FILTER-AND-FORWARD DISTRIBUTED BEAMFORMING FOR TWO-WAY RELAY NETWORKS WITH FREQUENCY SELECTIVE CHANNELS

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ABSTRACT

In filter-and-forward (FF) based two-phase two-way relay networks, each transmission for data exchange between two transceivers consists of only two phases. In the first phase, both transceivers transmit their data simultaneously to the relays. The received signal of each relay is filtered with a finite impulse response (FIR) filter to compensate for the frequency selectivity of the channels, and then, the output of the filter is forwarded to both transceivers in the second phase. A new approach to distributed cooperative beamforming for such two-way relay networks with frequency selective channels is proposed This approach maximizes the lower signal-to-interference-plus-noise (SINR) of the two transceivers subject to a constraint on the total transmitted power. In the proposed approach, the transmitted powers of the transceivers as well as the coefficients of the relay filters are optimized. Simulation results demonstrate that using an FF relaying strategy can significantly improve the received SINR as compared to the traditional amplify-and-forward relaying approach.

***Index Terms*—** Cooperative communications, distributed beamforming, two-way relay networks, filter-and-forward protocol

1. INTRODUCTION

In relay networks, different users share their communication resources to help each other in data transmission. Such relay networks have recently attracted much interest in the literature not only because these networks can exploit cooperative spatial diversity of different users in the network, but also because they can extend the coverage of wireless communication systems.

Different relaying strategies have been proposed to achieve cooperative diversity. Amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) relaying protocols have been widely used in relay networks. Due to its simplicity, the AF relaying protocol has become one of the most popular relaying strategies. In the AF protocol, the relay nodes forward properly scaled and phase-shifted versions of their received signals to the receiver. However, in the case of frequency selective channels, the AF relaying strategy is not efficient in suppressing the significant amount of inter-symbol interference (ISI) caused by channel frequency selectivity. To circumvent such a problem, a filter-and-forward (FF) relaying strategy has been proposed in [1] as an extension of the AF protocol. In the FF relaying protocol, the received signal at each relay is passed through a finite impulse response (FIR) filter, and then, the output of this filter is retransmitted to the receiver.

Recently, several distributed beamforming approaches [1]-[3] have been developed for one-way relay networks, where a source transmits data to a destination with the help of multiple relays. In such one-way relay networks, the data flows in one direction, i.e., from the source to the destination. In the case when two transceivers wish to exchange their data, several relaying schemes have been developed [4]-[10]. In this paper, we consider a bandwidth efficient two-phase two-way relaying scheme [4]-[10]. In such two-way relay networks, two transceivers simultaneously transmit their data to the relays in the first phase. The received signals at the relays are processed and then forwarded to both transceivers in the second phase.

Distributed beamforming techniques that use the AF relaying strategy have been recently developed for two-phase two-way relay networks with frequency flat channels [9]-[10]. However, these AF-based beamforming techniques can hardly suppress the ISI when the channels are frequency selective. To combat ISI, in this paper we propose a FF-based distributed beamforming methods for the two-phase two-way relay networks with frequency selective channels. In the proposed method, we consider maximizing the lower signal-to-ISI-plus-noise-ratio (SINR) of the two transceivers subject to a constraint of the total power consumed in the network. We develop a semi-closed-form solution which yields the beamforming weights in a closed form *given that the transmit power of one of the transceivers is known.* We also show that the transmit power of each of the two transceivers can be obtained using a one-dimensional search. Simulation results show that the SINRs of the receivers can be significantly improved by using such an FF relaying strategy instead of the traditional AF relaying approach.

2. SIGNAL MODEL

As shown in Fig. 1, a half-duplex two-phase two-way relay network with frequency selective channels is considered in this paper. Two transceivers and R relay nodes are equipped with a single antenna and each relay deploys an FIR filter. Each transmission between two transceivers consists of two phases. In the first phase, both transceivers simultaneously broadcast their data to the relays. The signal received at each relay is passed through the relay FIR filter to compensate for the frequency selective transceiver-to-relay and relay-to-transceiver channels. In the second phase, the filtered signal at each relay is retransmitted back to both transceivers. With the assumption that both transceivers have the full knowledge of the channel state information (CSI), the transceivers independently compute the coefficients of each relay filter using the available CSI and a certain beamforming criterion. It is also assumed that there exists a low-rate feedback link for the transceivers to send back the optimal coefficients of the relay filters to the relays.

