BEAMFORMER DESIGN EXPLOITING BLIND SOURCE EXTRACTION TECHNIQUES

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ABSTRACT

Recently we have shown how a blind source extraction (BSE) algorithm can be equipped with some prior information about mixing parameters of the desired source in order to extract this source. The prior information, which may contain errors, is used to construct a matrix from linear combinations of correlation matrices. The extraction filter is easily obtained from the specific eigenstructure of this matrix.

Here we project the beamformer design problem onto the above mentioned BSE algorithm by parameterizing the mixing system. We show in three ways that the proposed method is efficient and flexible. First, with one procedure an LCMV and MVDR beamformer can be obtained. Second, by taking only two appropriate linear combinations of correlation matrices, which may be interpreted as *selection beamformers*, the desired source can be selected. Third, selection beamformers can be designed for a subset of sensors while the final beamformer exploits data from all sensors.

Index Terms— Beamforming, blind signal extraction, second order statistics, prior information, direction of arrival

1. INTRODUCTION

During the last decades, beamformers have been designed for several sensor array signal processing applications, each with different characteristics such as fixed, adaptive, and blind beamformers [1, 2].

We focus on an intermediate approach where we incorporate available prior information into a blind signal processing (BSP) framework, which leads to a source extraction algorithm. Typically, in a BSP scenario the order in which sources are extracted is unknown because of a permutation indeterminacy [3]. To tackle this problem we suggest to incorporate information about the desired source into a blind source extraction (BSE) algorithm such that directly the desired source can be extracted [4]. A strength of this method is that the extraction filters are independent from errors in the a priori information that is used, as long as the desired source is selected. In the context of blind beamforming especially the work in [4] is interesting since it exploits prior information about the mixing parameters of the desired source.

The contribution of this work is threefold. First, we generalize the work from [4] such that the algorithm can handle complex mixtures and signals. Second, we show that two types of extraction filters can be obtained. Typically, an extraction filter has the requirement that the contribution of all undesired sources has to be zero, which corresponds to a linear constraint minimum variance (LCMV) extraction filter or beamformer. We show that with the same procedure and similar effort also a minimum variance distortionless response (MVDR) solution can be obtained. The MVDR extraction filter or beamformer has the requirement that the desired source is extracted without distortion while the output power is minimized, i.e., a tradeoff between interference and noise reduction is made. Third, the beamformer design problem is projected onto the BSE problem by parameterizing the mixing system. Exploiting this parametrization we are able to improve the efficiency and flexibility of the proposed method. In [4] linear combinations of correlation matrices are taken based on an a priori guess of the mixing parameters that correspond to the desired source. The number of linear combinations that have to be taken is depending on the number of sources. In this work we show that a linear combination can be interpreted as a *selection beamformer*. By choosing two appropriate selection beamformers, based on the parameterized structure of the problem and a guess of the direction of arrival (DOA) of the desired source, we are able to identify the desired LCMV and MVDR beamformers. These selection beamformers can even be designed if the positions of only a subset of sensors are available, which leads to linear combinations of only a few correlation matrices.

The structure in this work is as follows. In Section 2 we generalize the BSE algorithm from [4] to complex mixtures. In Section 3 we project the beamformer design problem onto the BSE algorithm. In Section 4 the validity of this method is illustrated by means of a computer simulation. Finally, in Section 5 conclusions and suggestions for future research are given.

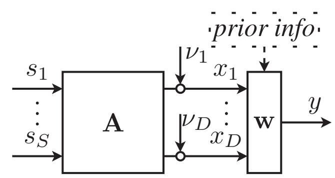


Fig. 1. Model of the BSE problem where D sensors observe S latent source signals that are mixed by a complex mixing system $\mathbf{A}$. An extraction filter $\mathbf{w}$ that extracts the desired source has to be identified; therefore, prior information about the desired source is required.

