MATCHED DIRECTION BEAMFORMING BASED ON SIGNAL SUBSPACE

Jie Zhuang, Ping Huang, and Wei Huang

School of Communication and Information Engineering University of Electronic Science and Technology of China (UESTC) Chengdu, 611731, China. e-mail: jz@uestc.edu.cn

ABSTRACT

The actual manifold vector (or steering vector) of the signal of interest (SOI) is often imprecise in practical applications. However the true manifold vector can be expressed as a product of a known matrix and an unknown coordinate vector in many cases. This model can accommodate many manifold uncertainties, for instance, the look direction error, local scattering, etc. Matched direction beamformer (MDB) is referred to as the beamformer resolving the signal that is drawn from an unknown direction inside the known subspace. The main contribution of this paper is to propose a new MDB that can estimates the coordinate vector associated with the SOI, without using the knowledge of the interference subspace (IS). Moreover the proposed approach is robust to the dimension overestimation of the signal subspace.

Index Terms— Array beamforming, matched direction beamforming (MDB), signal subspace, look direction error.

1. INTRODUCTION

Consider an array with N sensors impinged by one signal of interest (SOI) and M interference signals. The M + 1 signals are assumed to be narrow-band, uncorrelated with each other and located in the far field of the array. The covariance matrix of the received $N \times 1$ array signal vector $\mathbf{x}(t)$, denoted by **R**, may be assumed to have the following form:

$$\mathbf{R} = \mathcal{E}\{\mathbf{x}(t)\mathbf{x}^{H}(t)\} = \sigma_{0}^{2}\mathbf{a}_{0}\mathbf{a}_{0}^{H} + \sum_{i=1}^{M}\sigma_{i}^{2}\mathbf{a}_{i}\mathbf{a}_{i}^{H} + \sigma_{n}^{2}\mathbf{I}$$
(1)

where $\mathcal{E}\{\cdot\}$ denotes the expectation operator, $(\cdot)^H$ represents Hermitian transpose, $\mathbf{a} \in \mathcal{C}^{N \times 1}$ stands for the array manifold vector (or steering vector), and \mathbf{I} is the identity matrix. We use subscript 0 to indicate the entities related to the SOI. $(\sigma_0^2, \{\sigma_i^2\}_{i=1}^M)$ are the powers of the SOI and the M interference signals respectively. σ_n^2 denotes the power of the additive white Gaussian noise. In practical applications, the covariance matrix can be estimated by using K received snapshots $\{\mathbf{x}(t_k)\}_{k=1}^K$ as follows:

$$\widehat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(t_k) \mathbf{x}^H(t_k)$$
(2)

Due to many practical reasons, the manifold vector \mathbf{a}_0 is not perfectly known [1]. For instance, the look direction error can result in the difference between the actual manifold and the presumed (or nominal) manifold. The straightforward consequence of this mismatch is to cause a substantial performance degradation of conventional adaptive beamformers [2]. However, in many cases this unknown manifold vector \mathbf{a}_0 can be modeled to lie in a known *p*-dimension linear subspace $\langle \mathbf{H} \rangle$, but how to combine \mathbf{a}_0 using the base of $\langle \mathbf{H} \rangle$ is otherwise unknown [3, 4]. This means \mathbf{a}_0 may be written as

$$\mathbf{a}_0 = \mathbf{H}\mathbf{b}_0 \tag{3}$$

where the matrix $\mathbf{H} \in \mathcal{C}^{N \times p}$ is known a *priori* but the coordinate vector $\mathbf{b}_0 \in \mathcal{C}^{p \times 1}$ is unknown. The most obvious example may be the case of the look direction error. Using the Taylor expansion and retaining the terms up to the second order, the actual manifold can be approximated by [5, 3]

$$\mathbf{a}(\theta_0) \approx \mathbf{a}(\overline{\theta}_0) + (\theta_0 - \overline{\theta}_0)\dot{\mathbf{a}}(\overline{\theta}_0) + \frac{(\theta_0 - \overline{\theta}_0)^2}{2}\ddot{\mathbf{a}}(\overline{\theta}_0) \quad (4)$$

