (\mathbf{y}_1, \dots, \mathbf{y}_n)$  of sensor observations takes the form  $\mathbf{Q}_{\mathbf{y}^n} = \{q_{\mathbf{y}^n}(\sigma_v, \sigma_u)\}_{u,v=1}^{|\Sigma|}$ , where

$$q_{\mathbf{y}^n}(\sigma_v, \sigma_u) = \frac{1}{n} \sum_{i=1}^n \mathbf{1}\{\mathbf{y}_{i-1} = \sigma_v, \mathbf{y}_i = \sigma_u\}. \quad (2)$$

The conditional empirical probability of transitioning from state  $\sigma_v$  to  $\sigma_u$ , as derived from the sequence  $\mathbf{y}^n$ , is

$$q_{\mathbf{y}^n}(\sigma_u | \sigma_v) = \frac{q_{\mathbf{y}^n}(\sigma_v, \sigma_u)}{q_{\mathbf{y}^n}(\sigma_v)},$$

where  $q_{\mathbf{y}^n}(\sigma_v) = \sum_{u=1}^{|\Sigma|} q_{\mathbf{y}^n}(\sigma_v, \sigma_u)$ . A convex relative entropy measure for Markov chains against a Markov source  $\mathbf{\Pi}$  is given by [6]

$$I_2(\mathbf{Q} | \mathbf{\Pi}) = \sum_{v=1}^n q(\sigma_v) H_2(q(\cdot | \sigma_v) | \pi(\sigma_v, \cdot))$$

where  $H_2(q(\cdot | \sigma_v) | \pi(\sigma_v, \cdot)) = \sum_{u=1}^{|\Sigma|} q(\sigma_u | \sigma_v) \log \frac{q(\sigma_u | \sigma_v)}{\pi(\sigma_u | \sigma_v)}$ .

Consider the subproblem of using the sensor readings from a sensor at location  $B_k$  to determine if the CBRN release corresponds to scenario  $L_i$  or  $L_j$ . The Generalized

Likelihood Ratio Test (GLRT) compares the normalized generalized log-likelihood ratio

$$X_{ijk}(\mathbf{y}^{k,n}) = \frac{1}{n} \log \frac{\sup_{\theta_i \in \Omega_i} P_{\theta_i}^k(\mathbf{y}^{k,n})}{\sup_{\theta_j \in \Omega_j} P_{\theta_j}^k(\mathbf{y}^{k,n})}$$

to a threshold  $\lambda$  and declares  $L_i$  to be the release scenario whenever

$$\mathbf{y}^{k,n} \in \mathcal{S}_{ijk,n}^{GLRT} = \{\mathbf{y}^n | X_{ijk}(\mathbf{y}^n) \geq \lambda\}, \quad (3)$$

and  $L_j$  otherwise. Selection of the value for  $\lambda$  in (3) is conducted through evaluation of decision error probabilities.

The GLRT described in (3) can make two types of error. Namely, the decision test can declare  $L_j$  to be the release scenario when in actuality the release scenario was  $L_i$  and vice versa. The probabilities of these errors are represented as,

$$\begin{aligned} \alpha_{ijk,n}^{GLRT}(\theta_j) &= \mathbf{P}_{\theta_j}[\mathbf{y}^{k,n} \in \mathcal{S}_{ijk,n}^{GLRT}], \\ \beta_{ijk,n}^{GLRT}(\theta_i) &= \mathbf{P}_{\theta_i}[\mathbf{y}^{k,n} \notin \mathcal{S}_{ijk,n}^{GLRT}], \end{aligned}$$

where  $\mathbf{P}_{\theta}$  is a probability evaluated assuming that  $\mathbf{y}^{k,n}$  is drawn from  $P_{\theta}^k(\cdot)$ .

Certainly some values of  $\lambda$  in (3) are better than others. To evaluate the GLRT's performance, we will use the generalized Neyman-Pearson criterion.

**Definition 1** The decision rule  $\{\mathcal{S}_{ijk,n}\}$  is generalized Neyman-Pearson (GNP) optimal if it satisfies

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \log \alpha_{ijk,n}^{\mathcal{S}}(\theta_j) < \lambda, \forall \theta_j \in \Omega_j$$

and maximizes  $-\limsup_{n \rightarrow \infty} \frac{1}{n} \log \beta_{ijk,n}^{\mathcal{S}}(\theta_i)$  uniformly  $\forall \theta_i \in \Omega_i$ .

The following theorem, whose proof has been omitted for brevity, establishes the optimality of the GLRT in the Neyman-Pearson sense.

