A ROBUST APPROACH TO OPTIMUM WIDELY LINEAR MVDR BEAMFORMER

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ABSTRACT

In many array processing, the received signals are nonstationary, or in particular, noncircular. Widely linear minimum variance distortionless response (WL MVDR) beamformers can exploit the noncircularity of received signals and improve the performance of the conventional MVDR beamformer. However, in the optimum WL MVDR beamformer, the array steering vector (ASV) and the signal noncircularity coefficient should be known *a priori* for the signal of interest. This requirement puts strict limitation to the implementation of this beamformer. We therefore in this paper propose a robust approach to the optimal WL MVDR beamformer that can deal with the uncertainties in the ASV and noncircularity coefficient. Two variants of the proposed approach are developed based on the treatment of the uncertainties. By doing so, the requirement on the exact information is relaxed while the performance improvement can still be obtained. Simulation studies are also provided to illustrate the performance of the proposed approach.

***Index Terms*—** Array signal processing, widely linear, MVDR, robust beamforming.

1. INTRODUCTION

In array signal processing, beamforming is a widely used technique to enhance a spatially propagating signal of interest (SOI) in the presence of spatial interference signals plus noise [1]. The conventional beamforming approaches based on second-order (SO) property have been mainly focusing on stationary observations [2], resulting linear and time invariant (TI) beamformers. One well-known optimum beamformer is the minimum variance distortionless response (MVDR) beamformer proposed by Capon [3]. However, when it comes to SO nonstationary signals, conventional linear and TI approaches like MVDR beamformer turn out to be suboptimal. And in many cases, SO nonstationary signals are also SO noncircular, which may happen in many cases as stated in [4].

To exploit the noncircularity of observations, widely linear (WL) filter based approaches have been proposed and shown with improved performance (see, for example [4–7] and papers therein). Particularly, paper [4] proposed a method of WL MVDR beamformer for the reception of an unknown signal corrupted by SO noncircular interferences. This WL MVDR beamformer shows performance improvement over conventional MVDR beamformer in the steady state and has the potential of processing up to $2(N-1)$ rectilinear interferences from an array of N sensor elements. To further exploit the noncircularity of SOI, an optimal WL MVDR beamformer is proposed in papers [8, 9]. By this approach, signal component contained in the conjugate of SOI is also retrieved. Thus, the output SINR is further improved compared with WL MVDR beamformer.

To fully benefit from the optimal WL MVDR beamformer, *a priori* knowledge on the SOI array steering vector (ASV) and its noncircularity coefficient is assumed to be available or preestimated. However, in many cases of practical importance, there are some difficulties in determining both the exact ASV and the actual noncircularity coefficient for SOI, which gives rise to the uncertainties in beamforming. For example, the ASV uncertainties may be due to the fact that the array response is not well calibrated, or that the direction of arrival (DOA) of SOI is not accurately estimated. And the uncertainty of noncircularity may be due to partial information on the waveform, phase offset, and even frequency offset of SOI. These kinds of uncertianties will seriously degrade the performance of the WL MVDR beamformer, leading it to perform even worse than MVDR beamformer.

To address the uncertainties in the WL MVDR based beamforming, we propose a robust approach to the optimal WL MVDR beamformer in this paper. Motivated by conventional robust beamforming approaches for SO stationary observations, which has been extensively studied in recent years (see, for example, [10–12] and papers therein), we first formulate the robust WL MVDR beamforming problem by maximizing the WL MVDR beamformer output power subject to the constraint on the augmented steering vector mismatch. The augmented steering vector error is deduced from the uncertainties in SOI ASV and the noncircularity coefficient. As formulating the whole uncertainty in the form of mismatch in augmented steering vector does not exploit its structure information, we then propose the second approach by imposing this structure property in the robust optimization formulation.

2. OPTIMAL WIDELY LINEAR MVDR

2.1. Signal Model

Suppose an array of N antennas is used for receiving narrowband signals and the array output is a complex vector denoted by $\mathbf{x}(t)$ and comprises of the contribution from SOI and interferences-plus-noise

$$\mathbf{x}(t) = \mathbf{a}s(t) + \mathbf{v}(t) \tag{1}$$

where $s(t)$ is the complex envelope of SOI (zero-mean and potentially SO noncircular), $\mathbf{v}(t)$ is the interference-plus-noise component, and $\mathbf{a}$ is the SOI steering vector with the ℓ_2-norm $\|\mathbf{a}\| = N$. It is further assumed that the interference signals contributing to $\mathbf{v}(t)$ are assumed to be zero-mean, potentially SO noncircular and statistically uncorrelated with $s(t)$.