The reciprocal channels between Transceiver 1 (T_1) and the relays and those between Transceiver 2 (T_2) and the relays can be modeled as linear FIR filters, the lth effective tap of which are $R \times 1$ vectors $\mathbf{f}_l = [f_{l,1}, \cdots, f_{l,R}]^T$ and $\mathbf{g}_l = [g_{l,1}, \cdots, g_{l,R}]^T$, $l = -N, \cdots, N$, respectively, where $N = (L-1)/2$ and L is the channel length and $(\cdot)^T$ denotes the transpose. In this paper, we assume that the lengths of the transceiver-to-relay channels as well as the

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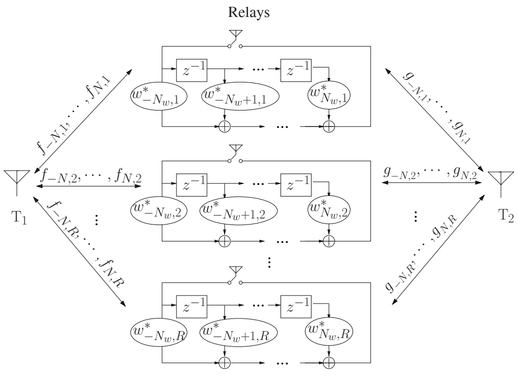

Fig. 1. System model of the two-way relay network.

lengths of the relay filters are all odd numbers. We also assume that all transceiver-to-relay channels have the same length. It is, however, straightforward to extend our results to the case when the channel and filter lengths are even numbers and/or when the transceiver-to-relay channels have different lengths.

In the first stage of each transmission, both transceivers transmit their signals to the relays. The $R \times 1$ vector of signals received at the relays at time sample n can be written as

$$\mathbf{r}(n) = \sum_{l=-N}^{N} \mathbf{f}_l s_1(n-l) + \sum_{l=-N}^{N} \mathbf{g}_l s_2(n-l) + \boldsymbol{\eta}(n) \qquad (1)$$

where $s_1(n)$ and $s_2(n)$ denote the signals transmitted by T_1 and T_2, respectively, and $\boldsymbol{\eta}(n) = [\eta_1(n), \cdots, \eta_R(n)]^T$ is the $R \times 1$ vector of relay noises.

At each relay, the received signal passes through the relay FIR filter. In the second stage of each transmission, the output vector of the relay filters $\mathbf{t}(n) = [t_1(n), \cdots, t_R(n)]^T$ is retransmitted back to the transceiver, which can be expressed as

$$\mathbf{t}(n) = \sum_{q=-N_w}^{N_w} \mathbf{W}_q^H \mathbf{r}(n-q) \qquad (2)$$

where $\mathbf{W}_q = \mathrm{diag}\{w_{q,1}, \cdots, w_{q,R}\}$ is the diagonal matrix of the relay filter impulse responses corresponding to the qth effective filter taps of the relays, $(\cdot)^H$ denotes the Hermitian transpose, $N_w = (L_w - 1)/2$, and L_w is the length of the relay FIR filter. Hereafter, for any vector $\mathbf{x}$, $\mathrm{diag}\{\mathbf{x}\}$ denotes the diagonal matrix containing the entries of $\mathbf{x}$ on its main diagonal and for any square matrix $\mathbf{X}$, $\mathrm{diag}\{\mathbf{X}\}$ denotes the vector formed from the diagonal entries of $\mathbf{X}$.