where θ_0 and $\overline{\theta}_0$, respectively, represent the true and nominal direction-of-arrival (DOA) of the SOI. $\dot{\mathbf{a}}(\overline{\theta}_0)$ and $\ddot{\mathbf{a}}(\overline{\theta}_0)$ denote the first- and second-order derivatives of the array manifold with respect to the nominal DOA $\overline{\theta}_0$. Thus we can derive a matrix $\mathbf{H}_1 = [\mathbf{a}(\overline{\theta}_0) \ \dot{\mathbf{a}}(\overline{\theta}_0)]$ such that $\mathbf{a}_0 \in \langle \mathbf{H}_1 \rangle$. Also, we may have the following approximations:

$$\mathbf{a}(\overline{\theta}_{0}) = \mathbf{a}(\theta_{0} - \Delta\theta_{0})$$
(5a)
$$\approx \mathbf{a}(\theta_{0}) - \Delta\theta_{0}\dot{\mathbf{a}}(\theta_{0}) + \frac{(\Delta\theta_{0})^{2}}{2}\ddot{\mathbf{a}}(\theta_{0})$$
(5b)
$$\mathbf{a}(\overline{\theta}_{0} \pm \Delta\theta) = \mathbf{a}(\theta_{0} \pm \Delta\theta - \Delta\theta_{0})$$
(5b)

$$\approx \mathbf{a}(\theta_0) + (\pm \Delta \theta - \Delta \theta_0) \dot{\mathbf{a}}(\theta_0) + \frac{(\pm \Delta \theta - \Delta \theta_0)^2}{2} \ddot{\mathbf{a}}(\theta_0)$$

where $\Delta \theta_0 = \theta_0 - \overline{\theta}_0$ is unknown but $\Delta \theta$, related to the expected range of the DOA of the SOI, can be set a value even if the true DOA is unavailable. In the simulation section, for instance, we set $\Delta \theta = 2^\circ$. After some straightforward algebraic manipulations, we can obtain

$$\mathbf{a}(\theta_{0}) \approx \frac{(\Delta\theta)^{2} - (\Delta\theta_{0})^{2}}{(\Delta\theta)^{2}} \mathbf{a}(\overline{\theta}_{0}) \qquad (6)$$
$$+ \frac{(\Delta\theta_{0})^{2} + \Delta\theta\Delta\theta_{0}}{2(\Delta\theta)^{2}} \mathbf{a}(\overline{\theta}_{0} + \Delta\theta)$$
$$+ \frac{(\Delta\theta_{0})^{2} - \Delta\theta\Delta\theta_{0}}{2(\Delta\theta)^{2}} \mathbf{a}(\overline{\theta}_{0} - \Delta\theta)$$

It is clear that $\mathbf{a}(\theta_0)$ also belongs to the known linear subspace $\langle \mathbf{H}_2 \rangle$, where $\mathbf{H}_2 = [\mathbf{a}(\overline{\theta}_0) \quad \mathbf{a}(\overline{\theta}_0 - \Delta\theta) \quad \mathbf{a}(\overline{\theta}_0 + \Delta\theta)]$. Note that both \mathbf{H}_1 and \mathbf{H}_2 can be computed prior to the beamformer processing if the presumed DOA $\overline{\theta}_0$ and array geometry information are available. The example of local scattering is discussed in [3, 6] and more practical examples can be found in [4].