2. BSE FOR COMPLEX MIXTURES

2.1. Model and assumptions

A model of the BSE problem is depicted in Figure 1. The D observed sensor signals $x_1, \cdots, x_D$ contain complex mixtures of S, possibly complex, source signals $s_1, \cdots, s_S$ and may be contaminated by additive noise. The relation between the observed sensor signals and the source signals is as follows:

$$\mathbf{x}[n] = \mathbf{A}\mathbf{s}[n] + \boldsymbol{\nu}[n] = \sum_{j=1}^{S} \mathbf{a}^j s_j[n] + \boldsymbol{\nu}[n] \quad \forall\, n \in \mathbb{Z} \qquad (1)$$

where the column vectors $\mathbf{x}[n] = \left[x_1[n], \cdots, x_D[n]\right]^T$, $\mathbf{s}[n] = \left[s_1[n], \cdots, s_S[n]\right]^T$, and $\boldsymbol{\nu}[n] = \left[\nu_1[n], \cdots, \nu_D[n]\right]^T$ contain respectively the sensor, source, and noise signals at discrete time index n with a sampling rate such that no aliasing occurs. The complex mixing system is represented by the matrix $\mathbf{A} = \left[\mathbf{a}^1, \cdots, \mathbf{a}^S\right]$ from which mixing column vector $\mathbf{a}^j = \left[a_1^j, \cdots, a_D^j\right]^T$ corresponds to source j for $j \in [1, S]$, with $(\cdot)^T$ representing a transpose without conjugation. In this work, without loss of generality, we assume to have the same number of sensors as sources for simplicity; however, we keep using the symbols D and S for transparency.

The assumptions on the second order statistics (SOS) of the signals are the same as in [4], i.e., the source signals are assumed to be uncorrelated w.r.t. each other and w.r.t. the noise signals. Additionally, we assume to know a noise-free region of support (NF-ROS), which consists of correlation time-lag pairs for which only the source signals are correlated [4, 5]. Finally, we assume that the source autocorrelation functions are linearly independent in the NF-ROS. For a mathematical description of these assumptions we refer to [4].

The objective is to identify a filter $\mathbf{w}$ that extracts the desired source signal from the observations. The relation between the output of the extraction filter $y[n]$ and the observed signals is

$$y[n] = \mathbf{w}^H\mathbf{x}[n] = \mathbf{w}^H\mathbf{A}\mathbf{s}[n] + \mathbf{w}^H\boldsymbol{\nu}[n] \qquad (2)$$

where $(\cdot)^H$ represent the Hermitian transpose.

Notation in this paper is as follows. Matrices and vectors are represented by bold, upper case and lower case letters such as matrix $\mathbf{A}$ and vector $\mathbf{w}$, respectively. A matrix can be decomposed in its row and column vectors. In order to distinguish between the row and column vectors of a matrix we use sub- and superscript indices, respectively. Thus, the j in $\mathbf{a}^j$ tells us that the vector represents the j'th column of the matrix $\mathbf{A}$. Similarly, both sub- and superscript indices are used together to represent a single element in a matrix, i.e., the element in row i and column j of matrix $\mathbf{A}$ is a_i^j. The symbol for the imaginary unit is $\jmath = \sqrt{-1}$ and $\bar{(\cdot)}$ represents a complex conjugate. Further notation is addressed at occurrence.

2.2. Identifying extraction filters

The extraction algorithm uses the following set of sensor correlation matrices:

$$\mathbf{C}_i^x = \begin{bmatrix} r_{i1}^x[\Omega_1] & r_{i1}^x[\Omega_2] & \cdots & r_{i1}^x[\Omega_K] \\ \vdots & \vdots & \ddots & \vdots \\ r_{iD}^x[\Omega_1] & r_{iD}^x[\Omega_2] & \cdots & r_{iD}^x[\Omega_K] \end{bmatrix} \quad \forall\, i \in [1, D] \quad (3)$$

where the sensor correlation function for time-lag pair $\Omega_\kappa = \{n, k\}_\kappa$ is defined as follows:

$$r_{i_1 i_2}^x[\Omega_\kappa] = \mathbb{E}\left\{x_{i_1}[n]\bar{x}_{i_2}[n-k]\right\} \quad \forall\, 1 \le i_1, i_2 \le D \qquad (4)$$

where $\mathbb{E}$ is the mathematical expectation operator.

Since the correlation functions are evaluated in the NF-ROS we obtain the following noise-free structure in the correlation matrices:

$$\mathbf{C}_i^x = \bar{\mathbf{A}}\,\mathrm{diag}\left(\mathbf{a}_i\right)\mathbf{C}^s \quad \forall\, i \in [1, D] \qquad (5)$$

where $\bar{\mathbf{A}}$ is the $D \times S$ conjugate without transpose mixing matrix, $\mathrm{diag}\left(\mathbf{a}_i\right)$ is an $S \times S$ matrix with the elements of the i'th row vector of the mixing matrix on the diagonal, and $\mathbf{C}^s$ is the $S \times K$ source autocorrelation matrix with the following structure:

$$\mathbf{C}^s = \begin{bmatrix} r_{11}^s[\Omega_1] & r_{11}^s[\Omega_2] & \cdots & r_{11}^s[\Omega_K] \\ \vdots & \vdots & \ddots & \vdots \\ r_{SS}^s[\Omega_1] & r_{SS}^s[\Omega_2] & \cdots & r_{SS}^s[\Omega_K] \end{bmatrix} \qquad (6)$$

