A CIRCULARITY-BASED DOA ESTIMATION METHOD UNDER COEXISTENCE OF NONCIRCULAR AND CIRCULAR SIGNALS

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ABSTRACT

In this paper, we consider the direction of arrival (DOA) estimation problem under the coexistence of noncircular and circular signals. By exploiting the difference between the circularity of noncircular and circular signals, a method is proposed, which estimates the DOAs of noncircular and circular signals separately. The maximum number of detectable directions by the proposed method is twice that by the MUSIC method. Furthermore, since the proposed method resolves noncircular and circular signals based on the circularity difference rather than the DOA difference, the proposed method performs well regardless of the DOA separation between noncircular and circular signals. Simulation results illustrate the effectiveness of the proposed method.

Index Terms**—**Direction of arrival (DOA) estimation, Noncircular signals, MUSIC

1. INTRODUCTION

Array processing plays an important role in fulfilling the increased demands of wireless communication services [1]. However, the increase in system capacity by using array processing techniques is limited to the number of array elements since the maximum number of detectable directions by most of existing DOA estimation methods (e.g., MUSIC [2] and ESPRIT [3]) is less than the number of array elements. Although the methods based on high-order statistics overcome the problem above to some extent [4], [5], they require high computational cost.

It should be emphasized that the DOA estimation methods above did not exploit the noncircularity property of noncircular signals (e.g., amplitude modulated (AM) and binary phase shift keying (BPSK) modulated signals) which are often used in many systems such as satellite systems. Only recently have some works [6]-[9] been proposed to estimate the DOAs of noncircular signals. By exploiting the noncircularity property of noncircular signals, they increase the number of detectable directions and improve the estimation accuracy as well. However, they cannot deal with the more realistic scenario that noncircular and circular signals coexist. To cope with this case, a method has been proposed in [10], which is based on the vector composed of the array output vector and its conjugate counterpart. Compared with the MUSIC method, this method increases the number of detectable directions and improves the estimation accuracy. However, since the method produces noncircular and circular signals simultaneously, it fails when noncircular and circular signals are spatially close to each other. To overcome this problem, we propose a method for estimating the DOAs of noncircular and circular signals separately, based on the fact that the unconjugated spatial covariance matrix of noncircular signals equals nonzero and the counterpart of circular signals equals zero, under the assumption that the signals are independent of each other. Since the proposed method resolves noncircular and circular signals by exploiting the difference between the circularity of noncircular and circular signals instead of the DOA difference, it performs well regardless of the DOA separation between the aforementioned signals. In addition, the maximum number of detectable directions by the proposed method is twice that by the MUSIC method and larger than that by the method in [10]. The price of the proposed method is that in large DOA separation, its estimation accuracy is lower than that of the MUSIC method.

2. PRELIMINARIES

2.1. Array data model

In this paper, the superscripts *, T, and H represent the conjugate, transpose, and conjugate transpose operations, respectively. Consider the array with M sensors located on the same plane, where the first sensor is taken as reference and the coordinates of the mth sensor are denoted by (x_m, y_m). Suppose that there are K narrow-band far-field signals $s_k(t)$ with the center wavelength λ and the directions θ_k for $k=1,\cdots,K$. The vector of M sensor outputs can then be expressed as [10]

$$\mathbf{r}(t)=\sum_{k=1}^{K}\mathbf{a}(\theta_k)s_k(t)\delta_k e^{j\phi_k}+\mathbf{n}(t) \tag{1}$$

where $\mathbf{n}(t)$ is a circular complex white Gaussian noise vector with zero-mean and spatially uncorrelated with $\mathrm{E}[\mathbf{nn}^{\mathrm{T}}]=\mathbf{0}$ and $\mathrm{E}[\mathbf{nn}^{\mathrm{H}}]=\sigma_n^2\mathbf{I}$, δ_k and ϕ_k are the unknown amplitude and phase parameters, respectively, and $\mathbf{a}(\theta_k)$ is the steering vector for the kth signal, given by $\mathbf{a}(\theta_k)=[1, e^{-j2\pi d_{2,k}/\lambda},\cdots,e^{-j2\pi d_{M,K}/\lambda}]^{\mathrm{T}}$ where $d_{m,k}$ is the distance from the first sensor to the mth sensor along θ_k, defined by $d_{m,k}=x_m\cos\theta_k+y_m\sin\theta_k$. For simplicity we omit the time variable. Using matrix notation, (1) is rewritten as