**Theorem 1** The GLRT with a threshold  $\lambda$  is asymptotically optimal under the GNP criterion if and only if

$$\inf_{\mathbf{Q} \in \mathcal{C}_{ijk}} I_2(\mathbf{Q} | \mathbf{\Pi}) \geq \inf_{\mathbf{Q} \in \mathcal{A}_{ijk}} I_2(\mathbf{Q} | \mathbf{\Pi}) \forall \theta_i \in \Omega_i \quad (4)$$

where  $\mathcal{A}_{ijk} = \{\mathbf{Q} | \inf_{\theta_j} I_2(\mathbf{Q} | \mathbf{\Pi}_{\theta_j}) < \lambda\}$  and

$$\begin{aligned} \mathcal{C}_{ijk} &= \{\mathbf{Q} | \inf_{\theta_j \in \Omega_j} I_2(\mathbf{Q} | \mathbf{\Pi}_{\theta_j}) - \\ &\quad \inf_{\theta_i \in \Omega_i} I_2(\mathbf{Q} | \mathbf{\Pi}_{\theta_i}) \leq \inf_{\theta_j \in \Omega_j} I_2(\mathbf{Q} | \mathbf{\Pi}_{\theta_j})\}. \end{aligned}$$

Furthermore, assuming (4) is satisfied

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \log \alpha_{ijk,n}^{GLRT}(\theta_j) \leq -\lambda,$$

for all  $\theta_j \in \Omega_j$ , and

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \log \beta_{ijk,n}^{GLRT}(\theta_i) \leq - \inf_{\mathbf{Q} \in \mathcal{A}_{ijk}} I_2(\mathbf{Q} | \mathbf{\Pi}),$$

for all  $\theta_i \in \Omega_i$ .

Now, in the event that the GLRT is not asymptotically optimal (i.e., condition (4) does not hold,) information on the error probabilities can still be gleaned. Define the set  $\mathcal{D}_{ijk} = \{\mathbf{Q} \mid \inf_{\theta_j} I_2(\mathbf{Q} \mid \mathbf{\Pi}_{\theta_j}) - \inf_{\theta_i} I_2(\mathbf{Q} \mid \mathbf{\Pi}_{\theta_i}) < \lambda\}$ . Note,

$$\mathbf{y}^{k,n} \in \mathcal{S}_{ijk,n}^{GLRT} \Leftrightarrow \mathbf{Q}_{\mathbf{y}^{k,n}} \notin \mathcal{D}_{ijk}.$$

So, by Theorem 1, we have

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \log \alpha_{ijk,n}^{GLRT}(\theta_j) \leq - \inf_{\mathbf{Q} \notin \mathcal{D}_{ijk}} I_2(\mathbf{Q} \mid \mathbf{\Pi}_{\theta_j}), \quad \forall \theta_j \in \Omega_j$$

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \log \beta_{ijk,n}^{GLRT}(\theta_i) \leq - \inf_{\mathbf{Q} \in \mathcal{D}_{ijk}} I_2(\mathbf{Q} \mid \mathbf{\Pi}_{\theta_i}), \quad \forall \theta_i \in \Omega_i.$$

We can therefore determine the asymptotic bound on the natural logarithm of  $\beta_{ijk,n}^{GLRT}$  for any  $\theta_i \in \Omega_i$  by the nonlinear program

$$Z_{ijk}(\lambda, \theta_i) = \min_{\mathbf{Q}} I_2(\mathbf{Q} \mid \mathbf{\Pi}_{\theta_i})$$

$$\text{st } \begin{aligned} g(\mathbf{Q}; \Omega_i, \Omega_j) &\leq \lambda \\ \mathbf{A}\mathbf{q} &\leq \mathbf{0}, \end{aligned} \quad (5)$$

where

$$g(\mathbf{Q}; \Omega_i, \Omega_j) = \min_{\theta_j \in \Omega_j} I_2(\mathbf{Q} \mid \mathbf{\Pi}_{\theta_j}) - \min_{\theta_i \in \Omega_i} I_2(\mathbf{Q} \mid \mathbf{\Pi}_{\theta_i}).$$

The constraints that  $\mathbf{Q}$  be shift invariant and represent valid transition probabilities for a Markov chain are linear and are represented by the system of linear inequalities, where  $\mathbf{q}$  is a ‘‘vectorization’’ of  $\mathbf{Q}$ .

Using (5),  $Z_{ijk}(\lambda) = \min_{\theta_i \in \Omega_i} Z_{ijk}(\lambda, \theta_i)$  is the exponent of the worst case probability of the GLRT, using information from sensor  $k$ , declaring the release scenario is  $L_i$  when the actual release scenario is  $L_j$ . Likewise,  $Z_{jik}(\lambda)$  is the exponent of the worst case probability of the GLRT declaring release scenario  $L_j$  when the actual release scenario is  $L_i$ .

Since  $Z_{ijk}(\cdot)$  is a nonincreasing, non-negative function, with  $\lim_{\lambda \rightarrow \infty} Z_{ijk}(\lambda) = 0$ , there exists  $\lambda_{ijk}^* \geq 0$  such that  $Z_{ijk}(\lambda_{ijk}^*) = \lambda_{ijk}^*$ . Allow

$$d_{ijk} = \max\{\lambda_{ijk}^*, \lambda_{jik}^*\} \quad (6)$$

and let  $P_{ijk,n}^{(e)}$  represent the GLRT’s maximum probability of error. The above leads to the following proposition.