The received signals at the transceivers can be written as

$$y_1(n) = \sum_{m=-N}^{N} \mathbf{f}_m^T \mathbf{t}(n-m) + \upsilon_1(n) \qquad (3)$$

$$y_2(n) = \sum_{m=-N}^{N} \mathbf{g}_m^T \mathbf{t}(n-m) + \upsilon_2(n) \qquad (4)$$

where $\upsilon_1(n)$ and $\upsilon_2(n)$ are the receiver noises at T_1 and T_2, respectively. Using (1) and (2), we can rewrite (3) as

$$\begin{aligned} y_1(n) &= \sum_{m=-N}^{N} \mathbf{f}_m^T \sum_{q=-N_w}^{N_w} \mathbf{W}_q \sum_{l=-N}^{N} \mathbf{f}_l s_1(n-l-q-m) \\ &+ \sum_{m=-N}^{N} \mathbf{f}_m^T \sum_{q=-N_w}^{N_w} \mathbf{W}_q \sum_{l=-N}^{N} \mathbf{g}_l s_2(n-l-q-m) \\ &+ \sum_{m=-N}^{N} \mathbf{f}_m^T \sum_{q=-N_w}^{N_w} \mathbf{W}_q \boldsymbol{\eta}(n-q-m) + \upsilon_1(n). \end{aligned} \qquad (5)$$

It can be seen from (5) that the signal received at T_1 includes the self signals of T_1 transmitted earlier to the relays. Using the assumptions that the transceivers have the full CSI knowledge and that they compute the coefficients of the relay filters, the self signal component can be subtracted from (5) and the residual signal can be written as

$$\tilde{y}_1(n) = \mathbf{w}^H \boldsymbol{\mathcal{G}}_1 \breve{\mathbf{F}}_2 \breve{\mathbf{s}}_2(n) + \mathbf{w}^H \boldsymbol{\mathcal{G}}_1 \tilde{\mathbf{I}} \breve{\boldsymbol{\eta}}(n) + \upsilon_1(n) \qquad (6)$$

and similarly, the residual signal at T_2 can be expressed as

$$\tilde{y}_2(n) = \mathbf{w}^H \boldsymbol{\mathcal{G}}_2 \breve{\mathbf{F}}_1 \breve{\mathbf{s}}_1(n) + \mathbf{w}^H \boldsymbol{\mathcal{G}}_2 \tilde{\mathbf{I}} \breve{\boldsymbol{\eta}}(n) + \upsilon_2(n) \qquad (7)$$

where

$$\begin{aligned}
&\mathbf{w} \triangleq [\mathbf{w}_{-N_w}^T, \cdots, \mathbf{w}_{N_w}^T]^T, \ \mathbf{w}_q \triangleq \mathrm{diag}\{\mathbf{W}_q\}, \ q = -N_w, \cdots, N_w \\
&\boldsymbol{\mathcal{G}}_i \triangleq [\mathbf{I}_{L_w} \otimes \mathbf{G}_{i,-N}, \cdots, \mathbf{I}_{L_w} \otimes \mathbf{G}_{i,N}], \ i = 1, 2, \\
&\mathbf{G}_{1,l} \triangleq \mathrm{diag}\{\mathbf{f}_l\}, \ \mathbf{G}_{2,l} \triangleq \mathrm{diag}\{\mathbf{g}_l\}, \ l = -N, \cdots, N \\
&\breve{\mathbf{F}}_i \triangleq [\breve{\mathbf{F}}_{i,-N}^T, \cdots, \breve{\mathbf{F}}_{i,N}^T]^T, \ i = 1, 2, \\
&\breve{\mathbf{F}}_{i,l} \triangleq [\overbrace{\mathbf{0}_{RL_w \times 1}, \cdots, \mathbf{0}_{RL_w \times 1}}^{(N+l) \text{ columns}}, \boldsymbol{\mathcal{F}}_i, \overbrace{\mathbf{0}_{RL_w \times 1}, \cdots, \mathbf{0}_{RL_w \times 1}}^{(N-l) \text{ columns}}], \\
&\boldsymbol{\mathcal{F}}_i \triangleq [\boldsymbol{\mathcal{F}}_{i,-N_w}^T, \cdots, \boldsymbol{\mathcal{F}}_{i,N_w}^T]^T, \\
&\boldsymbol{\mathcal{F}}_{i,l} \triangleq [\overbrace{\mathbf{0}_{R \times 1}, \cdots, \mathbf{0}_{R \times 1}}^{(N_w+l) \text{ columns}}, \mathbf{F}_i, \overbrace{\mathbf{0}_{R \times 1}, \cdots, \mathbf{0}_{R \times 1}}^{(N_w-l) \text{ columns}}], \\
&\mathbf{F}_1 \triangleq [\mathbf{f}_{-N}, \cdots, \mathbf{f}_N], \quad \mathbf{F}_2 \triangleq [\mathbf{g}_{-N}, \cdots, \mathbf{g}_N] \\
&\breve{\mathbf{s}}_i(n) \triangleq [s_i(n+2N+N_w), \cdots, s_i(n-2N-N_w)]^T, i = 1, 2 \\
&\tilde{\mathbf{I}} \triangleq [\breve{\mathbf{I}}_{-N}^T, \cdots, \breve{\mathbf{I}}_N^T]^T, \\
&\breve{\mathbf{I}}_l \triangleq [\overbrace{\mathbf{0}_{RL_w \times R}, \cdots, \mathbf{0}_{RL_w \times R}}^{(N+l) \text{ blocks}}, \mathbf{I}_{RL_w}, \overbrace{\mathbf{0}_{RL_w \times R}, \cdots, \mathbf{0}_{RL_w \times R}}^{(N-l) \text{ blocks}}] \\
&\breve{\boldsymbol{\eta}}(n) \triangleq [\boldsymbol{\eta}^T(n+N_w+N), \cdots, \boldsymbol{\eta}^T(n-N_w-N)]^T
\end{aligned}$$