$$\mathbf{r}=\mathbf{ABs}+\mathbf{n} \tag{2}$$

where $\mathbf{A}=[\mathbf{a}(\theta_1),\cdots,\mathbf{a}(\theta_K)]$, $\mathbf{B}=diag\{\delta_1 e^{j\phi_1},\cdots,\delta_K e^{j\phi_K}\}$, and $\mathbf{s}=[s_1,\ \cdots, s_K]^{\mathrm{T}}$.

The conjugated covariance matrix of the array output vector can be written as

$$\mathbf{R}=\mathrm{E}[\mathbf{rr}^{\mathrm{H}}]=\mathbf{ABR}_{\mathrm{s}}\mathbf{B}^{\mathrm{H}}\mathbf{A}^{\mathrm{H}}+\sigma_n^2\mathbf{I} \tag{3}$$

where $\mathrm{E}[\bullet]$ denotes mathematical expectation and $\mathbf{R}_{\mathrm{s}}=\mathrm{E}[\mathbf{ss}^{\mathrm{H}}]$.

Based on (1) and due to $\mathrm{E}[\mathbf{nn}^{\mathrm{T}}]=\mathbf{0}$, the unconjugated covariance matrix (also named as the elliptic covariance matrix [7]) of the array output vector can be expressed as [6]

$$\mathbf{R}'=\mathrm{E}[\mathbf{rr}^{\mathrm{T}}]=\mathbf{ABR}'_{\mathrm{s}}\mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} \tag{4}$$

where $\mathbf{R}'_{\mathrm{s}} = \mathrm{E}[\mathbf{s}\mathbf{s}^{\mathrm{T}}]$.

2.2. Circularity

Based on the literature [6] and [12], it is known that the elliptic covariance for a random variable h is defined by

$$\mathrm{E}[hh] = \rho_h \sigma_h^2 e^{j\varphi_h} \tag{5}$$

where ρ_h is the non-circularity rate of h and $0 \le \rho_h \le 1$, φ_h is the non-circularity phase, and $\sigma_h^2 = \mathrm{E}[hh^*]$.

Based on [10] and [11], it is known that a complex random variable h is named to be circular if $\mathrm{E}[h] = 0$ and $\mathrm{E}[hh] = 0$. Circularity is a common hypothesis for narrowband signal analysis. However, in many modern telecommunication systems or satellite systems, noncircular sources (such as AM and BPSK modulated signals) are often employed.

As mentioned in [10], it is more realistic that some users transmit circular signals while others send noncircular signals. In this paper, we consider the problem of DOA estimation in this more general scenario and develop a method for estimating the DOAs of circular and noncircular signals separately in Sec. 3.

In addition, similarly to [7]-[10], we consider that noncircular sources emit AM/BPSK modulated signals, which implies that the non-circularity rate ρ of the noncircular sources equals one.

3. THE PROPOSED METHOD

3.1. Derivation

Assumption 1: The signals are zero-mean and independent of each other, which leads to $\mathrm{E}[s_k(t)s_l^{\mathrm{H}}(t)] = 0$ and $\mathrm{E}\left[s_k(t)s_l(t)\right] = 0$ where $l \ne k$.

For clarity, we use subscripts nc and c to represent the quantities corresponding to noncircular and circular sources, respectively. Denote the number of noncircular and circular sources with K_{nc} and K_{c}, respectively, with $K = K_{\mathrm{nc}} + K_{\mathrm{c}}$. Let

$$\mathbf{A}_{\mathrm{nc}} = [\mathbf{a}(\theta_{\mathrm{nc},1}),\cdots,\mathbf{a}(\theta_{\mathrm{nc},K_{\mathrm{nc}}})]$$

$$\mathbf{A}_{\mathrm{c}} = [\mathbf{a}(\theta_{\mathrm{c},1}),\cdots,\mathbf{a}(\theta_{\mathrm{c},K_{\mathrm{c}}})]$$