Also, in [7, 8] the flat ellipsoidal uncertainty set has been investigated, in which the true manifold vector is expressed as

$$\mathbf{a}_{0} = \overline{\mathbf{a}}_{0} + \mathbf{B}\mathbf{b}_{fe} = \begin{bmatrix} \overline{\mathbf{a}}_{0} & \mathbf{B} \end{bmatrix} \begin{bmatrix} 1 & \mathbf{b}_{fe}^{T} \end{bmatrix}^{T}, \quad \|\mathbf{b}_{fe}\| \le 1 \quad (7)$$

where $\overline{\mathbf{a}}_0$ stands for the nominal manifold vector of the SOI, the known $N \times (p-1)$ matrix **B** is full column rank and the vector \mathbf{b}_{fe} is unknown. Clearly the flat ellipsoidal uncertainty set can be transformed to the model of (3) with $\mathbf{H} = \begin{bmatrix} \overline{\mathbf{a}}_0 & \mathbf{B} \end{bmatrix}$ and $\mathbf{b}_0 = \begin{bmatrix} 1 & \mathbf{b}_{fe}^T \end{bmatrix}^T$.

In this paper, we assume that the coordinate vector \mathbf{b}_0 stays unchanged over K snapshots and hence the contribution due to the SOI in **R** is rank-1. The so-called *matched direction beamforming* (MD-B) is referred to as the beamformer resolving the signal that is drawn from an unknown direction $\langle \mathbf{Hb}_0 \rangle$ inside the known subspace $\langle \mathbf{H} \rangle$ [3, 4]. The problem to be addressed in this paper is that we estimate the vector \mathbf{b}_0 by using the signal subspace instead of the interference subspace (IS).

2. PREVIOUS WORK

In [3], it is assumed that the IS $\langle \mathbf{A}_I \rangle$ is exactly known (where $\mathbf{A}_I = [\mathbf{a}_1, \dots, \mathbf{a}_M]$). Then it is shown that the vector \mathbf{b}_0 can be estimated by

$$\mathbf{b}_0 = \beta \mathcal{P}\{(\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H \mathbf{R} \mathbf{G}\}$$
(8)

where $\mathcal{P}\{\cdot\}$ denotes the principal eigenvector of the matrix between braces. β can be chosen such that $\|\mathbf{a}_0\|^2 = \|\mathbf{H}\mathbf{b}_0\|^2 = N$. Note that β does not affect the array output signal-to-interference-plusnoise ratio (SINR). The matrix **G** is defined as

$$\mathbf{G} = \mathbf{P}_{\mathbf{A}_{I}}^{\perp} \mathbf{H} \tag{9}$$

where $\mathbf{P}_{\mathbf{A}_{I}}^{\perp} = \mathbf{I} - \mathbf{A}_{I} (\mathbf{A}_{I}^{H} \mathbf{A}_{I})^{-1} \mathbf{A}_{I}^{H}$ stands for the projection operator onto the complement subspace of $\langle \mathbf{A}_{I} \rangle$. The problem in (8) is that the perfect knowledge of the IS is required for its implementation. In many practical applications, unfortunately, this assumption is not always valid or the IS cannot be estimated easily.

In [4], a generalized sidelobe canceller (GSC) structured MDB is presented. Initially, an error covariance is formed by

$$\mathbf{R}_{ee} = \left(\mathbf{H}^H \mathbf{R}^{-1} \mathbf{H}^H\right)^{-1} \tag{10}$$

where the columns of \mathbf{H} are required to be orthonormal columns. For the general \mathbf{H} the Gram-Schmidt algorithm [9] may be used to render the columns of \mathbf{H} orthonormal. In the presence of one interference signal only, the error covariance becomes [4]

$$\mathbf{R}_{ee} = \sigma_0^2 \mathbf{b}_0 \mathbf{b}_0^H + \eta \mathbf{H}^H \mathbf{a}_1 \mathbf{a}_1^H \mathbf{H} + \sigma_n^2 \mathbf{I}$$
(11)