$\mathbf{I}_N$ is the $N \times N$ identity matrix, $\otimes$ denotes the Kronecker product and $\mathbf{0}_{N \times M}$ is the $N \times M$ matrix of zeros. The interested readers are referred to [10] for the details of the derivations from (5) to (6).

From (6) and (7), we can see that $\mathbf{w}^H \boldsymbol{\mathcal{G}}_2 \breve{\mathbf{F}}_1$ and $\mathbf{w}^H \boldsymbol{\mathcal{G}}_1 \breve{\mathbf{F}}_2$ can be regarded as the equivalent channels from T_2 to T_1 and from T_1 to T_2, respectively. It has been proved in [10] that these two equivalent channels are equal to each other, i.e.,

$$\mathbf{H} \triangleq \boldsymbol{\mathcal{G}}_1 \breve{\mathbf{F}}_2 = \boldsymbol{\mathcal{G}}_2 \breve{\mathbf{F}}_1. \qquad (8)$$

From the definition of $\breve{\mathbf{s}}_i(n)$, we can see that the middle element of $\breve{\mathbf{s}}_i(n)$ is the desired signal at time sample n, i.e. $s_i(n)$, and the other elements are ISIs. Let $\mathbf{h}$ denote the middle column of $\mathbf{H}$ and let $\bar{\mathbf{H}}_1$ and $\bar{\mathbf{H}}_2$ denote the left and right parts of $\mathbf{H}$, respectively, i.e., $\mathbf{H} = [\bar{\mathbf{H}}_1, \mathbf{h}, \bar{\mathbf{H}}_2]$. Then, (6) and (7) can be rewritten as

$$\begin{aligned} \tilde{y}_1(n) &= \mathbf{w}^H [\bar{\mathbf{H}}_1, \mathbf{h}, \bar{\mathbf{H}}_2] \begin{bmatrix} \bar{\mathbf{s}}_{2,1}(n) \\ s_2(n) \\ \bar{\mathbf{s}}_{2,2}(n) \end{bmatrix} + \mathbf{w}^H \boldsymbol{\mathcal{G}}_1 \tilde{\mathbf{I}} \breve{\boldsymbol{\eta}}(n) + \upsilon_1(n) \\ &= \underbrace{\mathbf{w}^H \mathbf{h} s_2(n)}_{\text{signal}} + \underbrace{\mathbf{w}^H \bar{\mathbf{H}} \bar{\mathbf{s}}_2(n)}_{\text{ISI}} + \underbrace{\mathbf{w}^H \boldsymbol{\mathcal{G}}_1 \tilde{\mathbf{I}} \breve{\boldsymbol{\eta}}(n) + \upsilon_1(n)}_{\text{noise}} \end{aligned} \qquad (9)$$