$$\mathbf{B}_{\mathrm{nc}} = diag\{\delta_{\mathrm{nc},1}e^{j\phi_{\mathrm{nc},1}},\cdots,\delta_{\mathrm{nc},K_{\mathrm{nc}}}e^{j\phi_{\mathrm{nc},k_{\mathrm{nc}}}}\}$$

$$\mathbf{B}_{\mathrm{c}} = diag\{\delta_{\mathrm{c},1}e^{j\phi_{\mathrm{c},1}},\cdots,\delta_{\mathrm{c},K_{\mathrm{c}}}e^{j\phi_{\mathrm{c},k_{\mathrm{c}}}}\}$$

$$\mathbf{R}'_{\mathrm{nc},s} = diag\{\mathrm{E}[s_{\mathrm{nc},1}s_{\mathrm{nc},1}],\cdots,\mathrm{E}[s_{\mathrm{nc},K_{nc}}s_{\mathrm{nc},K_{\mathrm{nc}}}]\}$$

$$\mathbf{R}'_{\mathrm{c},s} = diag\{\mathrm{E}[s_{\mathrm{c},1}s_{\mathrm{c},1}],\cdots,\mathrm{E}[s_{\mathrm{c},K_{\mathrm{c}}}s_{\mathrm{c},K_{\mathrm{c}}}]\}.$$

Based on Assumption 1, $\mathbf{R}'_{\mathrm{s}}$ is diagonal. Thus, from (4), we can write the matrix $\mathbf{R}'$ as two parts: one corresponds to noncircular signals and the other corresponds to circular signals

$$\mathbf{R}' = \mathbf{A}_{\mathrm{nc}}\mathbf{B}_{\mathrm{nc}}\mathbf{R}'_{\mathrm{nc},s}\mathbf{B}_{\mathrm{nc}}^{\mathrm{T}}\mathbf{A}_{\mathrm{nc}}^{\mathrm{T}} + \mathbf{A}_{\mathrm{c}}\mathbf{B}_{\mathrm{c}}\mathbf{R}'_{\mathrm{c},s}\mathbf{B}_{\mathrm{c}}^{\mathrm{T}}\mathbf{A}_{\mathrm{c}}^{\mathrm{T}} \tag{6}$$

Due to the circularity of $s_{\mathrm{c},k}$, $\mathrm{E}[s_{\mathrm{c},k}s_{\mathrm{c},k}] = 0$. Based on the definition of $\mathbf{R}'_{\mathrm{c},s}$, $\mathbf{R}'_{\mathrm{c},s} = \mathbf{0}$. Then, from (6), we have

$$\mathbf{R}' = \mathbf{A}_{\mathrm{nc}}\mathbf{B}_{\mathrm{nc}}\mathbf{R}'_{\mathrm{nc},s}\mathbf{B}_{\mathrm{nc}}^{\mathrm{T}}\mathbf{A}_{\mathrm{nc}}^{\mathrm{T}} \tag{7}$$

Let $\mathbf{R}'_{\mathrm{nc},s1} = \mathbf{B}_{\mathrm{nc}}\mathbf{R}'_{\mathrm{nc},s}\mathbf{B}_{\mathrm{nc}}^{\mathrm{T}}$. (7) can be rewritten as

$$\mathbf{R}' = \mathbf{A}_{\mathrm{nc}}\mathbf{R}'_{\mathrm{nc},s1}\mathbf{A}_{\mathrm{nc}}^{\mathrm{T}} \tag{8}$$

Let

$$\mathbf{R}_{\mathrm{nc},s} = diag\{\mathrm{E}[s_{\mathrm{nc},1}s_{\mathrm{nc},1}^*],\cdots,\mathrm{E}[s_{\mathrm{nc},K_{\mathrm{nc}}}s_{\mathrm{nc},K_{\mathrm{nc}}}^*]\}$$

$$\mathbf{R}_{\mathrm{c},s} = diag\{\mathrm{E}[s_{\mathrm{c},1}s_{\mathrm{c},1}^*],\cdots,\mathrm{E}[s_{\mathrm{c},K_{\mathrm{c}}}s_{\mathrm{c},K_{\mathrm{c}}}^*]\}.$$