where $\eta = \frac{\sigma_1^2 \sigma_n^2}{\sigma_n^2 + \sigma_1^2 \mathbf{a}_1^H \mathbf{P}_H^{\perp} \mathbf{a}_1}$ represents the level how the interference is suppressed by the GSC beamformer. We can see that \mathbf{b}_0 can be approximated by the principal eigenvector of \mathbf{R}_{ee} up to a scaling factor if the value of η is sufficiently small. This implies that the interference manifold \mathbf{a}_1 is far from the subspace $\langle \mathbf{H} \rangle$ and therefore the interference is ideally suppressed by the GSC. However, when \mathbf{a}_1 is relatively close to $\langle \mathbf{H} \rangle$ and thus η becomes a significant number, then the effects of the interferences on \mathbf{R}_{ee} cannot be neglected any more. The immediate consequence is that the principal eigenvector contains significant contributions from the interferences. On the other hand, the SOI contributes not only to the principal eigenvector but also the other dominant eigenvectors. In order to deal with this situation, [4] suggests that, instead of one eigenvector, a set of dominant eigenvectors of \mathbf{R}_{ee} should be utilized to restore the SOI, which is the so-called multirank minimum variance distortionless response (MVDR) beamforming. Nevertheless all the dominant eigenvectors of \mathbf{R}_{ee} are contaminated by the interferences in such case.

3. PROPOSED MATCHED DIRECTION BEAMFORMER

Performing eigen-decomposition on R yields:

$$\mathbf{R} = \sum_{i=1}^{N} \lambda_i \mathbf{e}_i \mathbf{e}_i^H = \sum_{i=1}^{M+1} \lambda_i \mathbf{e}_i \mathbf{e}_i^H + \sum_{i=M+2}^{N} \lambda_i \mathbf{e}_i \mathbf{e}_i^H \qquad (12)$$

where λ_i is the *i*th largest eigenvalue and \mathbf{e}_i denotes the associated eigenvector. The M + 1 dominant eigenvectors $\mathbf{E}_s = [\mathbf{e}_1 \dots \mathbf{e}_{M+1}]$, associated with the largest M + 1 eigenvalues, are referred to as the signal-subspace eigenvectors. The rest eigenvectors $\mathbf{E}_n = [\mathbf{e}_{M+2} \dots \mathbf{e}_N]$ are referred to as the noise-subspace eigenvectors. Next we will develop an estimator of \mathbf{b}_0 without recourse to the IS.

Firstly a matrix \mathbf{A}_{ℓ} is formed by all the M + 1 dominant eigenvectors except \mathbf{e}_{ℓ} (where $\ell \in [1, M + 1]$), i.e.,

$$\mathbf{A}_{\ell} = [\mathbf{e}_1, \dots, \mathbf{e}_{\ell-1}, \mathbf{e}_{\ell+1}, \dots, \mathbf{e}_{M+1}]$$
(13)

Then a matrix U is defined as

$$\mathbf{U} = \mathbf{P}_{\mathbf{A}_{e}}^{\perp} \mathbf{H} \tag{14}$$

We can rewritten \mathbf{e}_{ℓ} as follows:

$$\mathbf{e}_{\ell} = \frac{\mathbf{e}_{\ell} (\mathbf{e}_{\ell}^{H} \mathbf{e}_{\ell})^{-1} \mathbf{e}_{\ell}^{H} \mathbf{a}_{0}}{\mathbf{e}_{\ell}^{H} \mathbf{a}_{0}} = \frac{\mathbf{P}_{\mathbf{e}_{\ell}} \mathbf{a}_{0}}{\mathbf{e}_{\ell}^{H} \mathbf{a}_{0}}$$
(15)

where the fact $(\mathbf{e}_{\ell}^{H} \mathbf{e}_{\ell})^{-1} = 1$ is used in the above. The notation $\mathbf{P}_{\mathbf{A}} = \mathbf{A}(\mathbf{A}^{H}\mathbf{A})^{-1}\mathbf{A}^{H}$ stands for the projection operator onto the subspace $\langle \mathbf{A} \rangle$. \mathbf{e}_{ℓ} is the combination of the manifold vectors of the SOI and all the interferences and therefore the inequality $\mathbf{e}_{\ell}^{H}\mathbf{a}_{0} \neq 0$ holds. Denoting the projection onto the noise-subspace $\langle \mathbf{E}_{n} \rangle$ by \mathbf{P}_{n} (i.e., $\mathbf{P}_{n} = \mathbf{E}_{n}(\mathbf{E}_{n}^{H}\mathbf{E}_{n})^{-1}\mathbf{E}_{n}^{H}$), we have

$$\mathbf{P}_{n}\mathbf{a}_{0} = \mathbf{0}$$
$$\mathbf{P}_{\mathbf{e}_{\ell}} + \mathbf{P}_{\mathbf{A}_{\ell}} + \mathbf{P}_{n} = \mathbf{I}$$
(16)

