DOA ESTIMATION OF AMPLITUDE MODULATED SIGNALS WITH LESS ARRAY SENSORS THAN SOURCES

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ABSTRACT

This paper addresses the Direction-of-Arrival (DOA) estimation problem for amplitude modulated signals whose number is more than that of the array sensors. The proposed method is based on an idea of virtual array. For source signals with amplitude modulation, such as binary phase shift keying (BPSK) and M-ary amplitude shift keying (M-ASK), we show that introducing in virtual array actually gives rise to processing the fourth-order moments of array output, which is related to higher-order statistics (HOS) techniques. While traditional HOS methods in array processing mainly exploit higher-order cumulants of the received data, we propose a DOA estimation method based on the fourth-order moments, which is of lower computational load than the fourth-order cumulants. Simulation results demonstrate the effectiveness of the proposed method for estimating DOAs of more source signals than array elements.

***Index Terms*—** DOA estimation, amplitude modulated signal, virtual array, under-determined

1. INTRODUCTION

In the field of array signal processing, addressing the direction-of-arrival (DOA) estimation problem has been widely studied [1]. However, classical DOA estimation methods always assume the number of sources is smaller than the number of array sensors.

Being one way out for handling the case of more sources than sensors (*i.e.*, under-determined case), higher-order statistics (HOS) techniques were introduced into array processing to deal with DOA estimation problem for non-Gaussian signals. The HOS methods give rise to higher resolution and increased degree-of-freedom. In [2] an HOS based MUSIC algorithm was proposed which utilizes fourth-order cumulants in lieu of the traditional second-order moments based approaches. It is demonstrated in [3] and [4] that the effective array aperture can be increased and the Gaussian noise can be suppressed by using higher-order cumulants. In [5] and [6], DOA estimation problems based on the fourth-order and higher order statistics are generalized as a uniform framework, leading to the concept of virtual array with an arbitrary even order ($2q$th order). Based on the $2q$th order virtual array, the $2q$-MUSIC algorithm has been proposed in [7], which can be regarded as a generalization of the standard MUSIC algorithm. Due to the increased degree-of-freedom, the HOS methods can estimate DOAs of more sources than array sensors. Most recently, a new DOA estimation approach based on the Khatri-Rao subspace is proposed in [8]. Although no higher-order cumulants are used, the Khatri-Rao subspace approach can also perform under-determined DOA estimation, assuming signals are quasi-stationary (for instance, speech and audio signals).

In this paper we deal with the DOA estimation problem for amplitude modulated signals in the under-determined case. A concept of virtual array and the procedure of constructing it are proposed, resulting in the processing of fourth-order moment of array output data. Based on the fourth-order moments, an MVDR type method is utilized to perform DOA estimation. The proposed higher-order moment based method is of lower computational complexity than calculating the fourth-order cumulants as in the traditional HOS methods, meanwhile it can also address under-determined DOA estimation cases and need not to know *a priori* the number of sources.

2. SIGNAL MODEL

Consider a uniform linear array (ULA) consisting of M omni-directional antennas with half-wavelength interelement spacing. There are n narrowband plane waves impinging on the array from angles θ_i, $(i = 1, ..., n)$ which are measured with respect to the broadside. The complex envelopes of these plane waves are $s_i(t)$ and they are assumed to be uncorrelated with each other. Moreover, to facilitate the following proposed DOA estimation method, here we assume that the source signals are amplitude modulated, such as binary phase shift keying (BPSK) and M-ary amplitude shift keying (M-ASK). Thereby the complex envelopes $s_i(t)$'s are real valued, *i.e.*, $s_i^*(t) = s_i(t)$, where the superscript '*' represents the complex conjugate.

The steering vector corresponding to each of the above source signals is respectively expressed as

$$\mathbf{a}(\theta_i) = \left[1, e^{-j\pi \sin\theta_i}, \cdots, e^{-j(M-1)\pi \sin\theta_i}\right]^T, \tag{1}$$

where $i = 1, ..., n$, $j = \sqrt{-1}$. And the baseband output at each sensor is denoted as $x_i(t)$. Then according to the definition above, the baseband array output can be written as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{e}(t), \tag{2}$$

where $\mathbf{x}(t) = [x_1(t), ..., x_M(t)]^T$, $\mathbf{s}(t) = [s_1(t), ..., s_n(t)]^T$, $\mathbf{A} = [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_n)]$ denotes the array manifold matrix, and $\mathbf{e}(t) \in \mathbb{C}^{M\times 1}$ is temporally and spatially white noise which is uncorrelated from the source signals. The array output vector $\mathbf{x}(t)$ is called a *snapshot*, and in this paper it is also termed the *actual snapshot* when necessary.