Similarly, since $\mathbf{R}_{\mathrm{s}}$ is diagonal, we write the matrix $\mathbf{R}$ in (3) as

$$\mathbf{R} = \mathbf{A}_{\mathrm{nc}}\mathbf{B}_{\mathrm{nc}}\mathbf{R}_{\mathrm{nc},s}\mathbf{B}_{\mathrm{nc}}^{\mathrm{H}}\mathbf{A}_{\mathrm{nc}}^{\mathrm{H}} + \mathbf{A}_{\mathrm{c}}\mathbf{B}_{\mathrm{c}}\mathbf{R}_{\mathrm{c},s}\mathbf{B}_{\mathrm{c}}^{\mathrm{H}}\mathbf{A}_{\mathrm{c}}^{\mathrm{H}} + \sigma_n^2\mathbf{I} \tag{9}$$

Let $\mathbf{R}_{\mathrm{nc},s1} = \mathbf{B}_{\mathrm{nc}}\mathbf{R}_{\mathrm{nc},s}\mathbf{B}_{\mathrm{nc}}^{\mathrm{H}}$ and $\mathbf{R}_{\mathrm{c},s1} = \mathbf{B}_{\mathrm{c}}\mathbf{R}_{\mathrm{c},s}\mathbf{B}_{\mathrm{c}}^{\mathrm{H}}$. (9) can be rewritten as

$$\mathbf{R} = \mathbf{A}_{\mathrm{nc}}\mathbf{R}_{\mathrm{nc},s1}\mathbf{A}_{\mathrm{nc}}^{\mathrm{H}} + \mathbf{A}_{\mathrm{c}}\mathbf{R}_{\mathrm{c},s1}\mathbf{A}_{\mathrm{c}}^{\mathrm{H}} + \sigma_n^2\mathbf{I} \tag{10}$$

Next, we estimate the DOAs of noncircular signals based on $\mathbf{R}'$. The singular value decomposition (SVD) of $\mathbf{R}'$ is computed as

$$\mathbf{R}' = \begin{bmatrix}\mathbf{U}_{\mathrm{nc},1} & \mathbf{U}_{\mathrm{nc},2}\end{bmatrix}\begin{bmatrix}\mathbf{\Lambda}_{\mathrm{nc}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{\mathrm{nc},1}^{\mathrm{H}}\\ \mathbf{V}_{\mathrm{nc},2}^{\mathrm{H}}\end{bmatrix} \tag{11}$$

where $\mathbf{\Lambda}_{\mathrm{nc}}$ is a K_{nc}-dimensional diagonal matrix and its diagonal elements are composed of the nonzero singular values.

Let $\mathbf{R}''_{\mathrm{nc},s} = \mathbf{R}'_{\mathrm{nc},s1}\mathbf{A}_{\mathrm{nc}}^{\mathrm{T}}\mathbf{A}_{\mathrm{nc}}^*(\mathbf{R}'_{\mathrm{nc},s1})^{\mathrm{H}}$. Using (8) and (11), we have

$$\mathbf{R}'(\mathbf{R}')^{\mathrm{H}} = \mathbf{A}_{\mathrm{nc}}\mathbf{R}''_{\mathrm{nc},s}\mathbf{A}_{\mathrm{nc}}^{\mathrm{H}} \tag{12}$$

$$\mathbf{R}'(\mathbf{R}')^{\mathrm{H}} = \begin{bmatrix}\mathbf{U}_{\mathrm{nc},1} & \mathbf{U}_{\mathrm{nc},2}\end{bmatrix}\begin{bmatrix}\mathbf{\Lambda}_{\mathrm{nc}}^2 & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{U}_{\mathrm{nc},1}^{\mathrm{H}}\\ \mathbf{U}_{\mathrm{nc},2}^{\mathrm{H}}\end{bmatrix} \tag{13}$$

Based on [2] and from (12) and (13), we have that all the columns of $\mathbf{A}_{\mathrm{nc}}$ are orthogonal to all the columns of $\mathbf{U}_{\mathrm{nc},2}$, i.e.,

$$\mathbf{U}_{\mathrm{nc},2}^{\mathrm{H}}\mathbf{a}(\theta_{\mathrm{nc},k}) = \mathbf{0},\ \text{for}\ k = 1,\cdots,K_{\mathrm{nc}} \tag{14}$$

Then, we define a spatial spectrum for noncircular signals as

$$P_{\mathrm{nc}}(\theta) = \left\|\mathbf{U}_{\mathrm{nc},2}^{\mathrm{H}}\mathbf{a}(\theta)\right\|^{-2} \tag{15}$$

where $\|\bullet\|$ denotes 2-norm of a vector.