Then pre-multiplying (15) by \mathbf{U}^H produces

$$\mathbf{U}^{H}\mathbf{e}_{\ell} = \frac{1}{\mathbf{e}_{\ell}^{H}\mathbf{a}_{0}}\mathbf{U}^{H}\left(\mathbf{I}-\mathbf{P}_{\mathbf{A}_{\ell}}-\mathbf{P}_{n}\right)\mathbf{a}_{0}$$

$$= \frac{1}{\mathbf{e}_{\ell}^{H}\mathbf{a}_{0}}\left(\mathbf{U}^{H}\mathbf{a}_{0}-\mathbf{U}^{H}\mathbf{P}_{\mathbf{A}_{\ell}}\mathbf{a}_{0}\right)$$

$$= \frac{1}{\mathbf{e}_{\ell}^{H}\mathbf{a}_{0}}\left(\mathbf{U}^{H}\mathbf{a}_{0}-\mathbf{H}^{H}\mathbf{P}_{\mathbf{A}_{\ell}}^{\perp}\mathbf{P}_{\mathbf{A}_{\ell}}\mathbf{a}_{0}\right)$$

$$= \frac{1}{\mathbf{e}_{\ell}^{H}\mathbf{a}_{0}}\mathbf{U}^{H}\mathbf{a}_{0} \qquad (17)$$

where the properties of $\mathbf{P}_{\mathbf{A}_{\ell}}^{\perp} = (\mathbf{P}_{\mathbf{A}_{\ell}}^{\perp})^{H}$ and $\mathbf{P}_{\mathbf{A}_{\ell}}^{\perp}\mathbf{P}_{\mathbf{A}_{\ell}} = \mathbf{0}$ are used in the above. Finally, we pre-multiply (17) by $(\mathbf{U}^{H}\mathbf{U})^{-1}$ and

insert (3) to obtain

$$(\mathbf{U}^{H}\mathbf{U})^{-1}\mathbf{U}^{H}\mathbf{e}_{\ell}$$

$$= \frac{1}{\mathbf{e}_{\ell}^{H}\mathbf{a}_{0}}(\mathbf{U}^{H}\mathbf{U})^{-1}\mathbf{U}^{H}\mathbf{a}_{0}$$

$$= \frac{1}{\mathbf{e}_{\ell}^{H}\mathbf{a}_{0}}(\mathbf{U}^{H}\mathbf{U})^{-1}\mathbf{H}^{H}\mathbf{P}_{\mathbf{A}_{\ell}}^{\perp}\mathbf{H}\mathbf{b}_{0}$$

$$= \frac{1}{\mathbf{e}_{\ell}^{H}\mathbf{a}_{0}}(\mathbf{U}^{H}\mathbf{U})^{-1}\mathbf{H}^{H}\mathbf{P}_{\mathbf{A}_{\ell}}^{\perp}\mathbf{P}_{\mathbf{A}_{\ell}}^{\perp}\mathbf{H}\mathbf{b}_{0}$$

$$= \frac{1}{\mathbf{e}_{\ell}^{H}\mathbf{a}_{0}}(\mathbf{U}^{H}\mathbf{U})^{-1}\mathbf{U}^{H}\mathbf{U}\mathbf{b}_{0}$$