Therefore, the covariance matrix of the array output is defined as

$$\mathbf{R}_{xx} = E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma_e^2\mathbf{I}, \tag{3}$$

where $\mathbf{R}_{ss} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is the source covariance matrix, and σ_e^2 is the noise power.

3. CONSTRUCTION OF VIRTUAL ARRAY

The virtual array used for proposed under-determined DOA estimation method is constructed in two steps. In the first step, Kronecker product is used to obtain the initial virtual snapshot. In the second step, a dimension-reduction operation is performed on the results of the first step to produce the virtual array with interference-cancellation, which is used for the subsequent DOA estimation. The following subsections describe the construction procedures in detail.

3.1. Kronecker Product of Actual Snapshot

Given the actual snapshot in (2), by utilizing Kronecker product, a set of virtual snapshot can be constructed as

$$\begin{aligned}\mathbf{x}_v(t) &= \mathbf{x}(t) \otimes \mathbf{x}^*(t) \\ &= (\mathbf{A} \otimes \mathbf{A}^*) \cdot (\mathbf{s}(t) \otimes \mathbf{s}^*(t)) + \tilde{\mathbf{e}}(t), \end{aligned} \tag{4}$$

where $\otimes$ denotes the Kronecker product, and

$$\begin{aligned}\tilde{\mathbf{e}}(\mathbf{t}) = &\ (\mathbf{I}_M \otimes \mathbf{A}^*) \cdot (\mathbf{e}(t) \otimes \mathbf{s}^*(t)) \\ &+ (\mathbf{A} \otimes \mathbf{I}_M) \cdot (\mathbf{s}(t) \otimes \mathbf{e}^*(t)) + \mathbf{e}(t) \otimes \mathbf{e}^*(t) \end{aligned} \tag{5}$$

denotes the terms that contribute as noise.

Moreover, $(\mathbf{A} \otimes \mathbf{A}^*) \cdot (\mathbf{s}(t) \otimes \mathbf{s}^*(t))$ in (4) is regarded as (virtual) signal term which can be denoted as $\tilde{\mathbf{s}}(t)$ and written more explicitly as

$$\begin{aligned}\tilde{\mathbf{s}}(t) = &\sum_{i=1}^{n} (\mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_i)) |s_i(t)|^2 \\ &+ \sum_{i=1}^{n} \sum_{\substack{k=1 \\ k \neq i}}^{n} (\mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_k)) s_i(t) s_k^*(t). \end{aligned} \tag{6}$$

Recall that the complex envelope of each source $s_i(t), i = 1, ..., n$, is real-valued, hence $s_i(t)s_k^*(t) = s_k(t)s_i^*(t)$ for $1 \leq i, k \leq n$, and $|s_i(t)|^2 = s_i^2(t)$. Therefore certain terms in (6), such as $(\mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_k))\, s_i(t)s_k^*(t)$ and $(\mathbf{a}(\theta_k) \otimes \mathbf{a}^*(\theta_i))\, s_k(t)s_i^*(t)$, can be combined, and then (6) can be further expressed as:

$$\tilde{\mathbf{s}}(t) = \sum_{i=1}^{n} \mathbf{a}_v(\theta_i) s_i^2(t) + \sum_{i=1}^{n} \sum_{k=i+1}^{n} \mathbf{b}_v(\theta_i, \theta_k) s_i(t) s_k(t), \tag{7}$$

where $\mathbf{a}_v(\theta_i) = \mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_i)$ and $\mathbf{b}_v(\theta_i, \theta_k) = \mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_k) + \mathbf{a}(\theta_k) \otimes \mathbf{a}^*(\theta_i)$, $1 \leq i, k \leq n$. Then according to (4) and (7), the virtual snapshot in (4) can be rewritten as

$$\mathbf{x}_v(t) = \tilde{\mathbf{s}}(t) + \tilde{\mathbf{e}}(t) = \mathbf{A}_v \cdot \mathbf{s}_v(t) + \tilde{\mathbf{e}}(t), \tag{8}$$

where

$$\mathbf{A}_v = [\mathbf{a}_v(\theta_1), \cdots, \mathbf{a}_v(\theta_n), \mathbf{b}_v(\theta_1, \theta_2), \cdots, \mathbf{b}_v(\theta_{n-1}, \theta_n)]$$

and

$$\mathbf{s}_v(t) = \left[s_1^2(t), \cdots, s_n^2(t), s_1(t)s_2(t), \cdots, s_{n-1}(t)s_n(t)\right]^T.$$

It is observed that (8) resembles the original array signal model in (2), thus can be regarded as a virtual array signal model with the number of (virtual) source signals $n(n+1)/2$ and the size of (virtual) array M^2.