From (14), it follows that $P_{\mathrm{nc}}(\theta)$ has a peak at the direction $\theta_{\mathrm{nc},k}$. Thus, by searching the peaks of $P_{\mathrm{nc}}(\theta)$, $\theta_{\mathrm{nc},k}$ is estimated as

$$\theta_{\mathrm{nc},k} = \arg\max_{\theta} P_{\mathrm{nc}}(\theta) \tag{16}$$

Next, we estimate the DOAs of circular signals based on the estimates of $\theta_{\mathrm{nc},k}$. Based on (8), we can estimate $\mathbf{R}'_{\mathrm{nc},s1}$ as

$$\mathbf{R}'_{\mathrm{nc},s1} = \mathbf{A}_{\mathrm{nc}}^{\dagger}\mathbf{R}'\left(\mathbf{A}_{\mathrm{nc}}^{\mathrm{T}}\right)^{\dagger} \tag{17}$$

where $(\bullet)^{\dagger}$ represents the pseudo inverse operation, $\mathbf{A}_{\mathrm{nc}}^{\dagger} = \left(\mathbf{A}_{\mathrm{nc}}^{\mathrm{H}}\mathbf{A}_{\mathrm{nc}}\right)^{-1}\mathbf{A}_{\mathrm{nc}}^{\mathrm{H}}$, and $\left(\mathbf{A}_{\mathrm{nc}}^{\mathrm{T}}\right)^{\dagger} = \mathbf{A}_{\mathrm{nc}}^*\left(\mathbf{A}_{\mathrm{nc}}^{\mathrm{T}}\mathbf{A}_{\mathrm{nc}}^*\right)^{-1}$ [13].

Denote the kth diagonal element of $\mathbf{R}'_{\mathrm{nc},s1}$ as $\mathrm{R}'_{\mathrm{nc},s1}(k,k)$. Due to $\mathbf{R}'_{\mathrm{nc},s1} = \mathbf{B}_{\mathrm{nc}}\mathbf{R}'_{\mathrm{nc},s}\mathbf{B}_{\mathrm{nc}}^{T}$, we have

$$\mathrm{R}'_{\mathrm{nc},s1}(k,k) = \delta_{\mathrm{nc},k}^2 e^{j2\phi_{\mathrm{nc},k}}\mathrm{E}[s_{\mathrm{nc},k}s_{\mathrm{nc},k}]. \tag{18}$$

In addition, based on Sec. 2.2, when noncircular sources emit AM/BPSK modulated signals, we have

$$\mathrm{E}[s_{\mathrm{nc},k}s_{\mathrm{nc},k}] = \sigma_{\mathrm{nc},k}^2 e^{j\varphi_{\mathrm{nc},k}} \tag{19}$$

where $\varphi_{\mathrm{nc},k}$ is the noncircularity phase, and $\sigma_{\mathrm{nc},k}^2 = \mathrm{E}[s_{\mathrm{nc},k}s_{\mathrm{nc},k}^*]$.

Substituting (19) into (18), we have

$$\delta_{\mathrm{nc},k}^2\sigma_{\mathrm{nc},k}^2 = |\mathrm{R}'_{\mathrm{nc},s1}(k,k)| \tag{20}$$

Due to $\mathbf{R}_{\mathrm{nc},s1} = \mathbf{B}_{\mathrm{nc}}\mathbf{R}_{\mathrm{nc},s}\mathbf{B}_{\mathrm{nc}}^{\mathrm{H}}$ and $\mathbf{R}_{\mathrm{nc},s}$ is diagonal, we have

$$\mathbf{R}_{\mathrm{nc},s1} = diag\{\delta_{\mathrm{nc},1}^2\sigma_{\mathrm{nc},1}^2,\cdots,\delta_{\mathrm{nc},K_{\mathrm{nc}}}^2\sigma_{\mathrm{nc},K_{\mathrm{nc}}}^2\} \tag{21}$$