$$= \frac{1}{\mathbf{e}_{\ell}^{H}\mathbf{a}_{0}}\mathbf{b}_{0} \qquad (18)$$

where the idempotent property of projection operator $\mathbf{P}_{\mathbf{A}_{\ell}}^{\perp} = \mathbf{P}_{\mathbf{A}_{\ell}}^{\perp} \mathbf{P}_{\mathbf{A}_{\ell}}^{\perp} \mathbf{P}_{\mathbf{A}_{\ell}}^{\perp}$ is utilized. It is obvious that the vector \mathbf{b}_{0} can be estimated by

$$\mathbf{b}_0 = \beta (\mathbf{U}^H \mathbf{U})^{-1} \mathbf{U}^H \mathbf{e}_\ell = \beta \mathbf{U}^\dagger \mathbf{e}_\ell \tag{19}$$

where $\mathbf{U}^{\dagger} = (\mathbf{U}^{H}\mathbf{U})^{-1}\mathbf{U}^{H}$ denotes the pseudo-inverse of \mathbf{U} . The proposed weight vector for the MDB can be constructed as

$$\mathbf{w}_{prop} = \beta \mathbf{R}^{-1} \mathbf{H} \mathbf{b}_0 = \beta \mathbf{R}^{-1} \mathbf{H} \mathbf{U}^{\dagger} \mathbf{e}_{\ell}$$
(20)

which is a Capon-type beamformer.

Remark 1: The assumption made in this paper is that the signalsubspace eigenvectors $[\mathbf{e}_1, \ldots, \mathbf{e}_{M+1}]$ are available, which is a quite weak assumption compared with that in [3] where it assumes that the IS, $\langle \mathbf{A}_I \rangle$, is perfectly known.

Remark 2: From (19), we observe that the proposed estimator is not related to the interference manifold vectors and therefore, theoretically, provides accurate estimation even when the interference signals are very close to the SOI. In comparison, the estimator of (11) in [4] is sensitive to the interferences close to the SOI.

Here we discuss two factors that are possible to restrict the accuracy of the estimator in (19). In practice, the covariance matrix \mathbf{R} is estimated by (2). If the number of snapshots acquired by the array is quite small, the orthogonality between $\hat{\mathbf{P}}_n$ and \mathbf{a}_0 may be impaired due to the effect of finite snapshots, which leads to $\hat{\mathbf{P}}_n \mathbf{a}_0 \neq \mathbf{0}$. Another factor is that the manifold vector \mathbf{a}_0 is not completely located in the subspace $\langle \mathbf{H} \rangle$. In the example of look direction error (see (4) and (5)), for instance, \mathbf{H} is formed by discarding the higher orders. In such case, the actual manifold may be modeled as

$$\mathbf{a}_0 = \mathbf{H}\mathbf{b}_0 + \Delta_0 \tag{21}$$

where $\mathbf{b}_0 = \mathbf{H}^{\dagger} \mathbf{a}_0$. Taking the effect of finite snapshots into account and inserting (21) into (17) and then (18), the estimated $\hat{\mathbf{b}}_0$ becomes

$$\widehat{\mathbf{b}}_{0} = \beta \left(\mathbf{b}_{0} + \widehat{\mathbf{U}}^{\dagger} \Delta_{0} - \widehat{\mathbf{U}}^{\dagger} \widehat{\mathbf{P}}_{n} \mathbf{a}_{0} \right)$$
(22)

Remark 3: Consider the case when the signal-subspace dimension (M + 1) is overestimated. The projector $\widehat{\mathbf{P}}_n$ with reduceddimension is still orthogonal to \mathbf{a}_0 , which implies that the estimator in (19) still works. Therefore the proposed method is robust to the overestimation of the signal-subspace dimension. If the signalsubspace dimension is underestimated, the estimated noise subspace containing the signal eigenvector(s) will have the SOI component, meaning that $\widehat{\mathbf{P}}_n$ is not orthogonal with \mathbf{a}_0 . Hence the proposed method fails in the case of underestimation.

