

# POLYNOMIAL-PHASE SIGNAL SOURCE TRACKING USING AN ELECTROMAGNETIC VECTOR-SENSOR

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## ABSTRACT

A pre-processing technique is developed to track a polynomial-phase signal using an electromagnetic vector-sensor, which can be collocated or spatially-spread. The performance of the single-forgetting-factor algorithm incorporating the proposed pre-processing approach is improved significantly, and it even surpasses the performance of the multiple-forgetting-factor algorithm in a polynomial-phase source scenario. Simulation results verify the efficacy of the proposed technique.

**Index Terms**— Antenna arrays, radar tracking, direction of arrival estimation, FM radar.

## 1. INTRODUCTION

This paper develops a pre-processing technique to enhance the source-tracking performance in a polynomial-phase source scenario using a single vector-sensor. The proposed approach is based on the following facts: 1) The “vector-cross-product” algorithm inspired by the Poynting theorem is used for source-tracking using a single electromagnetic vector-sensor; 2) The  $q$ -order difference-function of a  $q$ -order polynomial-phase signal’s phase is a constant.

The polynomial phase-signal can be completely polarized, partially polarized or unpolarized, and the electromagnetic vector-sensor can be collocated or spatially-spread. The proposed approach has been used for direction-finding and polarization estimation with a single polarized vector-sensor in [1].

### 1.1. “Vector-Cross-Product” Based Source-Tracking with an Electromagnetic Vector-Sensor

An electromagnetic vector-sensor comprises a dipole triad and a loop triad. The dipole triad is composed of three orthogonally-collocated dipoles which are used to measure the three components of the signal’s electric field, and the loop triad is composed of three orthogonally-collocated loops which are used to measure the three components of the signal’s magnetic field. Since the electromagnetic vector-sensor can measure both the electric field and the magnetic field of the source, the Poynting vector can be derived from the

vector-cross-product of the measurements for the two fields. This vector-cross-product algorithm has been investigated extensively for direction-finding in [2–4]. These dipoles and loops, even spatially-spread [5, 6] (satisfying some conditions), can also be used to estimate the Poynting vector of the source.

This vector-cross-product approach has been investigated for source-tracking in [7]. The advantages of the source-tracking algorithm proposed in [7] are summarized as follows: 1) It can be used for various types of the sources and it is independent of the source’s frequency-spectrum. It can be used for both the completely/partially polarized signal and the unpolarized signal. 2) It has a low computational complexity. 3) Only a single vector-sensor is sufficient to track the source.

### 1.2. Polynomial Phase Signal

The polynomial-phase signal (PPS) has wide applications in radar, sonar and communication systems. Different signals are used in these systems with the phase as a continuous function of time. This function on a closed interval can be approximated by polynomials from the Weierstrass theorem [8]. The polynomial-phase signal can be modeled in continuous time as:

$$s(t; \psi) = \sqrt{\mathcal{P}} \exp\{j \varphi(t; \psi)\}, \quad (1)$$

$$\varphi(t; \psi) = b_0 + b_1 t + b_2 t^2 + \cdots + b_q t^q, \quad (2)$$

where  $\psi = (b_0, b_1, \dots, b_q)^T$  is a vector that contains the parameters in the polynomial phase  $\varphi(t; \psi)$ , with  $b_\ell$  ( $\ell = 0, 1, 2, \dots, q$ ) symbolizing the  $\ell$ -order coefficient,  $T$  denotes transposition, and  $q$  is the degree of the polynomial-phase signal. The initial phase of the polynomial-phase signal is  $b_0$ , and the power is  $\mathcal{P}$ .

It is notable that the phase of the signal,  $\varphi(t; \psi)$ , is a  $q$ -order polynomial of  $t$ . The  $q$ -order difference of  $\varphi(t; \psi)$  is thus a constant. Based on this property of the polynomial-phase signal and in order to extract the relationship in the adjacent measurement data vectors, we develop a pre-processing approach to enhance the efficacy of the source-tracking algorithm proposed in [7]. The following derivation will be based on the single-forgetting-factor (SFF) algorithm in order to save the computation workload.

