RELAY SELECTION IN MULTI-USER AMPLIFY-FORWARD WIRELESS RELAY NETWORKS

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ABSTRACT

For multi-user (MU) amplify-and-forward (AF) cooperative networks, their spectral efficiency can be upgraded within the orthogonal transmission of each source node to an assigned subset of all available relays while their information throughput can be improved through optimized power allocation. We consider the joint optimization in both relay assignment for each source-destination pair and power allocation, which is in fact among the hardest problems in optimization. This is the minimization of a nonconvex objective function subject to mixed integer constraints. The existing numerical algorithms could rarely address to its solutions through computationally affordable procedures. Even the conventional relaxation of the integer constraints by linear constraints does not lead to convex optimization, so the standard convexification does not work either. Nevertheless, we show that it can be effectively solved in the d.c. (difference of two convex) programming context. Numerical simulation confirms the effectiveness of our setting.

Index Terms— Amplify-and-Forward relay, relay selection, power allocation, maxmin SNR, d.c. programming

1. INTRODUCTION

Multi-user relay-assisted wireless communication systems have been a very active research area recently (see e.g. [1, 2] and references therein). Distributed relays are employed to assist communication between source-destination links. Cooperative diversity is also exploited to improve link quality and reliability, and to enlarge network coverage [3]. A variety of cooperative schemes have been proposed in the literature addressing different scenarios, among which, Amplify-and-Forward (AF) relaying schemes are of special interest due to its low complexity for implementation. To avoid interference among different transmission links, they are often made orthogonal [4] so the network spectral efficiency can be gradually deteriorated. This is particularly sensitive in a network with many nodes [5]. To address this issue, an effective approach is to assign only a limited subset of relays for each source-destination link. Indeed, the network coverage can be effectively expanded with the help of multi-relay selection in location-based networks [6]. However, most researches have been dedicated to single-relay schemes only and their results could not be easily extended to multi-relay cases (see e.g [3] and references therein). Beside unpractical exhaustive enumeration, authors in [3] and [7] proposed several heuristics, which activate a fixed number of relays with the strongest up-links or up-down links for each user. On the other hand, Phan et al [8] considered the joint power and relay selection problem and proposed an one-round power and selection alternation. With maximum-ratio-combining (MRC) assumed at all destination nodes, the optimization objective is shown convex and so the optimized power allocation for all available relays selected is solved firstly by means of convex programming. Next, based on the power ranking of the solution by the convex program, a number of the strongest relays for each user are selected. Lastly, under such selection, the optimal power allocation is alternated. It has been shown in [8] that this alternation is better than the random selection based approach.

In this paper, MRC at destinations is no longer employed so even the power allocation optimization for all relays selected is not convex and the above mentioned standard convex relaxation cannot be applied. Nevertheless, we show that the integer (discrete) constraint of relay selection can be represented by continuous d.c. (difference of two convex) sets [9], while the objective function in the joint power and relay selection is a d.c. function [9]. Furthermore, we show that the problem is losslessly transformed to the minimization of a d.c. function over convex constraints, which can be effectively solved by a tailored algorithm, which is another development in this paper.

The paper is organized as follows. Section 2 is devoted to the problem statement while Section 3 is devoted to its solution. Simulation is Section 4 shows the viability of our results. Some conclusions are drawn in Section 5.

2. MATHEMATICAL MODELING

Consider an AF relay network with M source nodes communicating in pairs to the other M destination nodes with the help of N relays. Orthogonal transmissions and time-multiplexing are enabled from the user side. Although bi-directional communication is enabled, due to the symmetry of network, we only study the communication from the M source to the M destination nodes. It is also worth mentioning that the proposed scheme also applies to reversed-directional communication (from destination to source nodes) given enough channel state information.

Let $s = (s_1, s_2, ..., s_M)^T$ be the vector of signals independently sent by M sources. Each component s_i is normalized to zero mean and variance $\mathbb{E}[|s_i|^2] = 1$. Let $h_n = (h_{n1}, h_{n2}, ..., h_{nM})^T \in \mathcal{C}^M$, $n = 1, 2, ..., N$ be the uplink channel vector from relay n to all users and $\ell_n = (\ell_{n1}, \ell_{n2}, ..., \ell_{nM})^T \in \mathcal{C}^M, n = 1, 2, ..., N$ be the corresponding down-link channel vector. Also let σ_r^2 and σ_d^2 be the variance of additive circularly symmetric white Gaussian noise at each relay and destination node, respectively. Suppose $\boldsymbol{x}_{nm}$ is the link between relay n and source/destionation m. Accordingly, $\boldsymbol{x}_{nm} = 1$ if and only if relay n is assigned to assist communication between source m and destination m (otherwise $\boldsymbol{x}_{nm} = 0$). For spectral efficiency, the number of relays for each user assistance is restricted,

$$\sum_{n=1}^{N} \boldsymbol{x}_{nm} \leq N_R,\ m = 1, 2, ..., M. \tag{1}$$

Under orthogonal transmission, the received signal at relay n from user m is $y_{nm} = \boldsymbol{x}_{nm} h_{nm} s_m + n_n$, which is then amplified by $\boldsymbol{\alpha}_{nm} \in C$ before being transmitted to its destination m. Practically, the following power constraint must be imposed

$$\sum_{m=1}^{M} (\boldsymbol{x}_{nm}|\boldsymbol{\alpha}_{nm}|)^2(|h_{nm}|^2 + \sigma_r^2) \leq P_n,\ n = 1, 2, ..., N. \tag{2}$$