$$\tilde{y}_2(n) = \mathbf{w}^H \mathbf{h} s_1(n) + \mathbf{w}^H \bar{\mathbf{H}} \bar{\mathbf{s}}_1(n) + \mathbf{w}^H \boldsymbol{\mathcal{G}}_2 \tilde{\mathbf{I}} \breve{\boldsymbol{\eta}}(n) + \upsilon_2(n) \qquad (10)$$

where

$$\begin{aligned} \bar{\mathbf{H}} &\triangleq [\bar{\mathbf{H}}_1, \bar{\mathbf{H}}_2] \\ \bar{\mathbf{s}}_{i,1}(n) &\triangleq [s_i(n+2N+N_w), \cdots, s_i(n+1)]^T \\ \bar{\mathbf{s}}_{i,2}(n) &\triangleq [s_i(n-1), \cdots, s_i(n-2N-N_w)]^T \\ \bar{\mathbf{s}}_i(n) &\triangleq [\bar{\mathbf{s}}_{i,1}^T(n), \bar{\mathbf{s}}_{i,2}^T(n)]^T, \ i = 1, 2. \end{aligned}$$

From (9) and (10), we can see that the signal, the ISI and the noise components at T_i can be defined as

$$
\begin{aligned}
y_{S,i}(n) &\triangleq \mathbf{w}^H \mathbf{h} s_j(n), &(11)\\
y_{I,i}(n) &\triangleq \mathbf{w}^H \bar{\mathbf{H}} \bar{\mathbf{s}}_j(n), &(12)\\
y_{N,i}(n) &\triangleq \mathbf{w}^H \boldsymbol{\mathcal{G}}_i \tilde{\mathbf{I}} \breve{\boldsymbol{\eta}}(n) + \upsilon_i(n) &(13)\\
& \quad i,j = 1,2,\ i \neq j\,.
\end{aligned}
$$

In the next section, we use the data model to develop our distributed Beamforming (equalization) technique.

3. MAXIMIZATION OF THE LOWER SINR

In this section, we consider two-way distributed beamforming problem that uses the transmitted powers of the transceivers as well as the relay filter coefficients as the design parameters. The beamforming problem maximizes the smaller received SINR of the transceivers subject to the total transmitted power constraint, which can be written as

$$
\max_{P_1>0, P_2>0, \mathbf{w}} \min\{\mathrm{SINR}_1, \mathrm{SINR}_2\} \text{ s.t. } P_1 + P_2 + P_r \leq P_{\max} \quad (14)
$$

where P_i and SINR_i, $i = 1,2$, denote the transmit power and SINR of T_i, respectively, P_r is the total relay transmitted power, and $P_{\max}$ denotes the maximal allowable total transmitted power of the relay network. Using (2), the total relay transmitted power can be written as [10]

$$
P_r = \mathrm{E}\{\mathbf{t}^H(n)\mathbf{t}(n)\} = \mathbf{w}^H \left(P_1 \mathbf{D}_1 + P_2 \mathbf{D}_2 + \sigma_\eta^2 \mathbf{I}_{RL_w}\right) \mathbf{w} \quad (15)
$$

where $\mathrm{E}\{\cdot\}$ denotes the statistical expectation, $\mathbf{D}_i \triangleq \sum_{m=1}^{R} (\mathbf{I}_{L_w} \otimes \mathbf{E}_m) \boldsymbol{\mathcal{F}}_i \boldsymbol{\mathcal{F}}_i^H (\mathbf{I}_{L_w} \otimes \mathbf{E}_m)^H$, $\mathbf{E}_m \triangleq \mathrm{diag}\{\mathbf{e}_m\}$ and $\mathbf{e}_m$ is the mth column of the identity matrix.