The adaptive beamformer is generally designed to be optimal based on the SO statistics. The SO statistics of the noncircular observation $\mathbf{x}(t)$ are defined by

$$\begin{aligned} \mathbf{R}_x &\stackrel{\text{def}}{=} <\mathrm{E}[\mathbf{x}(t)\mathbf{x}(t)^H]> &= \pi_s \mathbf{a}\mathbf{a}^H + \mathbf{R}_v \\ \mathbf{C}_x &\stackrel{\text{def}}{=} <\mathrm{E}[\mathbf{x}(t)\mathbf{x}(t)^T]> &= \pi_s \gamma_s \mathbf{a}\mathbf{a}^T + \mathbf{C}_v \end{aligned} \tag{2}$$

where $< \cdot >$ denotes the time-averaging operation with respect to t and $\mathbf{R}_v =< \mathrm{E}[\mathbf{v}(t)\mathbf{v}(t)^H] >$, $\mathbf{C}_v =< \mathrm{E}[\mathbf{v}(t)\mathbf{v}(t)^T] >$, π_s is the time-averaged power of the SOI $\pi_s \stackrel{\text{def}}{=}< \mathrm{E}[|s(t)|^2] >$, and $\gamma_s \stackrel{\text{def}}{=} < \mathrm{E}[s(t)^2] >/\pi_s$ is the noncircularity coefficient of the SOI. Notice that depending the form of $s(t)$, the value of $|\gamma_s|$ may range from 0 to 1. For instance, when the complex envelope of signal is on a line, $|\gamma_s|$ is 1 and it is said to be rectilinear.

To exploit the noncircularity of $\mathbf{x}(t)$, the beamformer design needs to be extended to include the conjugate component of the array output. Let $\tilde{\mathbf{x}}(t)$ be the extended array output built by stacking the array output and its conjugate component

$$\begin{aligned}\tilde{\mathbf{x}}(t) &\stackrel{\text{def}}{=} [\mathbf{x}(t)^T, \mathbf{x}(t)^H]^T \\ &= \tilde{\mathbf{a}}_1 s(t) + \tilde{\mathbf{a}}_2 s(t)^* + \tilde{\mathbf{v}}(t) = \tilde{\mathbf{A}}\tilde{\mathbf{s}}(t) + \tilde{\mathbf{v}}(t)\end{aligned} \tag{3}$$

where $\tilde{\mathbf{a}}_1 = [\mathbf{a}^T, \mathbf{0}_N^T]^T$, $\tilde{\mathbf{a}}_2 = [\mathbf{0}_N^T, \mathbf{a}^H]^T$, $\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}_1, \tilde{\mathbf{a}}_2]$ and $\tilde{\mathbf{s}}(t) = [s(t), s(t)^*]^T$. And the SO statistics of the extended array output $\tilde{\mathbf{x}}(t)$ is

$$\mathbf{R}_{\tilde{x}} =< \mathrm{E}[\tilde{\mathbf{x}}(t)\tilde{\mathbf{x}}(t)^H] > \; = \tilde{\mathbf{A}}\mathbf{R}_{\tilde{s}}\tilde{\mathbf{A}}^H + \mathbf{R}_{\tilde{v}} \tag{4}$$

where $\mathbf{R}_{\tilde{s}} =< \mathrm{E}[\tilde{\mathbf{s}}(t)\tilde{\mathbf{s}}(t)^H] >$, $\mathbf{R}_{\tilde{v}} =< \mathrm{E}[\tilde{\mathbf{v}}(t)\tilde{\mathbf{v}}(t)^H] >$.