From (20) and (21), it follows that

$$\mathbf{R}_{\mathrm{nc},s1} = diag\{|\mathrm{R}'_{\mathrm{nc},s1}(1,1)|,\cdots,|\mathrm{R}'_{\mathrm{nc},s1}(K_{\mathrm{nc}},K_{\mathrm{nc}})|\} \tag{22}$$

Therefore, based on (17), we can estimate $\mathbf{R}_{\mathrm{nc},s1}$ by (22). Let

$$\mathbf{R}_1 = \mathbf{A}_{\mathrm{nc}}\mathbf{R}_{\mathrm{nc},s1}\mathbf{A}_{\mathrm{nc}}^{\mathrm{H}} . \tag{23}$$

Then, using the DOA estimates of noncircular signals and the estimate of $\mathbf{R}_{\mathrm{nc},s1}$ above, we obtain the estimate of $\mathbf{R}_1$.

Based on (9), we have

$$\mathbf{R} - \mathbf{R}_1 = \mathbf{A}_{\mathrm{c}}\mathbf{R}_{\mathrm{c},s1}\mathbf{A}_{\mathrm{c}}^{\mathrm{H}} + \sigma_n^2\mathbf{I} \tag{24}$$

Observing the right side of (24), it is clear that the matrix $\mathbf{R} - \mathbf{R}_1$ has K_{c} signals eigenvectors related to K_{c} circular signals. Then, decomposing $\mathbf{R} - \mathbf{R}_1$, we obtain

$$\mathbf{R} - \mathbf{R}_1 = \sum_{k=1}^{K_{\mathrm{c}}} \beta_{\mathrm{c},k}\mathbf{u}_{\mathrm{c},k}\mathbf{u}_{\mathrm{c},k}^H + \sum_{k=K_{\mathrm{c}}+1}^{M} \beta_{\mathrm{c},k}\mathbf{u}_{\mathrm{c},k}\mathbf{u}_{\mathrm{c},k}^H \tag{25}$$

where the eigenvalues $\{\beta_{\mathrm{c},k}\}_{k=1}^{M}$ are listed in descending order. Then, $\beta_{\mathrm{c},k}$ $(\mathbf{u}_{\mathrm{c},k})$ $k \le K_{\mathrm{c}}$ are the signal eigenvalues (eigenvectors) and the remaining ones are the noise eigenvalues (eigenvectors).

Denote $\mathbf{U}_{\mathrm{c},2} = [\mathbf{u}_{\mathrm{c},K_{\mathrm{c}}+1},\cdots,\mathbf{u}_{\mathrm{c},M}]$. Based on [2] and from (24) and (25), $\mathbf{U}_{\mathrm{c},2}^{\mathrm{H}}\mathbf{a}(\theta_{\mathrm{c},k}) = \mathbf{0}$. Then, we define a spatial spectrum as

$$P_{\mathrm{c}}(\theta) = \left\|\mathbf{U}_{\mathrm{c},2}^{\mathrm{H}}\mathbf{a}(\theta)\right\|^{-2} \tag{26}$$

Similarly to noncircular signals, $\theta_{\mathrm{c},k}$ is estimated as

$$\theta_{\mathrm{c},k} = \arg\max_{\theta} P_{\mathrm{c}}(\theta) \tag{27}$$

It is noted that only a finite number of sampled data is available in practice. Thus, $\mathbf{R}$ and $\mathbf{R}'$ have to be estimated by $\widehat{\mathbf{R}} = (1/N_s)\sum_{n=1}^{N_s}\mathbf{r}(t_n)\mathbf{r}^{\mathrm{H}}(t_n)$ and $\widehat{\mathbf{R}'} = (1/N_s)\sum_{n=1}^{N_s}\mathbf{r}(t_n)\mathbf{r}^{\mathrm{T}}(t_n)$ (N_s is the number of samples). The perturbation of $\widehat{\mathbf{R}}$ (respectively, $\widehat{\mathbf{R}'}$) from its true value $\mathbf{R}$ (respectively, $\mathbf{R}'$) causes random perturbation of the parameter estimates from their true values. Through this paper, "^" denotes the estimate of the quantity over which it appears. Then, the proposed method is summarized below.