The received SINR at T_i can be expressed as

$$
\mathrm{SINR}_i = \frac{\mathrm{E}\{|y_{S,i}(n)|^2\}}{\mathrm{E}\{|y_{I,i}(n)|^2\} + \mathrm{E}\{|y_{N,i}(n)|^2\}}. \quad (16)
$$

Using (11), we can write the received signal power at T_i as

$$
\mathrm{E}\{|y_{S,i}(n)|^2\} = \mathrm{E}\{|\mathbf{w}^H \mathbf{h} s_j(n)|^2\} = P_j \mathbf{w}^H \mathbf{h}\mathbf{h}^H \mathbf{w}. \quad (17)
$$

Using (12), we obtain the received ISI power

$$
\begin{aligned}
\mathrm{E}\{|y_{I,i}(n)|^2\} &= \mathrm{E}\{\mathbf{w}^H \bar{\mathbf{H}} \bar{\mathbf{s}}_j(n) \bar{\mathbf{s}}_j^H(n) \bar{\mathbf{H}}^H \mathbf{w}\}\\
&= P_j \mathbf{w}^H \mathbf{Q}_I \mathbf{w} \qquad (18)
\end{aligned}
$$

where $\mathbf{Q}_I \triangleq \bar{\mathbf{H}}\bar{\mathbf{H}}^H$. Making use of (13), we also have

$$
\begin{aligned}
\mathrm{E}\{|y_{N,i}(n)|^2\} &= \mathrm{E}\{\mathbf{w}^H \boldsymbol{\mathcal{G}}_i \tilde{\mathbf{I}} \tilde{\boldsymbol{\eta}}(n) \tilde{\boldsymbol{\eta}}^H(n) \tilde{\mathbf{I}}^H \boldsymbol{\mathcal{G}}_i^H \mathbf{w}\} + \sigma_{\upsilon,i}^2\\
&= \sigma_\eta^2 \mathbf{w}^H \boldsymbol{\mathcal{G}}_i \tilde{\mathbf{I}} \tilde{\mathbf{I}}^H \boldsymbol{\mathcal{G}}_i^H \mathbf{w} + \sigma_{\upsilon,i}^2\\
&= \mathbf{w}^H \mathbf{Q}_{N,i} \mathbf{w} + \sigma_{\upsilon,i}^2 \qquad (19)
\end{aligned}
$$

where $\mathbf{Q}_{N,i} \triangleq \sigma_\eta^2 \boldsymbol{\mathcal{G}}_i \tilde{\mathbf{I}} \tilde{\mathbf{I}}^H \boldsymbol{\mathcal{G}}_i^H$ and $\sigma_{\upsilon,i}^2$ is the noise power at T_i. For the sake of simplicity, it is assumed that $\sigma_\eta^2 = \sigma_{\upsilon,i}^2 = 1$ in this paper. Also, it can be proved that $\mathbf{Q}_{\mathrm{N},i} = \mathbf{D}_i$ for $i = 1,2$.

Making use of (15)-(19) and employing an auxiliary variable $\tau > 0$, we can rewrite problem (14) as

$$
\begin{aligned}
&\max_{P_1>0, P_2>0, \tau>0, \mathbf{w}} \quad \tau \qquad (20)\\
&\text{s.t. } \frac{P_2 \mathbf{w}^H \mathbf{h}\mathbf{h}^H \mathbf{w}}{P_2 \mathbf{w}^H \mathbf{Q}_{\mathrm{I}} \mathbf{w} + \mathbf{w}^H \mathbf{D}_1 \mathbf{w} + 1} \geq \tau\\
&\quad \frac{P_1 \mathbf{w}^H \mathbf{h}\mathbf{h}^H \mathbf{w}}{P_1 \mathbf{w}^H \mathbf{Q}_{\mathrm{I}} \mathbf{w} + \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + 1} \geq \tau\\
&\quad P_1 + P_2 + P_1 \mathbf{w}^H \mathbf{D}_1 \mathbf{w} + P_2 \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + \mathbf{w}^H \mathbf{w} \leq P_{\max}\,.
\end{aligned}
$$