If the source autocorrelation functions are linearly independent in the NF-ROS and the number of time-lag pairs K is larger than or equal to the number of sources S then the source autocorrelation matrix has full row rank, which is required later on. We assume from now on that the number of time-lag pairs equals the number of sources, i.e., $K = S$; otherwise, if $K > S$ then reduction techniques can be used [5, 3]. For $K < S$ the problem is underdetermined and cannot be solved using these techniques.

The identification of the desired extraction filter is based on joint diagonalization of linear combinations of correlation matrices. Linear combinations of correlation matrices from (3) are defined as

$$\boldsymbol{\Gamma}_l = \sum_{i=1}^{D} \xi_i^l \mathbf{C}_i^x \quad \text{with} \quad \boldsymbol{\xi}^l = \left[\xi_1^l \,\cdots\, \xi_D^l\right]^T \quad \forall\, l \in [1, L] \qquad (7)$$

The column vectors $\boldsymbol{\xi}^l$ for $l \in [1, L]$ are used to calculate the l'th linear combination of correlation matrices. Later we show how to choose these vectors and benefit from their degrees of freedom.

The structure of a linear combination of correlation matrices is quite similar to (5), i.e.,

$$\boldsymbol{\Gamma}_l = \bar{\mathbf{A}}\,\mathrm{diag}\left(\left[\alpha_l^1 \,\cdots\, \alpha_l^S\right]\right)\mathbf{C}^s \qquad (8)$$

where $\alpha_l^j = \langle \boldsymbol{\xi}^l, \mathbf{a}^j \rangle \triangleq \sum_{i=1}^{D} \bar{\xi}_i^l a_i^j$.

Under the assumption that the source autocorrelation matrix is real, which can be accomplished for stationary complex sources by averaging the sensor correlation functions for the same positive and negative lag, we use the eigenvalue decomposition of the following matrix $\mathbf{M}$ to identify extraction filters:

$$\mathbf{M} = \sum_{l=2}^{L} \bar{\boldsymbol{\Gamma}}_l \left(\boldsymbol{\Gamma}_1\right)^{-1} \boldsymbol{\Gamma}_l \left(\bar{\boldsymbol{\Gamma}}_1\right)^{-1} \equiv \mathbf{A}\boldsymbol{\Lambda}\left(\mathbf{A}\right)^{-1} \qquad (9)$$

where $\boldsymbol{\Lambda}$ is a diagonal matrix containing the following eigenvalues

$$\lambda^j = \frac{\sum_{l=2}^{L} \left|\alpha_l^j\right|^2}{\left|\alpha_1^j\right|^2} \quad \text{with} \quad \alpha_l^j = \left\langle \boldsymbol{\xi}^l, \mathbf{a}^j \right\rangle \qquad (10)$$

and the equivalence follows from the structure in (8).

From (9) it follows easily that the left eigenvectors correspond to the rows from the inverse of the mixing matrix, while the right eigenvectors correspond to columns of the mixing matrix. This means that the LCMV solution is found if we select the appropriate left eigenvector. For the MVDR solution we have to select the appropriate right eigenvector and minimize the output power over the space that is orthogonal to the direction of that eigenvector.

With this identification procedure two inevitable indeterminacies known from blind signal processing show up. First, the eigenvectors can be identified up to an unknown scaling. In this work we deal with this scaling indeterminacy without using additional prior information by normalizing either the extraction filter length or the output power. Second, the identified eigenvectors are ordered arbitrarily. This means that we do not know which filter belongs to which source. In order to deal with this permutation problem some additional prior information is required.

2.3. Selecting the desired extraction filter

From Section 2.2 it follows that two types of BSE problems can be solved by performing an eigenvalue decomposition on the matrix $\mathbf{M}$ from (9). What follows as well is that eigenvector $\boldsymbol{\mu}^m$, i.e., the eigenvector that extracts the j'th source, has a corresponding eigenvalue λ^m that is only depending on the mixing column of source j and the vectors to take linear combinations. For the eigenvalues it holds thus that $\lambda^m = \lambda(\mathbf{a}^j, \boldsymbol{\xi}^l)$ for a single j and all $l \in [1, L]$.