Remark 4: In [4, 10], the situation where the coordinate vector \mathbf{b}_0 is not constant but random during the K snapshot observation

time due to the fast varying environment has been considered. Consequently the time-varying coordinate vector is modeled as $\mathbf{b}_0(t)$ and the contribution due to the SOI is given by

$$\mathbf{R}_0 = \sigma_0^2 \mathbf{H} \mathbf{R}_{bb} \mathbf{H} \tag{23}$$

where $\mathbf{R}_{bb} = \mathcal{E}\{\mathbf{b}_0(t) \mathbf{b}_0^H(t)\}$ is a full rank matrix (rank-*p*). In [4] only the simple case with known \mathbf{R}_{bb} is studied. Using *Corollary VI.2* of [10] one can estimate the rank-*p* matrix \mathbf{R}_0 without the knowledge of \mathbf{R}_{bb} . However, this approach cannot be employed in the scenario of this paper. This is because that when using *Corollary VI.2* of [10], \mathbf{R}_{bb} is required to be full rank such that its inversion \mathbf{R}_{bb}^{-1} exists. Nevertheless, in this paper we assume the coordinate vector \mathbf{b}_0 keeps unchanged during the *K* snapshot observation time, and therefore the matrix $\mathbf{R}_{bb} = \mathbf{b}_0 \mathbf{b}_0^H$ is rank-1 and not invertible.

4. SIMULATION STUDIES

In order to evaluate the effectiveness of the proposed MDB, three simulation studies have been carried out using a uniform linear array with N = 10 sensors and half-wavelength sensor spacing. The array operates in the presence of three equally-powered source signals where one is the SOI and two are interferences. It is assumed -3° pointing error in the SOI direction with the actual DOA $\theta_0 = 0^\circ$ and the nominal DOA $\overline{\theta}_0 = -3^\circ$. The matrix **H** is obtained by performing Gram-Schmidt algorithm on the matrix $[\mathbf{a}(\overline{\theta}_0) \quad \mathbf{a}(\overline{\theta}_0 - \Delta\theta) \quad \mathbf{a}(\overline{\theta}_0 + \Delta\theta)]$ where we set $\Delta\theta = 2^\circ$. The input signal-to-noise ratio (SNR) is 0dB. The DOA of the first interference signal is fixed at 20° throughout all the following examples. The three MDB algorithms simulated are: 1) the multirank MV-DR where \mathbf{b}_0 is obtained by computing the principal eigenvector of \mathbf{R}_{ee} in (10), 2) the MDB proposed in [3] (see (8)) where the perfect knowledge of the IS is assumed to be known a priori, 3) the proposed method where without loss of generality we choose $\ell = 1$. The weight vectors of these three MDB beamformer can be uniformly expressed as $\widehat{\mathbf{w}} = \widehat{\mathbf{R}}^{-1}\mathbf{H}\widehat{\mathbf{b}}_{0}$.

In the first example, the DOA of the second interference signal varies from -15° to -5° . The array output SINR performances of these methods versus the DOA of the second interference are displayed in Fig. 1. Compared with the multirank MVDR, Fig. 1 shows that the second interference has to be closer to the SOI before it leads to a SINR deterioration for the proposed method. In other words, the multirank MVDR method is more sensitive to the relatively close interference(s) than the other two methods.

Next, the robustness of the proposed algorithm to the overestimation or underestimation of the signal-subspace dimension is examined. The nominal signal-subspace dimension is changed from 1 to 9, while the actual dimension is 3. Fig. 2 illustrates that the SIN-R performance of the proposed method maintains when the nominal dimension is between 3 and 8. The proposed approach fails when the nominal dimension is underestimated (where the nominal dimension is 1 or 2) because the estimated $\hat{\mathbf{P}}_n$ contains the signal component which leads to $\hat{\mathbf{P}}_n \mathbf{a}_0 \neq \mathbf{0}$. When the nominal dimension is above 8, the proposed approach also fails which can be explained by the following fact. If the rank of the matrix \mathbf{A}_ℓ in (13) is over 8, then the rank of $\mathbf{P}_{\mathbf{A}_\ell}^{\perp}$ is N - 8 = 2. Thus the rank of the matrix product $\mathbf{U} = \mathbf{P}_{\mathbf{A}_\ell}^{\perp} \mathbf{H}$ is not greater than 2. Therefore the 3×3 matrix $\mathbf{U}^H \mathbf{U}$ becomes rank deficient and non-invertible.