It can be seen that the power constraint is satisfied with equality when the solution is optimal. Otherwise, SINR_1 and SINR_2 or the value of τ can be increased by augmenting P_1 and P_2 while the power constraint is still satisfied. This contradicts with the assumption of the optimality. It can also be seen that the two receive SINRs are balanced when the solution is optimal, that is, $\mathrm{SINR}_1 = \mathrm{SINR}_2$. This can be proved by contradiction as follows. Without losing generality, we can assume $\tau = \mathrm{SINR}_1 < \mathrm{SINR}_2$ when the solution is optimal. However, we can decrease the value of P_1 such that $\mathrm{SINR}_2 = \mathrm{SINR}_1$ and the power constraint is still satisfied with inequality, which contradict with our earlier conclusion that the power constraint should be satisfied with equality when the solution is optimal. The problem in (20) can be solved using sequential quadratic programming (SQP). However, we can also develop a more efficient method to solve the problem in (20) with a semi-closed form solution. Equating SINR_1 and SINR_2 leads us to the following relationship between P_1 and P_2:

$$
P_1(1 + \mathbf{w}^H \mathbf{D}_1 \mathbf{w}) = P_2(1 + \mathbf{w}^H \mathbf{D}_2 \mathbf{w})\,. \quad (21)
$$

In light of (21), we can re-write the optimization problem in (20) as

$$
\begin{aligned}
\max_{P_1>0} \max_{\mathbf{w}} \quad & \frac{P_1 \mathbf{w}^H \mathbf{h}\mathbf{h}^H \mathbf{w}}{P_1 \mathbf{w}^H \mathbf{Q}_{\mathrm{I}} \mathbf{w} + \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + 1}\\
\text{s.t.} \quad & \mathbf{w}^H (2P_1 \mathbf{D}_1 + \mathbf{I}) \mathbf{w} \leq P_{\max} - 2P_1\,. \quad (22)
\end{aligned}
$$

For any *feasible* value of P_1, the solution to the inner maximization in (22) is given by [3]

$$
\begin{aligned}
\mathbf{w}_{\mathrm{o}}(P_1) = \kappa(P_1)\sqrt{P_{\max} - 2P_1}\Big(2P_1 \mathbf{D}_1 + \mathbf{I}\\
+ (P_{\max} - 2P_1)(P_1 \mathbf{Q}_{\mathrm{I}} + \mathbf{D}_2)\Big)^{-1} \mathbf{h} \qquad (23)
\end{aligned}
$$

where

$$
\begin{aligned}
\kappa(P_1) = \Bigg(\mathbf{h}^H \Big(2P_1 \mathbf{D}_1 + \mathbf{I} + (P_{\max} - 2P_1)(P_1 \mathbf{Q}_{\mathrm{I}} + \mathbf{D}_2)\Big)^{-1} \times\\
(2P_1 \mathbf{D}_1 + \mathbf{I})\Big(2P_1 \mathbf{D}_1 + \mathbf{I} + (P_{\max} - 2P_1)(P_1 \mathbf{Q}_{\mathrm{I}} + \mathbf{D}_2)\Big)^{-1} \mathbf{h}\Bigg)^{-2}\\
(24)
\end{aligned}
$$

and the corresponding maximum achievable balanced SINR is expressed as

$$
\mathrm{SINR}_{\max}(P_1) = P_1(P_{\max} - 2P_1)\tilde{\lambda}(P_1) \quad (25)
$$

where $\tilde{\lambda}(P_1)$ is the largest eigenvalue of the matrix

$$
\begin{aligned}
\tilde{\mathbf{P}}(P_1) \triangleq \Big(\mathbf{I} + (P_{\max} - 2P_1) \mathbf{A}^{-\frac{1}{2}}(P_1)(P_1 \mathbf{Q}_{\mathrm{I}} + \mathbf{D}_2) \mathbf{A}^{-\frac{1}{2}}(P_1)\Big)^{-1}\\
\times \mathbf{A}^{-\frac{1}{2}}(P_1)\, \mathbf{h}\mathbf{h}^H \mathbf{A}^{-\frac{1}{2}}(P_1) \qquad (26)
\end{aligned}
$$

and $\mathbf{A}(P_1) \triangleq 2P_1 \mathbf{D}_1 + \mathbf{I}$. As the rank of $\tilde{\mathbf{P}}(P_1)$ is one, its largest eigenvalue is given by

$$
\tilde{\lambda}(P_1) = \mathbf{h}^H \big(\mathbf{A}(P_1) + (P_{\max} - 2P_1)(P_1 \mathbf{Q}_{\mathrm{I}} + \mathbf{D}_2)\big)^{-1} \mathbf{h}\,. \quad (27)
$$

As a result, the optimal value of P_1 can be obtained as

$$
\max_{P_1} \; P_1(P_{\max} - 2P_1)\tilde{\lambda}(P_1) \qquad \text{s.t.} \quad 0 \leq P_1 \leq P_{\max}/2\,. \quad (28)
$$

The optimization problem in (28) is one-dimensional and it can be efficiently solved using a search technique.