By choosing the vectors $\boldsymbol{\xi}^l$ for $l \in [1, L]$ based on prior information about the mixing column that corresponds to the desired source we can characterize the eigenvalue that corresponds to the desired source. By calculating its corresponding eigenvector we obtain the desired extraction filter.

Suppose we have a guess $\hat{\mathbf{a}}$ of the mixing column $\mathbf{a}^d$ that corresponds to the desired source $s_d[n]$, with $d \in [1, S]$, for which the following condition holds:

$$\frac{\left|\langle \hat{\mathbf{a}}, \mathbf{a}^d \rangle\right|}{||\mathbf{a}^d||} > \frac{\left|\langle \hat{\mathbf{a}}, \mathbf{a}^j \rangle\right|}{||\mathbf{a}^j||} \quad \forall 1 \leq j \neq d \leq S \tag{11}$$

This condition means that the angle between the guessed mixing column and each true mixing column has to be the smallest for the desired mixing column; in other words, the guessed mixing column has to be a 'good' guess.

If we now choose the vector $\boldsymbol{\xi}^1$ in the direction of the guessed mixing column with length one, i.e., $\boldsymbol{\xi}^1 = \hat{\mathbf{a}} / ||\hat{\mathbf{a}}||_2$ and we choose $D-1$ vectors $\boldsymbol{\xi}^2, \cdots, \boldsymbol{\xi}^D$ such that the $L = D$ vectors together form an orthonormal basis, i.e., $\langle \boldsymbol{\xi}^{l_1}, \boldsymbol{\xi}^{l_2} \rangle = \delta_{l_1}^{l_2}$, then we know that the smallest eigenvalue of the matrix $\mathbf{M}$ corresponds to the desired source.

Proof. The mixing column vectors $\mathbf{a}^j$ for $j \in [1, S]$ can be decomposed in terms of the basis vectors, i.e., $\mathbf{a}^j = \sum_{l=1}^{D} \alpha_l^j \boldsymbol{\xi}^l$ where $\alpha_l^j = \langle \boldsymbol{\xi}^l, \mathbf{a}^j \rangle$. From (11) it follows that $\left|\alpha_1^j\right|$ has the largest value for the desired source, i.e., $j = d$, if all mixing columns are normalized, which we may assume because of the scaling indeterminacy. The eigenvalues from (10) can be rewritten as follows:

$$\lambda^j = \frac{\sum_{l=2}^{L} \left|\alpha_l^j\right|^2}{\left|\alpha_1^j\right|^2} = \frac{\left|\left|\mathbf{a}^j\right|\right|_2^2 - \left|\alpha_1^j\right|^2}{\left|\alpha_1^j\right|^2} \tag{12}$$

Because we know that $\left|\alpha_1^j\right|$ has the largest value for the desired source, i.e., $j = d$, it follows that the numerator and denominator in (12) have respectively the smallest and highest value for the desired source. This means that the smallest eigenvalue of $\mathbf{M}$ corresponds to the desired source. □

One of the main advantages of this approach is that the prior guess is not required to be exact. Deviations from the actual mixing column are allowed and the limitations are fully specified by (11).

3. BEAMFORMER DESIGN AS A BSE PROBLEM

It follows from the previous sections that the generalization from the BSE algorithm for real mixing scenarios [4] to a complex mixing scenario algorithm is a natural procedure. Instead of transforming every facet of previous work we map the beamformer design problem onto the complex BSE algorithm and illustrate that an improved efficiency and increased flexibility compared to the non-parameterized algorithm can be obtained by exploiting the parameterized structure.

3.1. Mapping beamformer design onto BSE

In a beamforming context only the direction of arrival (DOA) gives information for far field sources, which is represented by the angle θ_j for $j \in [1, S]$. This DOA is used in a unit length, two dimensional vector $\mathbf{q}_j$ that points from the source into the direction of the sensor array. The positive x-axis and y-axis correspond to $0°$ and $90°$, respectively. This means that the DOA information vector is given as $\mathbf{q}_j = \mathbf{q}(\theta_j) = \left[-\cos\theta_j \;\; -\sin\theta_j\right]^T$ for $j \in [1, S]$. A guessed DOA of the desired source is represented by $\hat{\theta}$ and vector $\hat{\mathbf{q}} = \mathbf{q}(\hat{\theta})$.