Step 1) Decompose $\widehat{\mathbf{R}'}$ to get $\widehat{\mathbf{U}_{\mathrm{nc},2}}$ and then estimate the DOAs of noncircular signals as $\widehat{\theta_{\mathrm{nc},k}}$ by (16).

Step 2) using $\widehat{\theta_{\mathrm{nc},k}}$ from step 1, obtain $\widehat{\mathbf{R}_{\mathrm{nc},s1}}$ by (17) and (22).

Step 3) Based on $\widehat{\theta_{\mathrm{nc},k}}$ from step 1 and $\widehat{\mathbf{R}_{\mathrm{nc},s1}}$ from step 2, obtain $\widehat{\mathbf{R}_1}$ by (23). Decompose $\widehat{\mathbf{R}} - \widehat{\mathbf{R}_1}$ to get $\widehat{\mathbf{U}_{\mathrm{c},2}}$, and then estimate the DOAs of circular signals by (27) as $\widehat{\theta_{\mathrm{c},k}}$.

3.2. Considerations

1) The minimum number of antennas

From (11) and (25), it follows that $M > \max\{K_{\mathrm{nc}}, K_{\mathrm{c}}\}$ must be satisfied in order to resolve all noncircular and circular signals.

2) The maximum number of detectable directions

From (11), it follows that the maximum number $K_{\mathrm{nc,max}}$ of noncircular signals which can be detected by *M* antennas equals *M*-1. From (25), the maximum number $K_{\mathrm{c,max}}$ of detectable circular signals equals *M*-1. Thus, the maximum number of detectable directions $K_{\max}$ equals 2*M*-2 (i.e., $K_{\max} = K_{\mathrm{nc,max}} + K_{\mathrm{c,max}}$), which is twice that by the MUSIC method and larger than that by the method in [10] ($2M - 2 - K_{\mathrm{c}}$).

4. SIMULATIONS RESULTS

In this section, we compare the performance of the proposed method, the MUSIC method, and the method in [10] by simulations.

4.1. The maximum number of detectable directions

In this example, we study the case in which the number of signals goes beyond the traditional MUSIC limit. In simulations, circular sources are supposed to send QPSK symbols and noncircular source sends BPSK symbols. We employ a four-sensor uniform linear array (ULA) with interelement spacing $\lambda/2$ and consider six equal power sources (three noncircular sources with directions -30° -10°, and 20°, respectively, and three circular sources with directions -20°, 10°, and 30°, respectively). The SNR and the sample number are 20 dB and 300, respectively.

Based on Sec. 3.2, we have that the maximum number of detectable directions by the proposed method equals $2M - 2 = 6$. The spatial spectrums (i.e., $P_{\mathrm{nc}}(\theta)$ and $P_{\mathrm{c}}(\theta)$) for both estimators (16) and (27) are shown in Fig. 1. From Fig. 1, we can see that the noncircular estimator produces three peaks corresponding to three noncircular signals and so does the circular estimator. One thing to be emphasized that the MUSIC method fails in this case since the MUSIC method cannot estimate more than $M - 1 = 3$ different directions and the method in [10] also fails since $K_{\mathrm{nc}} + 2K_{\mathrm{c}} = 9$ is larger than $2M - 2 = 6$.

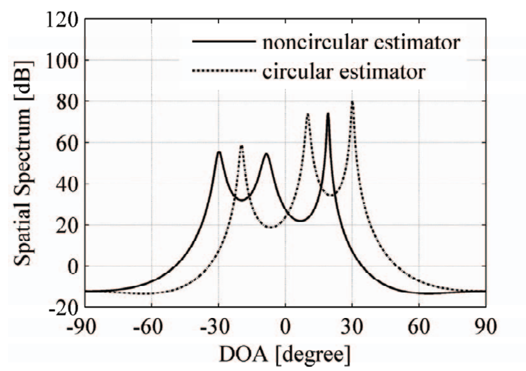

Fig. 1 Spatial spectrum when the number of signals is beyond the traditional MUISC limit.