3.2. Virtual Array with Interference-Cancellation

According to the virtual array signal model described by (8), we can see that although the array is virtually expanded to a size of M^2, there are also newly generated virtual source signals $s_i(t)s_k(t), i \neq k$. Obviously, virtual signals $s_i^2(t), i = 1, ..., n$, are directly related to each of the actual sources respectively. However, $s_i(t)s_k(t)$'s can be regarded as interference. Therefore, the increase of virtual array aperture comes with the cost of increased number of virtual interference.

On the other hand, the direct Kronecker product of snapshot actually brings in redundant phase information.

Therefore, to address the aforementioned problems of virtual interference and redundant phase, we construct a new virtual array structure which gains the advantage of increased degree-of-freedom, thus being able to deal with under-determined DOA problem.

3.2.1. Steering Vector with Redundancy-Reduction

We start by considering the phase redundancy reduction, but later it will be shown that the operation of reducing redundancy simultaneously leads to interference cancellation.

According to a basic algebraic relationship that $\mathrm{vec}(\mathbf{ab}^T) = \mathbf{b} \otimes \mathbf{a}$ for column vectors $\mathbf{a}$ and $\mathbf{b}$, the virtual steering vector $\mathbf{a}_v(\theta_i)$ in (7) can be expressed as

$$\mathbf{a}_v(\theta_i) = \mathrm{vec}\left(\mathbf{a}^*(\theta_i) \cdot \mathbf{a}^T(\theta_i)\right), \tag{9}$$

where '$\mathrm{vec}(\cdot)$' denotes the vectorization operator (in a column-wise manner). Therefore, we term the matrix $\mathbf{a}^*(\theta_i) \cdot \mathbf{a}^T(\theta_i)$ as *constructing matrix*. Based on the definition of $\mathbf{a}(\theta_i)$ in (1), the constructing matrix can be expressed in an exponential form as:

$$\mathbf{a}^*(\theta_i) \cdot \mathbf{a}^T(\theta_i) = \exp(j\mathbf{D}), \tag{10}$$

where '$\exp(\cdot)$' denotes performing entry-wise exponential function to a matrix, and

$$\mathbf{D} = \begin{bmatrix} 0 & \phi_i & \cdots & (M-1)\phi_i \\ -\phi_i & 0 & \cdots & (M-2)\phi_i \\ -2\phi_i & -\phi_i & \cdots & (M-3)\phi_i \\ \vdots & \vdots & \ddots & \vdots \\ -(M-1)\phi_i & -(M-2)\phi_i & \cdots & 0 \end{bmatrix},$$

where $\phi_i = -\pi \sin\theta_i$ represents the phase information that is related to the DOA θ_i. Actually there are only $(2M-1)$ distinct entries in this $M \times M$ matrix $\mathbf{D}$. Thus, more specifically, the M^2-length virtual steering vector $\mathbf{a}_v(\theta_i)$ contains only $(2M-1)$ distinct exponential terms: $e^{jm\phi_i}, m = -(M-1), \cdots, (M-1)$.

Hence, according to (9) and (10), we can reduce the dimension of $\mathbf{a}_v(\theta_i)$ by discarding the redundant elements in vector $\mathbf{a}_v(\theta_i)$ and preserving $(2M-1)$ distinct elements. This procedure is equivalent to selecting corresponding distinct entries from the constructing matrix $\mathbf{a}^*(\theta_i) \cdot \mathbf{a}^T(\theta_i)$, and there are various ways to perform the selection. To facilitate the under-determined DOA estimation, here we adopt a specific selection strategy by picking on the first column and the first row from the constructing matrix. And then by rearranging these selected terms by the order of increasing multiples of phase ϕ_i and forming a column vector, we construct the redundancy-reduction (also with reduced-dimension) steering vector as:

$$\mathbf{a}_r(\theta_i) = \left[\ e^{-j(M-1)\phi_i}, \cdots, 1, \cdots, e^{j(M-1)\phi_i}\ \right]^T. \tag{11}$$