Based on these assumptions the mixing column vectors are array response vectors, i.e., $\mathbf{a}^j = \mathbf{a}(\theta_j)$ for $j \in [1, S]$ with elements

$$a_i^j = a_i(\theta_j) = \exp\left(-\jmath 2\pi \, \mathbf{p}_i^T \mathbf{q}(\theta_j)\right) \tag{13}$$

where $\mathbf{p}_i$ for $i \in [1, D]$ are frequency-normalized sensor positions, i.e., the physical sensor positions divided by the wavelength.

Suppose that we have a guess $\hat{\theta}$ of the DOA θ_d of the desired source. In beamformer design, errors in the guess $\hat{\theta}$ is dealt with by diagonal loading or designing with uncertainty, which leads to a performance degradation [2]. If (11) holds for array response vector $\mathbf{a}(\hat{\theta})$, then we can choose $\boldsymbol{\xi}^1 = \mathbf{a}(\hat{\theta})/||\mathbf{a}(\hat{\theta})||_2$ and $D-1$ orthonormal vectors $\boldsymbol{\xi}^l$ for $l \in [2, D]$, which follows from Section 2.3. Using this procedure we identify the optimal beamformer independent of errors. Although the restriction in (11) seems rather complex, it becomes clearer when we interpret the vectors $\boldsymbol{\xi}^l$ as *selection beamformers*. The beampattern of selection beamformer $\boldsymbol{\xi}^l$ is as follows:

$$B(\boldsymbol{\xi}^l, \theta) = \left|\langle \boldsymbol{\xi}^l, \mathbf{a}(\theta) \rangle\right| = \left|(\boldsymbol{\xi}^l)^H \mathbf{a}(\theta)\right| \equiv |\alpha_l(\theta)| \tag{14}$$

Using this interpretation the restriction in (11) implies that the gain of selection beamformer $\boldsymbol{\xi}^1$ has to be higher for the DOA of the desired source than for the DOAs of the other sources. We use this interpretation to increase the efficiency and flexibility of the method.

3.2. Exploiting the parameterized structure

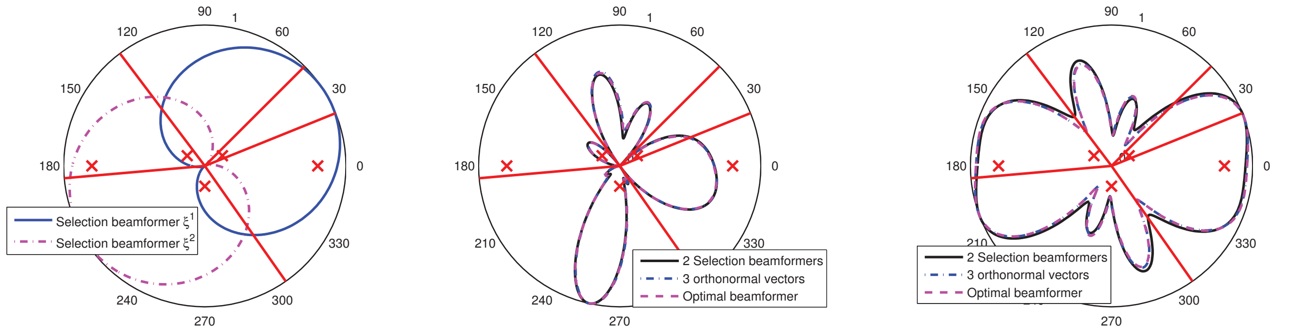
If we take only two linear combinations of correlation matrices then the eigenvalues of $\mathbf{M}$ as function of DOA angle θ become the ratio of two selection beampatterns, i.e.,

$$\lambda(\theta) = \left|\frac{\langle \boldsymbol{\xi}^2, \mathbf{a}(\theta) \rangle}{\langle \boldsymbol{\xi}^1, \mathbf{a}(\theta) \rangle}\right|^2 = \frac{B^2(\boldsymbol{\xi}^2, \theta)}{B^2(\boldsymbol{\xi}^1, \theta)} \tag{15}$$

If the following requirements hold for the selection beamformers: 1) $\boldsymbol{\xi}^1$ has the highest gain for $\hat{\theta}$, 2) $\boldsymbol{\xi}^2$ has the lowest gain for $\hat{\theta}$, and 3) $\boldsymbol{\xi}^1$ and $\boldsymbol{\xi}^2$ have symmetric and monotonically decreasing and increasing beampatterns, respectively, then we know that the smallest eigenvalue corresponds to the source with a DOA closest to $\hat{\theta}$.