Given that $\tilde{\mathbf{w}}$ denotes the $2N \times 1$ WL beamformer's weight, the output of the WL spatial filter is given by

$$y(t) = \tilde{\mathbf{w}}^H\tilde{\mathbf{x}}(t) = \tilde{\mathbf{w}}^H\tilde{\mathbf{a}}_1 s(t) + \tilde{\mathbf{w}}^H\tilde{\mathbf{a}}_2 s(t)^* + \tilde{\mathbf{w}}^H\tilde{\mathbf{v}}(t) \tag{5}$$

2.2. Optimal WL MVDR

Based on the signal model constructed, the WL MVDR beamformer is proposed in [4] by applying the MVDR concept to the augmented observation. Mathematically, the weight vector can be found by solving the following convex optimization

$$\min_{\tilde{\mathbf{w}}} \quad \tilde{\mathbf{w}}^H\mathbf{R}_{\tilde{x}}\tilde{\mathbf{w}} \qquad \text{subject to} \quad \tilde{\mathbf{A}}^H\tilde{\mathbf{w}} = \mathbf{f} \tag{6}$$

where $\mathbf{f} = [1, 0]^T$. The solution to this optimization can be obtained using the Lagrange multiplier and expressed as

$$\tilde{\mathbf{w}}_{mvdr1} = \mathbf{R}_{\tilde{x}}^{-1}\tilde{\mathbf{A}}[\tilde{\mathbf{A}}^H\mathbf{R}_{\tilde{x}}^{-1}\tilde{\mathbf{A}}]^{-1}\mathbf{f} = \mathbf{R}_{\tilde{v}}^{-1}\tilde{\mathbf{A}}[\tilde{\mathbf{A}}^H\mathbf{R}_{\tilde{v}}^{-1}\tilde{\mathbf{A}}]^{-1}\mathbf{f} \tag{7}$$

From the development of the WL MVDR, we see that it only exploits the noncircularity of interference and noise. Particularly, it only filters out the signal component associated with ASV $\tilde{\mathbf{a}}_1$ in (3). When $s(t)^*$ is uncorrelated with $s(t)$, the constraint $\tilde{\mathbf{w}}^H\tilde{\mathbf{a}}_2 = 0$ helps to reduce the interference. However, when $s(t)^*$ is correlated with $s(t)$, this constraint will lead to signal suppression. To avoid this problem, [8] proposed the optimal WL MVDR beamformer that can retrieve the signal component from $s(t)^*$ by further exploiting the noncircularity of SOI. Following the development in [8], the $s(t)^*$ can be decomposed as

$$s(t)^* = \gamma_s^* s(t) + [\pi_s(1 - |\gamma_s|^2)]^{1/2} s'(t) \tag{8}$$

where $s'(t)$ is orthogonal to $s(t)$. Thus, the signal model in (3) can be rewriten as:

$$\tilde{\mathbf{x}}(t) = \underbrace{(\tilde{\mathbf{a}}_1 + \gamma_s^*\tilde{\mathbf{a}}_2)}_{\tilde{\mathbf{a}}_\gamma} s(t) + \underbrace{[\pi_s(1 - |\gamma_s|^2)]^{1/2}\tilde{\mathbf{a}}_2 s'(t) + \tilde{\mathbf{v}}(t)}_{\tilde{\mathbf{v}}_\gamma(t)} \tag{9}$$

The optimal WL MVDR is then designed via solving [8]

$$\min_{\tilde{\mathbf{w}}} \quad \tilde{\mathbf{w}}^H\mathbf{R}_{\tilde{x}}\tilde{\mathbf{w}} \qquad \text{subject to} \quad \tilde{\mathbf{w}}^H\tilde{\mathbf{a}}_\gamma = 1 \tag{10}$$

By defining $\mathbf{R}_{\tilde{v}_\gamma}^{-1} =< \mathrm{E}[\tilde{\mathbf{v}}_\gamma(t)\tilde{\mathbf{v}}_\gamma(t)^H] >$ we arrive at the solution as

$$\tilde{\mathbf{w}}_{mvdr2} = [\tilde{\mathbf{a}}_\gamma\mathbf{R}_{\tilde{x}}^{-1}\tilde{\mathbf{a}}_\gamma]^{-1}\mathbf{R}_{\tilde{x}}^{-1}\tilde{\mathbf{a}}_\gamma = [\tilde{\mathbf{a}}_\gamma\mathbf{R}_{\tilde{v}_\gamma}^{-1}\tilde{\mathbf{a}}_\gamma]^{-1}\mathbf{R}_{\tilde{v}_\gamma}^{-1}\tilde{\mathbf{a}}_\gamma \tag{11}$$

3. ROBUST OPTIMAL WL MVDR BEAMFORMER

In this paper, we consider the performance degradation problem experienced by the optimal WL MVDR beamformer due to the mismatch between the actual and presumed knowledge of the parameters $(\mathbf{a}, \gamma_s)$. Although the authors in [8] suggested that these *a priori* knowledge can be estimated using blind identification approach (e.g. JADE algorithm, which works only when the SOI is rectilinear and the number of interference components is known) when the training sequence is not available, the estimation error resulting from the blind identification approach due to practical constraints may contribute to these mismatches. Hence, it is important to ensure the robustness of this beamformer design.