## 2. MEASUREMENT MODEL

When the source is moving, the direction-of-arrival and polarization of the source will be time-varying. With  $(\theta, \phi)$  to denote the elevation-angle and azimuth-angle of the source,  $(\gamma, \eta)$  to denote the polarization parameters of the source [9], the array-manifold of the moving source can be shown as:

$$\mathbf{a}(t) \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{e}(t) \\ \mathbf{h}(t) \end{bmatrix} \stackrel{\text{def}}{=} \begin{bmatrix} e_x(\theta(t), \phi(t), \gamma(t), \eta(t)) \\ e_y(\theta(t), \phi(t), \gamma(t), \eta(t)) \\ e_z(\theta(t), \phi(t), \gamma(t), \eta(t)) \\ h_x(\theta(t), \phi(t), \gamma(t), \eta(t)) \\ h_y(\theta(t), \phi(t), \gamma(t), \eta(t)) \\ h_z(\theta(t), \phi(t), \gamma(t), \eta(t)) \end{bmatrix}.$$

<sup>1</sup> From the vector-cross-product, the Poynting vector:

$$\mathbf{p}(t) = \frac{\mathbf{e}(t) \times \mathbf{h}^*(t)}{\|\mathbf{e}(t) \times \mathbf{h}^*(t)\|} = \begin{bmatrix} \sin \theta(t) \cos \phi(t) \\ \sin \theta(t) \sin \phi(t) \\ \cos \theta(t) \end{bmatrix} \quad (3)$$

$$\stackrel{\text{def}}{=} [u(t) \ v(t) \ w(t)]^T, \quad (4)$$

where  $\times$  denotes the vector-cross operation,  $*$  symbolizes the complex conjugation,  $\|\cdot\|$  represents the Frobenius norm, and  $T$  denotes the transposition.

The collected data-set at time  $t$  of the vector-sensor is:

$$\mathbf{y}(t) = \mathbf{a}(t)s(t; \boldsymbol{\psi}) + \mathbf{n}(t) \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{y}_e(t) \\ \mathbf{y}_h(t) \end{bmatrix}, \quad (5)$$

where  $\mathbf{n}(t)$  is the  $6 \times 1$  additive noise vector.

## 3. PROPOSED TECHNIQUE

Thanks to the vector-cross-product result in (3), the elevation-azimuth angle of the source can be estimated straightforwardly. The target of the source-tracking is to estimate the accurate  $\mathbf{p}(t)$ , (then the accurate arriving-angle) from the collected data set in (5).

### 3.1. Review the SFF Algorithm in [7]

Using an exponential window with a forgetting factor  $\lambda$ , the Poynting vector can be estimated by [7]:

$$\hat{\mathbf{p}}_N = \frac{\frac{1}{\sum_{n=1}^N \lambda^{-n}} \sum_{n=1}^N \lambda^{-n} \text{Re}\{\mathbf{y}_e(t) \times \mathbf{y}_h^*(t)\}}{\left\| \frac{1}{\sum_{n=1}^N \lambda^{-n}} \sum_{n=1}^N \lambda^{-n} \text{Re}\{\mathbf{y}_e(t) \times \mathbf{y}_h^*(t)\} \right\|} \stackrel{\text{def}}{=} [\hat{u}_N, \hat{v}_N, \hat{w}_N]^T, \quad (6)$$

where  $\text{Re}\{\cdot\}$  denotes the real-part of the complex number in  $\{\cdot\}$ . The direction-of-arrival (DOA) of the source can then be estimated by:  $\hat{\theta}_N = \arccos(\hat{w}_N)$ ,  $\hat{\phi}_N = \angle(\hat{u}_N + j\hat{v}_N)$ , where  $\angle$  denotes the complex angle of the ensuing number.

<sup>1</sup>For the entities in the array-manifold  $\mathbf{a}$ , please refer to [6, 9, 10].

The aim of this paper is to estimate the  $\hat{\mathbf{p}}_N$  more precisely, from the collected data-vector in (5) in a polynomial-phase source scenario. The following will show how and why.