Apparently, elements of $\mathbf{a}_r(\theta_i)$ form a subset of $\mathbf{a}_v(\theta_i)$'s elements. To express this relationship, we introduce a manipulation matrix $\mathbf{Q}$ to express this selection strategy:

$$\mathbf{a}_r(\theta_i) = \mathbf{Q} \cdot \mathbf{a}_v(\theta_i). \tag{12}$$

The size of matrix $\mathbf{Q}$ is $(2M-1) \times M^2$, and in each row there's only one non-zero entry (whose values is 1). The effect of $\mathbf{Q}$ is to select elements from vector $\mathbf{a}_v(\theta_i)$ according to aforementioned selection strategy of constructing redundancy-reduction structure. More specifically, matrix $\mathbf{Q}$ can be expressed as:

$$\mathbf{Q} = \left[\begin{array}{c:c} \begin{matrix} & & 1 \\ & \iddots & \\ 1 & & \end{matrix} & \mathbf{0}_{M\times(M^2-M)} \\ \hdashline \mathbf{0}_{(M-1)\times M} & \mathbf{I}_{M-1} \otimes \underbrace{[1, 0, \cdots, 0]}_{1\times M} \end{array}\right]. \tag{13}$$

Notice that the upper left block of $\mathbf{Q}$ is an anti-diagonal matrix.

3.2.2. *Cancellation of Interference*

Now we show that by performing the redundancy-reduction (also as dimension-reduction) operation, the virtual interference steering vectors $\mathbf{b}_v(\theta_i, \theta_k)$ in (7) can be expressed by $\mathbf{a}_r(\theta_i)$, thus equivalently being eliminated.

The redundancy-reduction operation on $\mathbf{b}_v(\theta_i, \theta_k)$ is performed with matrix $\mathbf{Q}$ as $\mathbf{Q} \cdot \mathbf{b}_v(\theta_i, \theta_k) = \mathbf{Q} \cdot (\mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_k)) + \mathbf{Q} \cdot (\mathbf{a}(\theta_k) \otimes \mathbf{a}^*(\theta_i))$. Parallel to Section 3.2.1, to illustrate this operation more clearly, we form a constructing matrix to construct the dimension-redundant vector $\mathbf{Q} \cdot (\mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_k))$.

Notice that $\mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_k) = \mathrm{vec}\left(\mathbf{a}^*(\theta_k) \cdot \mathbf{a}^T(\theta_i)\right)$, and then we have

$$\mathbf{a}^*(\theta_k) \cdot \mathbf{a}^T(\theta_i) = \exp(j\mathbf{E})$$

where

$$\mathbf{E} = \begin{bmatrix} 0 & \phi_i & \cdots & (M-1)\phi_i \\ -\phi_k & \phi_i - \phi_k & \cdots & (M-1)\phi_i - \phi_k \\ -2\phi_k & \phi_i - 2\phi_k & \cdots & (M-1)\phi_i - 2\phi_k \\ \vdots & \vdots & \ddots & \vdots \\ -(M-1)\phi_k & \phi_i - (M-1)\phi_k & \cdots & (M-1)(\phi_i - \phi_k) \end{bmatrix}.$$

In accordance with the construction of $\mathbf{a}_r(\theta_i)$ discussed in 3.2.1, we form the dimension-reduction version of $(\mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_k))$ by picking on the first column and first row from the matrix $\exp(j\mathbf{E})$, and then rearrange these terms into a column vector in the same manner as forming $\mathbf{a}_r(\theta_i)$ in (11). Therefore, we obtain

$$\begin{aligned} &\mathbf{Q} \cdot (\mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_k)) \\ &\quad = \left[e^{-j(M-1)\phi_k}, \cdots, e^{-j\phi_k}, 1, e^{j\phi_i}, \cdots, e^{j(M-1)\phi_i}\right]^T. \end{aligned} \tag{14}$$

Using (14), the result of dimension-deduction performed on interference steering vector $\mathbf{b}_v(\theta_i, \theta_k)$ is:

$$\begin{aligned} & \mathbf{Q} \cdot \mathbf{b}_v(\theta_i, \theta_k) \\ = \; & \mathbf{Q} \cdot (\mathbf{a}(\theta_i) \otimes \mathbf{a}^*(\theta_k)) + \mathbf{Q} \cdot (\mathbf{a}(\theta_k) \otimes \mathbf{a}^*(\theta_i)) \\ = \; & \Big[\left(e^{-j(M-1)\phi_k} + e^{-j(M-1)\phi_i}\right), \cdots, (1+1), \cdots, \\ & \left(e^{j(M-1)\phi_k} + e^{j(M-1)\phi_i}\right)\Big]^T \\ = \; & \left[e^{-j(M-1)\phi_i}, \cdots, 1, \cdots, e^{j(M-1)\phi_i}\right]^T \\ & + \left[e^{-j(M-1)\phi_k}, \cdots, 1, \cdots, e^{j(M-1)\phi_k}\right]^T \\ = \; & \mathbf{a}_r(\theta_i) + \mathbf{a}_r(\theta_k). \end{aligned} \tag{15}$$