Let $\bar{\mathbf{a}}$ denote the presumed SOI steering vector. Assume that $\bar{\mathbf{a}}$ and $\mathbf{a}$ is confined within a multi-dimensional sphere with radius ε_a. Also, we further assume that the mismatch between the presumed and actual noncircularity coefficient $(\gamma_s - \bar{\gamma}_s)$ has an absolute value less than the square-root of ε_γ.

$$\|\mathbf{a} - \bar{\mathbf{a}}\|^2 \le \varepsilon_a \quad \text{and} \quad |\gamma_s - \bar{\gamma}_s|^2 \le \varepsilon_\gamma \tag{12}$$

In the following, we propose two robust approaches to deal with the uncertainties in steering vector and noncircularity coefficient.

3.1. WL-RCB1: Robust Against the Whole Uncertainty in the Augmented ASV

We first approach the problem by translating the uncertainty constraints in (12) into the uncertainty for the *whole* extended steering vector. Let $\mathbf{e} \stackrel{\text{def}}{=} \mathbf{a} - \bar{\mathbf{a}}$ and $\gamma_\Delta \stackrel{\text{def}}{=} \gamma_s - \bar{\gamma}_s$ denote the SOI steering vector and noncircularity coefficient mismatches. The mismatch in $\tilde{\mathbf{a}}_\gamma$ is denoted as $\tilde{\mathbf{e}}_\gamma$ and can be calculated as

$$\begin{aligned}\tilde{\mathbf{e}}_\gamma \stackrel{\text{def}}{=} \tilde{\mathbf{a}}_\gamma - \bar{\tilde{\mathbf{a}}}_\gamma &= \begin{bmatrix} \mathbf{a} - \bar{\mathbf{a}} \\ (\bar{\gamma}_s^* + \gamma_\Delta^*)(\bar{\mathbf{a}}^* + \mathbf{e}^*) - \bar{\gamma}_s^*\bar{\mathbf{a}}^* \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{e} \\ \bar{\gamma}_s^*\mathbf{e}^* + \gamma_\Delta^*(\bar{\mathbf{a}} + \mathbf{e})^* \end{bmatrix}\end{aligned} \tag{13}$$

where $\bar{\tilde{\mathbf{a}}}_\gamma$ stands for the presumed extended steering vector. The whole uncertainty level then is evaluated by

$$\begin{aligned}\|\tilde{\mathbf{e}}_\gamma\|^2 &= \|\tilde{\mathbf{a}}_\gamma - \bar{\tilde{\mathbf{a}}}_\gamma\|^2 \\ &\le \|\mathbf{e}\|^2 + (\|\bar{\gamma}_s\mathbf{e}\| + \|\gamma_\Delta\bar{\mathbf{a}}\| + \|\gamma_\Delta\mathbf{e}\|)^2 \\ &\le \underbrace{\varepsilon_a + (\|\bar{\gamma}_s|\sqrt{\varepsilon_a} + \sqrt{N}\sqrt{\varepsilon_\gamma} + \sqrt{\varepsilon_\gamma}\sqrt{\varepsilon_a})^2}_{\epsilon_1}\end{aligned} \tag{14}$$

Then, the formulation of the robust Capon beamforming (RCB) can be applied to design the WL-RCB1 beamformer via the following optimization:

$$\begin{aligned}&\tilde{\mathbf{w}}_{wl-rcb1} = \arg\min_{\tilde{\mathbf{w}}}\max_{\tilde{\mathbf{a}}_\gamma} \quad \tilde{\mathbf{w}}^H\mathbf{R}_{\tilde{x}}\tilde{\mathbf{w}} \\ &\qquad \text{subject to} \quad \tilde{\mathbf{w}}^H\tilde{\mathbf{a}}_\gamma = 1, \quad \|\tilde{\mathbf{a}}_\gamma - \bar{\tilde{\mathbf{a}}}_\gamma\|^2 \le \epsilon_1\end{aligned} \tag{15}$$