### 3.2. The Pre-Processing Technique

With  $T_s$  to denote the sampling time-interval, consider there are  $M$  time samples. The collected  $6 \times M$  data set will be:

$$\mathbf{Y} = [\mathbf{y}(T_s), \mathbf{y}(2T_s), \dots, \mathbf{y}(MT_s)], \quad (7)$$

where  $\mathbf{y}(mT_s) = \mathbf{a}(mT_s)s(mT_s, \boldsymbol{\psi}) + \mathbf{n}(mT_s), \forall m = 1, 2, \dots, M$ . In order to simplify the exposition, consider the *noiseless* case in the following derivation. Let  $\mathbf{x}_{(q)}(t) = \mathbf{y}(t)$  and this  $\mathbf{x}_{(q)}(t)$  is then a  $6 \times 1$  vector. Let  $x_{i,q}(t)$  be the  $i$ th row of  $\mathbf{x}_{(q)}(t), \forall i = 1, 2, \dots, 6$ . Perform the following recursive computations in the box:

1) For the collected data-vector at time  $t$ ,

$$\mathbf{x}_{(q-1)}(t) = \mathbf{x}_{(q)}(t)x_{i,q}^*(t + T_s). \quad (8)$$

2) Repeat step 1) for  $q = q - 1$  until  $\mathbf{x}_{(0)}(t)$  is reached.

For a  $q$ -order PPS, in total there are  $q$  times recursive computations of step 1). For each recursive computation for step 1), the number of data-vector in  $\mathbf{Y}$  will decrease one. We presume the DOA and the polarization of the PPS remain the same during the sequential  $(q + 1)T_s$  time-intervals. By the described computations above, the first  $(q + 1)$  data-vector in (7),  $[\mathbf{y}(T_s), \mathbf{y}(2T_s), \dots, \mathbf{y}((q + 1)T_s)]$  will carry out one  $6 \times 1$  data-vector:

$$\mathbf{z}(T_s) = \mathbf{x}_{(0)}(T_s) = \tilde{\mathbf{a}}(T_s)e^{j(-1)^q b_q(q!)T_s^q}, \quad (9)$$

$$\tilde{\mathbf{a}}(T_s) \stackrel{\text{def}}{=} \mathbf{a}(T_s) \left( \left[ |\mathbf{a}(T_s)|_i \right]^{(2^{(q-1)}-1)} [\mathbf{a}(T_s)]_i^* \right), \quad (10)$$

where  $[\mathbf{a}]_i$  symbolizes the  $i$ th element of the vector in  $[\ ]$ . Similarly, from any  $(q + 1)$  contiguous data vectors in (7),  $[\mathbf{y}(nT_s), \mathbf{y}((n + 1)T_s), \dots, \mathbf{y}((n + q)T_s)]$ , we can obtain:

$$\mathbf{z}(nT_s) = \tilde{\mathbf{a}}(nT_s)e^{j(-1)^q b_q(q!)T_s^q} \quad (11)$$

$$\stackrel{\text{def}}{=} [\mathbf{z}_e^T(nT_s), \mathbf{z}_h^T(nT_s)]^T, \forall n = 1, \dots, (M - q).$$

In the *noisy case*, (9)-(11) will become approximated.

It is worth noting that  $\mathbf{z}(nT_s)$  is now independent of the signal model, and  $e^{j(-1)^q b_q(q!)T_s^q}$  is a constant and only depends on: 1) the sampling time-interval  $T_s$ , 2) the order of the polynomial-phase signal  $q$ , and 3) the highest order coefficient  $b_q$ , all of which are time independent. The  $\mathbf{z}(nT_s)$  can then be seen as a complex number  $c \stackrel{\text{def}}{=} e^{j(-1)^q b_q(q!)T_s^q}$ , multiplying the modified time-varying array-manifold  $\tilde{\mathbf{a}}(nT_s)$ , and this  $c$  is a constant. Hence, through the above processing, the source-tracking performance will be independent of the signal model and only depends on the noise. This is the issue investigated in [7].

The following problem is to adaptively estimate  $\mathbf{p}(nT_s)$  over  $n = 1, 2, \dots, (M-q)$ , from  $[\mathbf{z}(T_s), \mathbf{z}(2T_s), \dots, \mathbf{z}((M-q)T_s)]$ . The single-forgetting-factor (SFF) algorithm reviewed in Section 3.1 and the multiple-forgetting-factor (MFF) algorithm in [7] can be adopted to track the source. For the SFF algorithm:

$$\hat{\mathbf{p}}(nT_s) = \frac{\text{Re}\{\mathbf{z}_e(nT_s) \times \mathbf{z}_h^*(nT_s)\}}{\|\text{Re}\{\mathbf{z}_e(nT_s) \times \mathbf{z}_h^*(nT_s)\}\|}, \quad (12)$$

$$\hat{\mathbf{p}}_N = \frac{\sum_{n=0}^N \lambda^{-n} \hat{\mathbf{p}}(nT_s)}{\sum_{n=0}^N \lambda^{-n}}. \quad (13)$$