Of course, more advanced selection beamformers may be designed in order to select the desired source based on more advanced prior information about the DOAs of the desired and undesired sources. For example asymmetry in the selection beamformers can be incorporated to prefer an error on the left over an error on the right w.r.t. the guessed DOA.

However, here we suggest the design of selection beamformers that increase the efficiency of the method by using only a subset of the sensors from the sensor array. If for example the positions of only two sensors are known, then the selection beamformers can be designed for these two sensors. Subsequently, linear combinations of correlation matrices containing the data of all sensors are used to identify the final LCMV or MVDR beamformer. As a result, always the optimal beamformer for all sensors is identified even if prior information is available for only a subset of sensors.



(a) Selection beampatterns (b) LCMV objective (c) MVDR objective

Fig. 2. Simulation results for the LCMV and MVDR objectives. Red crosses represent the sensor positions, spokes represent the DOAs of the sources, and the guessed DOA was $\theta_0 = 30^\circ$. In (a) the selection beampatterns are depicted. The beampatterns in (b) and (c) are based on: two selection beamformers, linear combinations of three orthonormal vectors, and the optimal beamformer.

4. SIMULATION RESULTS

We illustrate the proposed method by means of a computer simulation. In the example scenario we assume that $S = 5$ far field, autoregressive sources, with different temporal structure and DOAs -175°, -55°, 22°, 42°, and 127°, are observed by $D = 5$ sensors. The sensor array setup is depicted as the red crosses in Figure 2 and the DOAs are represented by red spokes. Three sensors are placed as a circular array with mutual distance over wavelength of 1/4 and two sensors are placed at ± 0.8 in the x-axis direction. This means that spatial aliasing occurs for these two microphones.

From now on we assume that only the positions of the three sensors from the circular sub-array are known. With traditional beamforming techniques it is then only possible to use these three sensors to build a beamformer from the parameterized model. We show that with our method the desired LCMV and MVDR beamformer for five sensors can be extracted from a mixture of five sources using the DOA guess $\theta_0 = 30^\circ$ for only the sensors in the circular array.

Simulation results are depicted in Figure 2. We designed the smooth beamformers for the sub-array as

$$\begin{bmatrix}\boldsymbol{\xi}^1 & \boldsymbol{\xi}^2\end{bmatrix} = \left(\begin{bmatrix}\hat{\mathbf{a}}(\theta_0) & \hat{\mathbf{a}}(-\theta_0)\end{bmatrix}^\dagger\right)^H \tag{16}$$

with $(\cdot)^\dagger$ the Moore-Penrose pseudoinverse for which the beampatterns are depicted in Figure 2a. In Figure 2b and Figure 2c the LCMV and MVDR problems are solved, respectively. In each figure the first beamformer is designed by using two selection beamformers, the second beamformer is designed from three orthonormal vectors, and the third beamformer is the optimal beamformer calculated from the simulation setup having all information. White Gaussian noise signals were used, which had a power per sensor of $r_{ii}^{\nu}[0] = 0.01$; therefore, the NF-ROS was chosen as lags 1 up to 3.

We observe from the simulations that both techniques presented in this work select the desired source at 22° and find the optimal LCMV and MVDR beamformers that extract the desired source from noisy observations based on an a priori guess about the DOA of the desired source. Because of errors in the estimation of the correlation data the beamformers are not exactly the same. In order to discuss the extraction performance in terms of signal to noise ratio a more extensive performance study is required where also reduction techniques [5, 3] are incorporated; however, this was not the focus of this work.

5. CONCLUSIONS

We generalized the BSE algorithm for real mixtures such that it can deal with complex mixtures of complex signals. We have shown that based on an a priori guess of the mixing column of the desired source the desired beamformers can be found for two well known objectives, i.e., the LCMV and MVDR beamformers. An advantage of this method is that the a priori guess is not required to be exact in order to identify the desired beamformer. Deviations from the actual mixing column are allowed as long as the desired beamformer is selected. Using the parametrization we have shown that a more efficient and flexible algorithm can be used to extract a desired source. By means of a computer simulation the validity of this method is depicted.

Future research topics are as follows. Reduction techniques should be incorporated such that overdetermined mixtures can also be dealt with and such that the method becomes more robust to noise and estimation errors in the correlation data. Subsequently, a performance study and real-life experiments should illustrate the robustness and performance of the method.

6. REFERENCES

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