The optimization can be solved in two steps. For the first step, we fix $\tilde{\mathbf{a}}_\gamma$ and find the optimal $\tilde{\mathbf{w}}$, which gives rise to a solution like $\tilde{\mathbf{w}} = (\tilde{\mathbf{a}}_\gamma^H\mathbf{R}_{\tilde{x}}^{-1}\tilde{\mathbf{a}}_\gamma)^{-1}\mathbf{R}_{\tilde{x}}^{-1}\tilde{\mathbf{a}}_\gamma$. Then for the second step, we insert the $\tilde{\mathbf{w}}$ back into (15) and solve the actual $\tilde{\mathbf{a}}_\gamma$. After some simple mathematical operations, the optimization problem is reduced to

$$\min_{\tilde{\mathbf{a}}_\gamma} \quad \tilde{\mathbf{a}}_\gamma^H\mathbf{R}_{\tilde{x}}^{-1}\tilde{\mathbf{a}}_\gamma \quad \text{subject to} \quad \|\tilde{\mathbf{a}}_\gamma - \bar{\tilde{\mathbf{a}}}_\gamma\|^2 \le \epsilon_1 \tag{16}$$

which is in a stand form of RCB and therefore can be solved accordingly based on the solution in [12]. We denote the solution to (16) as $\hat{\tilde{\mathbf{a}}}_\gamma$ and the WL-RCB1 beamformer weight is given by

$$\mathbf{w}_{wl-rcb1} = \frac{\mathbf{R}_{\tilde{x}}^{-1}\hat{\tilde{\mathbf{a}}}_\gamma}{\hat{\tilde{\mathbf{a}}}_\gamma^H \mathbf{R}_{\tilde{x}}^{-1}\hat{\tilde{\mathbf{a}}}_\gamma} \tag{17}$$

3.2. WL-RCB2: Exploiting the ASV structure information

Recall that the optimal ASV has a structure like

$$\tilde{\mathbf{a}}_\gamma = \begin{bmatrix} \mathbf{a} \\ \mathbf{0} \end{bmatrix} + \gamma_s^* \begin{bmatrix} \mathbf{0} \\ \mathbf{a}^* \end{bmatrix} \tag{18}$$

Formulating the uncertainty as a whole in the form of the mismatch in the extended steering vector does not exploit this structural information. This way, the uncertainty level is evaluated conservatively by using the upper bound, thus leading to the overestimation of uncertainty. The implication of overestimating the uncertainty is the SINR performance loss from the degradation in the interference suppression capability of the robust beamformer design [13–17].

Here we propose a structure-aided uncertainty based WL robust beamformer that formulates the uncertainty constraints due to the steering vector and the noncircularity coefficient separately while maintaining the structural relationship between them. Applying the methodology leading to (16), the formulation of the robust optimal WL MVDR beamformer design with the structure-aided constraints can be written as

$$\min_{\mathbf{a},\gamma_s} \quad f(\mathbf{a},\gamma_s) \quad \text{subject to} \quad \begin{cases} \|\mathbf{a}-\bar{\mathbf{a}}\|^2 \le \varepsilon_a \\ |\gamma_s - \bar{\gamma}_s|^2 \le \varepsilon_\gamma \end{cases} \tag{19}$$

The objective function $f(\mathbf{a},\gamma_s)$ is given by the following expression

$$\begin{aligned} f(\mathbf{a},\gamma_s) &\stackrel{\text{def}}{=} \tilde{\mathbf{a}}_\gamma^H \mathbf{R}_{\tilde{x}}^{-1} \tilde{\mathbf{a}}_\gamma \\ &= (1+|\gamma_s|^2)\mathbf{a}^H\mathbf{D}\mathbf{a} + 2\Re(\gamma_s^* \mathbf{a}^H \mathbf{C}\mathbf{a}^*) \end{aligned} \tag{20}$$

where $\Re(x)$ stands for the real component of x. Note that we use the following definition of $\mathbf{R}_{\tilde{x}}^{-1}$ to obtain the simplified expression of $f(\mathbf{a},\gamma_s)$

$$\mathbf{R}_{\tilde{x}}^{-1} = \begin{pmatrix} \mathbf{D} & \mathbf{C} \\ \mathbf{C}^* & \mathbf{D}^* \end{pmatrix}. \tag{21}$$

Meanwhile, we observe that the objective function is a third-order one and is not guaranteed to be convex, convex optimization method can not be directly applied.