Therefore, the virtual interference steering vectors $\mathbf{Q} \cdot \mathbf{b}_v(\theta_i, \theta_k)$, $1 \le i, k \le n$, are the linear combinations of $\mathbf{a}_r(\theta_i)$, $1 \le i \le n$.

3.2.3. *Virtual Snapshot with Interference-Cancellation*

Next we perform the 'multiplying by $\mathbf{Q}$' operation on the virtual snapshot $\mathbf{x}_v(t)$ in (8), which will be shown to result in the interference-cancellation and increase of degree-of-freedom. Using matrix $\mathbf{Q}$, the dimension-reduced version of $\mathbf{x}_v(t)$ is expressed as

$$\mathbf{x}_r(t) = \mathbf{Q} \cdot \mathbf{x}_v(t) = \mathbf{Q} \cdot \mathbf{A}_v \cdot \mathbf{s}_v(t) + \mathbf{Q} \cdot \tilde{\mathbf{e}}(t). \tag{16}$$

Substituting the results of (12) and (15) into (16), we further obtain:

$$\begin{aligned} \mathbf{x}_r(t) &= \mathbf{Q} \cdot \mathbf{A}_v \cdot \mathbf{s}_v(t) + \mathbf{Q} \cdot \tilde{\mathbf{e}}(t) \\ &= [\, \mathbf{a}_r(\theta_1), \cdots, \mathbf{a}_r(\theta_n), (\mathbf{a}_r(\theta_1) + \mathbf{a}_r(\theta_2)), \\ &\quad \cdots, (\mathbf{a}_r(\theta_{n-1}) + \mathbf{a}_r(\theta_n)) \,] \cdot \mathbf{s}_v(t) + \mathbf{Q} \cdot \tilde{\mathbf{e}}(t). \end{aligned} \tag{17}$$

Note that the virtual array manifold matrix $\mathbf{Q} \cdot \mathbf{A}_v$ consists of $\mathbf{a}_r(\theta_i), i = 1, \cdots, n$, as well as the linear combinations of them. Then by combining the $\mathbf{a}_r(\theta_i)$'s, (17) can be rewritten as

$$\mathbf{x}_r(t) = \tilde{\mathbf{A}}_r \cdot \mathbf{s}_r(t) + \mathbf{Q} \cdot \tilde{\mathbf{e}}(t), \tag{18}$$

where

$$\tilde{\mathbf{A}}_r = [\, \mathbf{a}_r(\theta_1) \,, \, \cdots \,, \, \mathbf{a}_r(\theta_n) \,],$$

and

$$\mathbf{s}_r(t) = \sum_{i=1}^{n} s_i(t) \cdot [\, s_1(t), \cdots, s_n(t) \,]^T.$$

Comparing this virtual array signal model with the original signal model in (2), we see that the number of sources in the new model is the same as that in the original model, which equals to n, while the array size is enlarged to $(2M-1)$. Thus the increase of degree-of-freedom can lead to better DOA resolution and allow the array to deal with more source signals than actual sensors. More specifically, the virtual array is able to resolve up to $(2M-2)$ sources.

4. DOA ESTIMATION BY NON-REDUNDANT MVDR METHOD

Based on the previous results, we utilize MVDR method for DOA estimation. Firstly, the *covariance matrix* of the virtual array is defined as

$$\mathbf{R}_{rxrx} = E\left\{\mathbf{x}_r(t)\mathbf{x}_r^H(t)\right\}. \tag{19}$$

According to (4) and (16), (19) is rewritten as:

$$\mathbf{R}_{rxrx} = \mathbf{Q} \cdot E\left\{(\mathbf{x}(t)\mathbf{x}^H(t)) \otimes (\mathbf{x}(t)\mathbf{x}^H(t))\right\} \cdot \mathbf{Q}^H. \tag{20}$$