The following recursive relation is obtained:

$$\hat{\mathbf{p}}(nT_s) = \lambda \hat{\mathbf{p}}(nT_s - T_s) + (1 - \lambda) \hat{\mathbf{p}}(nT_s), \quad (14)$$

$$\forall n = 1, 2, \dots, N.$$

Then using this  $\hat{\mathbf{p}}(nT_s)$  in (14) to replace the  $\hat{\mathbf{p}}_N$  in (6) to estimate the DOA of the source will improve the tracking performance.

**Remarks:**

- In (8), step 1) to compute  $\mathbf{x}_{(0)}(t)$ , any one row in  $\mathbf{x}_{(q)}(t)$  can be used. This does not affect the following derivation. In cases when any one row is equal to zero, we can use any other nonzero entity.
- (8) in step 1) can be changed to:

$$\mathbf{x}_{(q-1)}(t) = \sum_{i=1}^6 \mathbf{x}_{(q)}(t) x_{i,q}^*(t + T_s). \quad (15)$$

Though (15) will increase the computation workload, it has the following advantages: a) preserving the signal, b) enhancing the noise cancellation, and c) avoiding the case when one row in  $\mathbf{x}_{(q)}(t)$  is equal to zero.

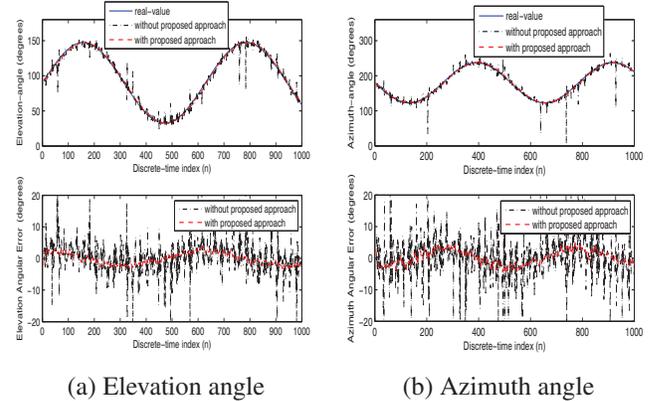
- For the MFF tracking approach in [7], the described pre-processing technique can also be adopted.
- The pre-processing technique proposed in this work and the tracking approaches proposed in [7] can be synergized with a Kalman filter to improve the tracking performance [7, 10, 11].

#### 4. SIMULATION

The efficacy of the proposed approach is demonstrated by Monte Carlo simulations. A second-order unity-power polynomial-phase signal (a.k.a. an LFM or a Chirp signal) is used with  $\{b_0 = 0.05, b_1 = 0.1, b_2 = 0.13\}$  impinging on an electromagnetic vector-sensor. The polarization of the signal is  $(\gamma = \pi/4, \eta = \pi/2)$ . Figure 1 plots the moving loci of the elevation-angle and the azimuth angle. The angular errors of the arriving-angle  $(\theta_r, \phi_r)$  are also plotted (the signal-to-noise ratio is set as 20dB).

Both the tracking results of SFF algorithm with and without incorporating the proposed pre-processing technique are presented. Figure 2 plots the corresponding results with a 4-order unity-power polynomial-phase signal with  $\{b_0 = 0.05, b_1 = 0.1, b_2 = 0.13, b_3 = 0.23, b_4 = 0.29\}$ . It can be seen clearly that the SFF algorithm incorporating the proposed technique outperforms its counterpart without incorporating the proposed technique. Figure 3 plots the corresponding results of a 2-order polynomial-phase signal with the MFF algorithm by setting  $\lambda_1 = 0.9, \lambda_2 = 0.8, \lambda_3 = 0.7$ . Table 1 summarizes the mean-values and standard-deviations of angular errors  $(\theta_r, \phi_r)$  with different source-tracking methods in the 2-order polynomial-phase signal scenario as in Figure 1. (For the meanings of  $\lambda_1, \lambda_2, \lambda_3$  of the MFF algorithm in Table 1, please refer to [7].) From the simulation results, it can be seen that with the proposed technique, the source tracking performance can be improved substantially with a lower computational complexity because the performance of the SFF approach outperforms the performance of the MFF approach. For the comparison of the computation workload between the SFF and the MFF methods, please refer to [7]. In addition, the standard-deviations of  $(\theta_r, \phi_r)$  with SFF algorithm decline when  $\lambda$  increases.