To solve this problem, we propose to modify the formulation by optimizing $(\tilde{\mathbf{a}}_\gamma, \gamma_s)$ instead of $(\mathbf{a}, \gamma_s)$. And the reformulated optimization can be expressed as:

$$\min_{\tilde{\mathbf{a}}_\gamma,\gamma_s} \quad \tilde{\mathbf{a}}_\gamma^H \mathbf{R}_{\tilde{x}}^{-1} \tilde{\mathbf{a}}_\gamma \quad \text{subject to} \quad \begin{cases} \|\mathbf{G}_1\tilde{\mathbf{a}}_\gamma - \bar{\mathbf{a}}\|^2 \le \varepsilon_a \\ \|\mathbf{G}_2\tilde{\mathbf{a}}_\gamma - \gamma_s^*\bar{\mathbf{a}}^*\|^2 \le \varepsilon_a|\gamma_s|^2 \\ |\gamma_s - \bar{\gamma}_s|^2 \le \varepsilon_\gamma \end{cases} \tag{22}$$

where $\mathbf{G1}$ and $\mathbf{G}_2$ are the selection matrices that select the top and bottom $(N \times 1)$ vector of $\tilde{\mathbf{a}}_\gamma$, respectively. The second constraint requires $|\gamma_s|$, which can be replaced by $|\bar{\gamma}_s|$ when unknown (as $|\gamma_s| \approx |\bar{\gamma}_s|$ in general). For this problem, it is convex and can be reformulated to be a second order cone (SOC) optimization problem and solved using some solver like SeDuMi [18]. According to [11, 12], the WL-RCB1 is more efficient than the WL-RCB2 in computation and can take the advantage of recursive eigendecomposition to update the beamformer weights. More details on the implmentation of SOC solver is omitted due to page restriction. After obtaining the ASV estimation by solving (22), we can construct the beamformer filter according to (17).

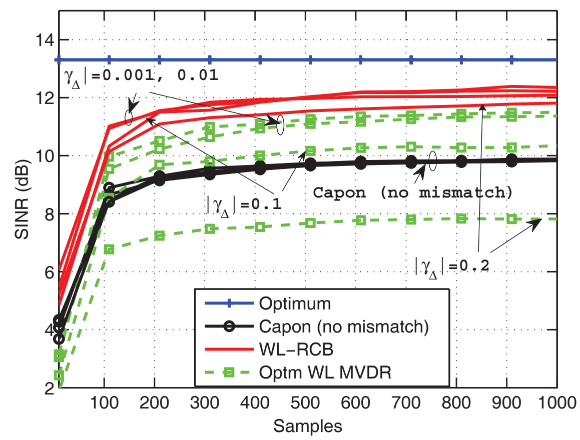

Fig. 1. Performance comparison.

4. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed robust WL MVDR algorithms with numerical simulations. We first examine the WL-RCB1, then followed by the WL-RCB2 for comparison. We take 50 Monte Carlo trials to generate the results.

4.1. Performance of WL-RCB1

We assume that $K = 2$ narrowband binary phase shift keying (BPSK) signals are impinging on a two-element array of half-wavelength inter-element distance from directions $\{0^o, 30^o\}$. The sampling rate is at 20×10^4. The normalized carrier frequencies are assumed to be $\{0, 0\}$. The baud rates are $\{1/5, 1/9\}$. The Nyquist-shaping pulse is taken as the pulse shaping function for the BPSK signals with the roll-off factor taken to be 1. We set the signal-to-noise-ratio (SNR) to be $\{10, 20\}$ dB. The SOI's initial phase is set to be $\pi/3$ so that the $\gamma_s = -0.5 + j0.866$. For the second signal,we set the initial phase to be 0. We assume that the ASV of SOI is known a priori and the noncircularity coefficient is subject to uncertainties. The nominal noncircularity coefficient is with error level $|\gamma_\Delta| = \epsilon_\gamma$. And the error γ_Δ is drawn from complex random Gaussian numbers. We set the value of ϵ_γ to be $\{0.001, 0.01, 0.1, 0.2\}$. We in this case study use the exact value of the error level to calculate the user parameter ϵ_1 for the WL-RCB1. We compare the proposed WL-RCB1 algorithm with Capon beamformer (known ASV), Optimal WL MVDR under uncertainty in noncircularity coefficient. The optimal Capon beamformer gives an SINR output 10.2 dB for this case, while the optimal WL-MVDR gives 13.3 dB SINR output. The result is shown in Fig. 1. From this result, we can see that although the optimal WL-MVDR can tolerate small uncertainty in the noncircularity coefficient, it's performance degrades heavily when the uncertainty is large. However, the robust WL-RCB1 outperforms the the other beamformers for all uncertainty levels.