**Proposition 1** *Suppose the sensor at  $B_k$  uses the GLRT and compares  $X_{ijk}(\mathbf{y}^{k,n})$  to  $d_{ijk}$ . Then, the maximum probability of error satisfies*

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \log P_{ijk,n}^{(e)} \leq -d_{ijk}.$$

So far all of the results presented for the source localization problem have concerned the binary GLRT in (3) when the real problem is to select a location that appears within the

scenarios in  $\mathcal{L}$ . One approach is to make a series of decisions of the form (3) until a single release location has been chosen.

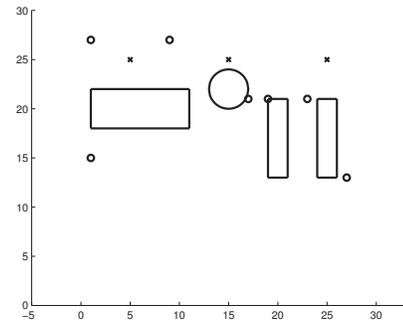
Assume without loss of generality that the sensors are placed at locations  $B_1, \dots, B_K$ . Using the observations from sensor  $B_{k^*}$ , where  $k^* = \operatorname{argmax}_k d_{12k}$ , a decision is made between scenarios  $L_1$  and  $L_2$  via the GLRT with threshold  $d_{12k^*}$ , as established in (3) and (6). This scenario is then compared with  $L_3$  by the GLRT, and so on, until  $N - 1$  decisions have been made and only one scenario from  $\mathcal{L}$  is accepted. The location associated with this selected scenario is declared the CBRN source location.

### 3. SENSOR PLACEMENT

Now that performance guarantees have been established for each sensor location in  $\mathcal{B}$ , the question of where to place  $K < M$  sensors can be addressed. Using the bound in Proposition 1, we can select the  $K$  elements of  $\mathcal{B}$  such that the worst case error when deciding between any two release scenarios in  $\mathcal{L}$  is minimized. This is done, as in [5], via a mixed integer linear program (MILP) which, although shown to be NP-hard, can be solved efficiently for large sets  $\mathcal{B}$  and  $\mathcal{L}$  using a special purpose algorithm [7].

### 4. NUMERICAL EXPERIMENTATION

To demonstrate the performance of the sensor placement approach of Section 3 and the CBRN source localization of Section 2, we simulated several release scenarios in the Los Alamos developed Quick Urban & Industrial Complex (QUIC) Dispersion Modeling System [8]. QUIC first solves the fluid dynamics problem of determining local wind eddies throughout a modeled three-dimensional, outdoor setting using the methods of R ockle [9]. Using the fluid flow solution, QUIC simulates the travel of CBRN particulates via Lagrangian random walk. Previously, the QUIC codes have been tested and validated for real-world situations [8].



**Fig. 1.** Model with optimal sensor placement solution.

In the following demonstrations, a three-dimensional model with several large obstructions (i.e., buildings in an urban setting) was modeled in QUIC. CBRN point releases originating from three release locations were simulated with wind originating from NNW, N, or NNE. 225 potential sensor locations are considered on an evenly spaced grid near ground level in the model. Figure 1 illustrates a cross section of the model with the locations of the obstructions (larger shapes) and releases (“x”s).

The probability laws for each combination of wind direction, release location, and sensor location are determined by Monte Carlo simulation. First sensor concentration observations are encoded into binary outputs indicating the presence of non-nil concentration. Considering the binary nature of the sensor readings, sensor measurement noise is accomplished via Bernoulli trials which result in flipping sensor observations. This sensor noise model was parameterized to result in one sensor observation per CBRN event to be erroneous on average. The resulting noisy sensor readings provide a wealth of observations from which probability laws can be established, allowing for solution of (5) and the determination of the  $d_{i,j,k}$  in (6). Analysis of the worst case probability of error by the number of sensors deployed revealed no improvement once seven sensors are utilized. The optimal placement solution is depicted in Figure 1 as the small circles.

A test set is constructed for each release location by first selecting a wind direction according to a “wind rose” which describes the likelihood of each wind direction. Then, following the same procedure as the simulations from which the probability laws were derived, test sensor observations are generated.

The ability of the localization methodology was also evaluated on each test scenario. The confusion matrix is presented in Table 1. The release locations are referred to as “L” for the left release location, “M” for the middle release location, and “R” for the right release location. The row corresponding to a particular location depicts the percentage of test scenarios determined to originate from each possible location. In total, all test locations were placed correctly with greater than 90% accuracy.

	L	M	R
L	0.91	0.08	0.01
M	0.01	0.95	0.04
R	0.00	0.03	0.97

**Table 1.** Confusion matrix for localization methodology.

## 5. CONCLUSION

We have presented an approach for CBRN source localization through a sequential GLRT paradigm is used to determine

where the CBRN event started and demonstrated promising performance in a numerical experiment. A clear future direction for work is in examining alternatives to the sequential GLRT, such as a maximum likelihood decision test in which all sensor observations are used in unison instead of sequentially.

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