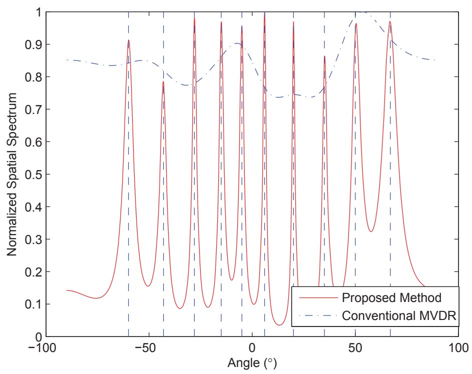

Fig. 1. DOA estimation by proposed method for 10 signals with 6 array sensors. Vertical dashed lines indicate true DOAs.

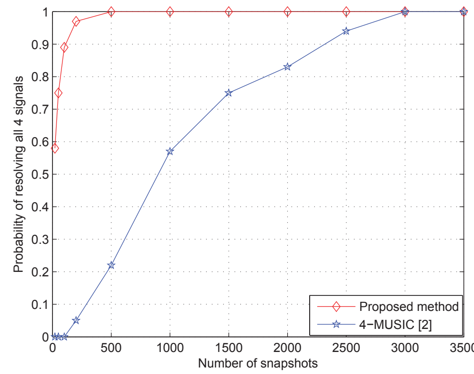

Fig. 2. Performance of resolving signals versus snapshots number. Four signals impinge on an array with 3 sensors.

Therefore all the entries of $\mathbf{R}_{rxrx}$ are the fourth-order moments of the actual array outputs $x_i(t), i = 1, \cdots, M$. In comparison with the traditional higher-order statistics methods which exploit higher-order cumulants, the calculation of fourth-order moments is obviously more computationally efficient.

Given the definition of covariance matrix for the virtual array, the corresponding MVDR spectrum function is expressed as

$$P_{r\_MVDR}(\theta) = \frac{1}{\mathbf{a}_r^H(\theta)\mathbf{R}_{rxrx}^{-1}\mathbf{a}_r(\theta)}, \tag{21}$$

and then the DOA estimates are given by the locations of the n highest peaks of the spacial spectrum function.

5. SIMULATION RESULTS

Simulation is performed to evaluate the proposed method. Here we consider amplitude modulated (AM) source signals impinging on a uniform linear array (ULA), and the interelement spacing is equal to half-wavelength. The signals' DOA angles are measured with respect to the broadside. Each receiver's signal is corrupted by additive white Gaussian noise, and the signal-to-noise ratio (SNR) is assumed to be 10dB.

Fig. 1 shows the ability of the proposed method for dealing with under-determined DOA estimation case, where 10 AM Gaussian signals impinge on a ULA with 6 sensors. The true DOAs are $\{\theta_1, \cdots, \theta_{10}\}$={-60°, -43°, -28°, -15°, -5°, 6°, 20°, 35°, 50°, 67°}. And 500 snapshots are used. In this under-determined case, the traditional DOA estimation methods based on second-order statistics cannot resolve all sources. And the fourth-order cumulant MUSIC [2] is also not applicable because the sources are Gaussian signals. The simulation result illustrates that, the proposed method gives DOA estimates for up to $(2M - 2)$ sources with M sensors.

In the second example, we consider the performance of the proposed method for resolving signals in under-determined case. Here, four AM signals that are zero-mean uniformly distributed impinge on a ULA with 3 sensors, The true DOAs are $\{\theta_1, \cdots, \theta_4\}$={-30°, -10°, 15°, 35°}. Fig. 2 shows the probability of resolving all the four sources as a function of snapshots, comparing with the fourth-order cumulant MUSIC (4-MUSIC) [2], which is a representative HOS method. The interference cancellation in 4-MUSIC is performed in a statistical sense, and the calculation of cumulants requires large number of snapshots; while the proposed virtual array model cancels the interference in a deterministic manner. As a result, simulation result shows that the proposed method resolves all sources in under-determined case using much less snapshots than 4-MUSIC, even if the number of sources is known to 4-MUSIC.

6. CONCLUSION

We address the DOA estimation problem for amplitude modulated signals in the under-determined case. Through mathematical analysis, we show that the proposed virtual array model increases the degree-of-freedom when source signals are amplitude modulated. We therefore propose a MVDR-type DOA estimation method that is based on the fourth-order moment, which is of lower computational load than the traditional fourth-order cumulant methods. Numerical examples are provided to verify the validity of the proposed method for dealing with under-determined case.

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