Finally the effect of finite snapshots is investigated. The covariance matrix \mathbf{R} is estimated by using (2). The dimension of the signal subspace is 3. All other parameters are the same as that in the second example. The average of 500 independent simulation runs is used to plot each simulation point in Fig. 3. The three methods converge to the steady values when the snapshot number is over 400. Also, Fig.3 indicates that the proposed method is better than the multirank MVDR.



Fig. 1. Array output SINR versus the azimuth of the second interference; the first example.



Fig. 2. Array output SINR versus the nominal signal-subspace dimension; the second example.

5. CONCLUSIONS

The unknown actual manifold vector is modeled as a vector lying in a known subspace. The main contribution of this paper is that we develop an estimator of the coordinate vector without the knowledge of the interference subspace. This means that the condition with the perfect knowledge of interference subspace in [3] may be relaxed. Instead, the proposed estimator takes use of the signal subspace which can be easily obtained by performing eigen-decomposition on the covariance matrix. Another advantage of the proposed approach is the robustness to the signal dimension overestimation. Simulations reveal that the proposed MDB achieves the similar performance to that with perfect knowledge of interference knowledge, and outperforms the multirank MVDR proposed in [4] in the situation when the interference is relatively close to the SOI. A future



Fig. 3. Array output SINR versus the snapshot number; the third example.

direction of research consists of improving the coordinate vector estimator in the case of small snapshot number.

6. REFERENCES

- [1] J. Li and P. Stoica, Eds., *Robust Adaptive Beamforming*, John Wiley & Sons, 2005.
- [2] H. Cox, "Resolving power and sensitivity to mismatch of optimum array processors," *The Journal of the Acoustical Society of America*, vol. 54, no. 3, pp. 771–785, 1973.
- [3] O. Besson, L.L. Scharf, and F. Vincent, "Matched direction detectors and estimators for array processing with subspace steering vector uncertainties," *IEEE Trans. Signal Processing*, vol. 53, no. 12, pp. 4453 – 4463, dec. 2005.
- [4] A. Pezeshki, B. D. Van Veen, Louis L. Scharf, H. Cox, and M. L. Nordenvaad, "Eigenvalue beamforming using a multirank mvdr beamformer and subspace selection," *IEEE Trans. Signal Processing*, vol. 56, no. 5, pp. 1954–1967, 2008.
- [5] M.H. Er and B.C. Ng, "A self-steered array system for microwave communications," *IEEE Trans. Antennas and Propagation*, vol. 42, no. 8, pp. 1192–1195, Aug 1994.
- [6] G. Fuks, J. Goldberg, and H. Messer, "Bearing estimation in a ricean channel i.i. inherent accuracy limitations," *IEEE Trans. Signal Processing*, vol. 49, no. 5, pp. 925–937, may 2001.
- [7] J. Li, P. Stoica, and Z. Wang, "On robust capon beamforming and diagonal loading," *IEEE Trans. Signal Process.*, vol. 51, no. 7, pp. 1702–1715, July 2003.
- [8] R.G. Lorenz and S.P. Boyd, "Robust minimum variance beamforming," *IEEE Trans. Signal Process.*, vol. 53, no. 5, pp. 1684–1696, May 2005.
- [9] G. H. Golub and C. F. Van Loan, *Matrix Computations*, The Johns Hopkins University Press, October 1996.
- [10] L.L. Scharf and M.L. McCloud, "Blind adaptation of zero forcing projections and oblique pseudo-inverses for subspace detection and estimation when interference dominates noise," *IEEE Trans. Signal Processing*, vol. 50, no. 12, pp. 2938 – 2946, dec 2002.