It is worth pointing out that the SFF and MFF algorithms can be used for arbitrary signal-model, but the proposed pre-processing technique can only be used in a polynomial-phase source scenario as discussed in this paper.



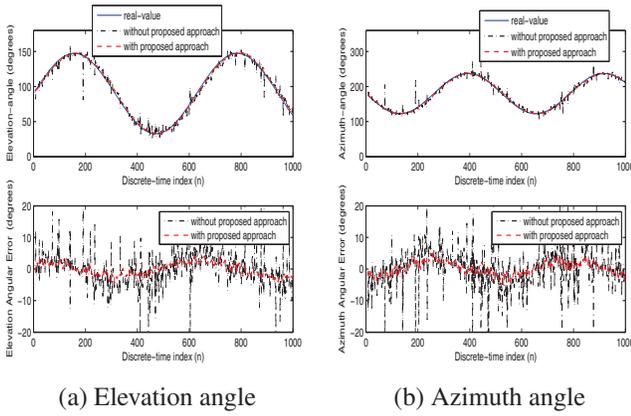
**Fig. 1:** SFF source-tracking of a 2-order polynomial-phase signal with  $\lambda = 0.8$ .

#### 5. CONCLUSION

A pre-processing technique is proposed to improve the source-tracking performance in a polynomial-phase signal scenario using a single electromagnetic vector sensor. This approach incorporating the SFF algorithm can offer a better performance than the MFF algorithm with a lower computation workload.

**Table 1:** Mean-Values and Standard-Deviations of  $(\theta_r, \phi_r)$  (in degree) of a 2-order Polynomial-Phase Signal

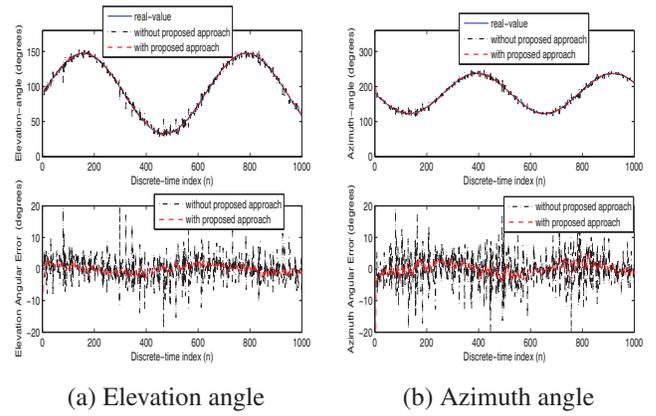
	$\lambda_1(\lambda)$	$\lambda_2$	$\lambda_3$	Mean of $\theta_r$	Std. Dev. of $\theta_r$	Mean of $\phi_r$	Std. Dev. of $\phi_r$
MFF without the proposed technique	0.9	0.8	0.7	0.045	3.707	0.0141	4.697
MFF with the proposed technique	0.9	0.8	0.7	-0.134	<b>1.232</b>	-0.0412	<b>1.759</b>
SFF without the proposed technique	0.9	—	—	0.118	7.341	0.493	10.657
SFF with the proposed technique	0.9	—	—	-0.314	3.596	0.422	4.180
SFF without the proposed technique	0.8	—	—	0.229	5.751	0.711	10.082
SFF with the proposed technique	0.8	—	—	-0.110	1.782	0.168	2.103
SFF without the proposed technique	0.7	—	—	0.107	3.581	0.052	4.847
SFF with the proposed technique	0.7	—	—	-0.035	<b>1.223</b>	0.072	<b>1.480</b>



**Fig. 2:** SFF source-tracking of a 4-order polynomial-phase signal with  $\lambda = 0.8$ .

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**Fig. 3:** MFF source-tracking of a 2-order polynomial-phase signal with  $\lambda_1 = 0.9, \lambda_2 = 0.8, \lambda_3 = 0.7$ .

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