4.2. Performance of WL-RCB2

In this case study, we further consider the steering vector mismatch. Steering vector uncertainty is due to DOA error in this study. We keep the same signal setup. The error in DOA of SOI is set to be 3°,

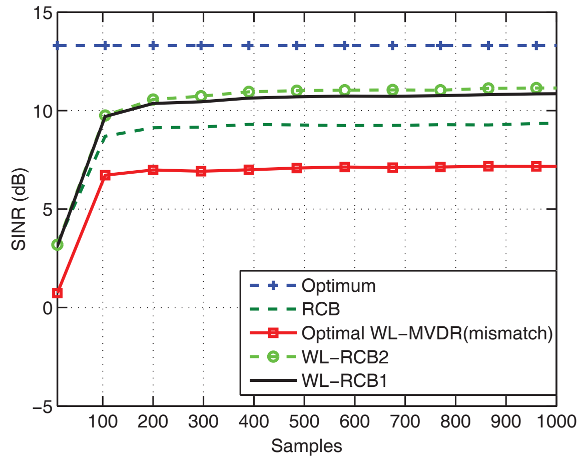

Fig. 2. SINR performance comparison.

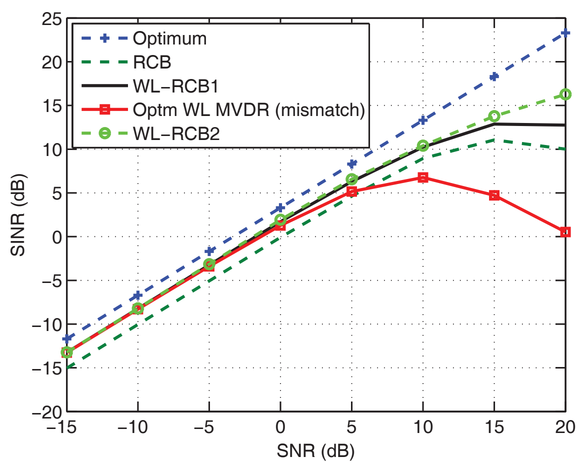

Fig. 3. Output SINR performance comparison.

and the $\epsilon_\gamma = 0.001$. Being blind to the exact errors in practice, we scale the exact errors in both $\mathbf{a}$ and γ_s by a factor of 1.2, in squared norm, to calculate the user parameters for all the robust algorithms to be examined. We compare the performance of RCB, the structure aided WL-RCB2, the WL-RCB1, and the optimal WL MVDR with mismatches. The result shown in Fig. 2 illustrates that under the uncertainties, optimal WL-MVDR no longer outperforms Capon beamformer. However, the proposed WL-RCB1 algorithm still outperforms Capon beamformer consistently. Meanwhile, the structure aided WL-RCB2 can further improve the performance by exploiting the structure information.

Furthermore, we investigate the algorithm's performance under different input SNR. We take 200 snapshots. The result is shown in Fig.3. We in this comparison takes the SINR output of RCB as a benchmark comparison. The results demonstrate that the proposed WL-RCB1 always outperforms the RCB and the optimal WL MVDR with mismatches. And the structure aided WL-RCB2 outperforms the WL-RCB1 for most SNRs.

5. CONCLUDING REMARKS

In this paper, we proposed a robust approach to the optimal WL MVDR beamformer under uncertainties in both the noncircularity coefficient and the ASV. WL-RCB1 algorithm was proposed to deal with both uncertainties by modeling the whole uncertainty in the augmented steering vector. WL-RCB2 algorithm was proposed to exploit the structure property of the augmented steering vector to individually deal with these two uncertainties. With experimental examples, we illustrated the effect of uncertainty on the optimal WL MVDR and the performance improvement brought by the proposed methods. Overall, the proposed approach can bring performance beyond conventional Capon beamformer.

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