4.2. The effect of DOA separation

In the first example, we employ a four-sensor ULA with interelement spacing $\lambda/2$. Assume there are two equal power sources: one source is noncircular with a direction θ_{nc}, and the other is circular with a direction θ_{c}. We define the Average Root Mean Square Error (ARMSE) of DOA estimates as $1/K\sum_{k=1}^{K}\mathrm{RMSE}\{\theta_k\}$. The SNR and the sample number are 10 dB and 300, respectively. θ_{nc} is 10° and θ_{c} varies from 10° to 40°. Thus, the DOA separation between θ_{c} and θ_{nc} (i.e., $\theta_{\mathrm{c}} - \theta_{\mathrm{nc}}$) varies from 0° to 30°. Based on 200 experiments, the ARMSE curves of the DOA estimates by the aforementioned methods versus the DOA separation are shown in Fig. 2.

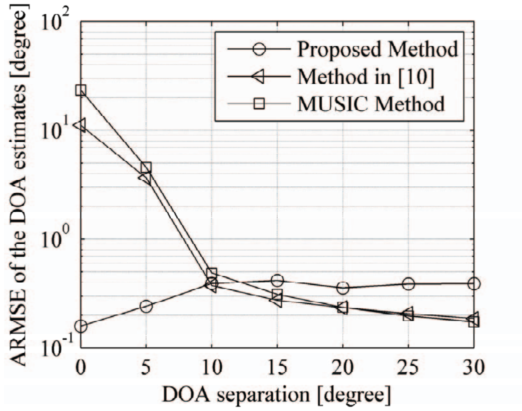

Fig. 2 ARMSE of the DOA estimates versus DOA separation.

From Fig. 2, it is shown that the performance of the proposed method basically keeps invariant, which is due to the fact that the proposed method separates noncircular and circular signals based on the circularity difference between noncircular and circular signals rather than the DOA difference. In addition, when the DOA separation is larger than 10°, the performance of the proposed method behaves worse than the MUSIC method and the method in [10]. This result arises from the fact that in finite samples, Assumption 1 is not satisfied and the non-diagonal elements of $\mathbf{R}'_{\mathrm{s}}$ are small but nonzero, so that (6) does not hold. In consequence, the matrix $\mathbf{R}'$ embodies some DOA information of circular signals except the DOA information of noncircular signals and so $\mathbf{R}-\mathbf{R}_1$ does, resulting in the deviation of the estimates of θ_{nc} and θ_{c} (obtained using $\mathbf{R}'$ and $\mathbf{R}-\mathbf{R}_1$, respectively) from their true values. On the other hand, the performance of the proposed method becomes better as the DOA separation decreases. This is because that when the DOA separation is small, the DOA information of circular signals included in the matrix $\mathbf{R}'$ can be equivalent to that of noncircular signals since the DOA of the circular signal is close to that of the noncircular signal, improving the estimation accuracy of the noncircular signal. Similarly, in small DOA separation, the estimation accuracy of the circular signal is improved. In contrast, the MUSIC method and the method in [10] perform worse with decreasing DOA separation. This is because the two aforementioned methods estimate the DOAs of noncircular and circular signals simultaneously and their performance is dependent on the DOA separation between noncircular and circular signals, leading to the degradation of the performance in small DOA separation. The performance degradation of the MUSIC method has been verified in [14].

5. CONCLUSIONS

In this paper, by exploiting the difference between the circularity of noncircular and circular signals, we propose a DOA estimation method under the coexistence of noncircular and circular signals. The proposed method has two advantages compared to the MUSIC method. The one is that the maximum number of detectable directions by the proposed method is twice that by the MUSIC method. The other is that the proposed method can resolve noncircular and circular signals regardless of the DOA separation between the two aforementioned signals. Since the proposed method increases the number of detectable directions, the users in mobile communications can be increased with a fixed number of antennas. Therefore, the proposed method may be a promising technique to future increase the system capacity in mobile communications with a small number of antennas since the trend towards smaller mobile stations limits the number of antennas. The drawback of the proposed method is that in large DOA separation, its estimation accuracy is lower than that of the MUSIC method.

6. ACKNOWLEDGEMENT

This work was supported by the Major State Basic Research Development Program of China (973 Program) under Grant 2011CB707001, the Natural Science Foundation of China under Grant 61101242, the National Science Fund for Distinguished Young Scholars under Grant 60825104, and the Fundamental Research Founds for the Central Universities Grant K50510020028.

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