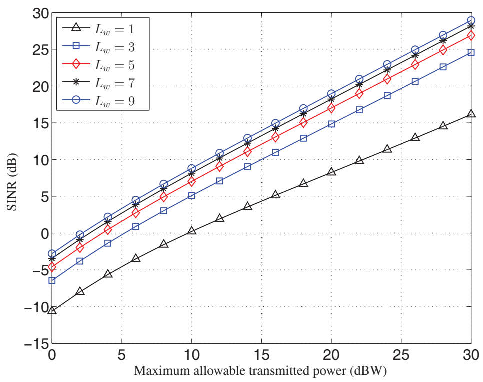

Fig. 2. The average values of minimal SINRs of T_1 and T_2 versus total transmitted power, obtained by solving (20), for different filter lengths; fourth example.

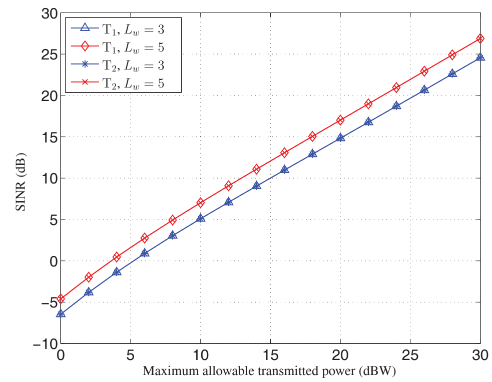

Fig. 3. The average values of SINRs of T_1 and T_2 versus total transmitted power, obtained by solving (20), for different filter lengths; fourth example.

4. SIMULATION RESULTS

In our simulations, we consider a relay network with $R = 10$ relays and quasi-static frequency selective channels with lengths $L = 5$. The channel impulse response coefficients are modeled as zero-mean complex Gaussian random variables with an exponential power delay profile $p(t) = \frac{1}{\sigma_t}\sum_{l=-N}^{N} e^{-t/\sigma_t}\delta(t - lT_s)$ [1], where σ_t denotes the delay spread and T_s is the symbol duration. In our simulations, $\sigma_t = 2T_s$ is used.

Fig. 2 shows the average values of the worst SINRs of T_1 and T_2 versus the maximal allowable total transmitted power for different relay filter lengths. From the figure, we can see that the received SINR increases monotonically with the length of the relay filter. In addition, $L_w = 1$ in this example corresponds to the beamforming technique of [9] that uses the AF relaying strategy. It can be seen that the proposed FF-based beamforming technique significantly outperforms its AF-based counterparts presented in [9] as the latter technique is not applicable in the case of frequency selective channels. Fig. 3 shows the received SINRs of T_1 and T_2 versus the maximal allowable total transmitted power for relay filter length $L_w = 3$ and $L_w = 5$. From this figure, we can see that the curves of the received SINRs of the two transceivers totally overlap each other, which verifies the SINR balancing property of this beamformer.

5. CONCLUSIONS

We have proposed a new approach to the problem of distributed beamforming for two-phase two-way relay networks in the case of frequency selective channels. Two transceivers simultaneously transmit their signals to multiple relays equipped with FIR filters to compensate for the time dispersive effects of frequency selective channels. The filtered signals are then retransmitted back to the transceivers. The proposed beamformer addresses the problem of optimally calculating the powers of the two transceivers as well as the coefficients of the relay filters under the criterion of maximizing the lower received SINR of two transceivers subject to a constraint on the total transmit power consumed in the whole network. This design approach is shown to lead to a semi-closed-form solution which is computationally efficient. Computer simulations have demonstrated that the proposed approaches have significant performance improvements compared to the existing amplify-and-forward relay beamforming techniques.

6. REFERENCES

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