With the relays allowed to send the signal immediately, the received signal at destination m is

$$y_{dm} = \sum_{n=1}^{N} \boldsymbol{x}_{nm} \boldsymbol{\alpha}_{nm} \ell_{nm}(h_{nm}s_m + n_n) + n_{dm}.$$

Using the polar representation $\boldsymbol{\alpha}_{nm} = |\boldsymbol{\alpha}_{nm}|e^{\jmath \arg(\boldsymbol{\alpha}_{nm})}$ it is clear that the optimal $\arg(\boldsymbol{\alpha}_{nm})$ is $-\arg(\ell_{nm}h_{nm})$ so $\boldsymbol{\alpha}_{nm} = |\boldsymbol{\alpha}_{nm}|e^{-\jmath \arg(\ell_{nm}h_{nm})}$ and

$$y_{dm} = \sum_{n=1}^{N} \boldsymbol{x}_{nm}|\boldsymbol{\alpha}_{nm}|(|\ell_{nm}h_{nm}|s_m + e^{-\jmath \arg(\ell_{nm}h_{nm})}n_n) + n_{dm} \tag{3}$$

Thus the signal-to-noise (SNR) at destination m is

$$\varphi_m(\boldsymbol{x}, \bar{\boldsymbol{\alpha}}) := \frac{\sum_{n=1}^{N} (\boldsymbol{x}_{nm}|\boldsymbol{\alpha}_{nm}|)^2 |\ell_{nm}h_{nm}|^2}{\sigma_r^2 \sum_{n=1}^{N} (\boldsymbol{x}_{nm}|\boldsymbol{\alpha}_{nm}|)^2 |\ell_{nm}|^2 + \sigma_d^2}$$

under the definition

$$\boldsymbol{x} = (\boldsymbol{x}_1, .., \boldsymbol{x}_N) \in \{0, 1\}^{N \times M}, \tag{4}$$

$\boldsymbol{x}_n = (\boldsymbol{x}_{n1}, ..., \boldsymbol{x}_{nM})^T \in \{0, 1\}^M$ and $\bar{\boldsymbol{\alpha}} = (\bar{\boldsymbol{\alpha}}_1, ..., \bar{\boldsymbol{\alpha}}_N)$, $\bar{\boldsymbol{\alpha}}_n = (|\boldsymbol{\alpha}_{n1}|, ..., |\boldsymbol{\alpha}_{nM}|)$.

The joint optimization in power allocation and relay selection can now formulated as

$$\max_{\boldsymbol{x}, \boldsymbol{\alpha}} \min_{m=1,2,...,M} \varphi_m(\boldsymbol{x}, \bar{\boldsymbol{\alpha}}) \quad \text{s.t.} \quad (1), (2), (4). \tag{5}$$

Beside the hard integer constraint $\boldsymbol{x} \in \{0, 1\}^{NM}$, one can see that the objective function in (5) is highly nonconvex, so (5) is a very hard optimization. The next section addresses computational solutions of this program.

3. MAXMIN SNR OPTIMIZATION

We now propose a program to tackle problem (5). Firstly, we show an effective d.c. representation for its objective function. Secondly, the integer constraints in (5) are equivalently represented by d.c. constraints [9]. Finally, (5) is exactly transformed to minimization of a d.c. objective function subject to convex only constraints with an iterative procedure for the optimal solution proposed.

3.1. D.C. Representation of Objective Function

By introducing a joint power allocation and relay selection variable $\bar{\boldsymbol{\alpha}}_{nm} = \boldsymbol{x}_{nm}|\boldsymbol{\alpha}_{nm}|$, problem (5) is equivalently rewritten by

$$\max_{\boldsymbol{x}, \boldsymbol{y}, \bar{\boldsymbol{\alpha}}} \min_{m=1,2,...,M} \varphi_m(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}_m) = \frac{\sum_{n=1}^{N} \bar{\boldsymbol{\alpha}}_{nm}^2 |\ell_{nm}h_{nm}|^2}{\sigma_r^2 \boldsymbol{y}_m + \sigma_d^2} \tag{6a}$$

s.t. : (1), (4);

$$\sum_{m=1}^{M} \bar{\boldsymbol{\alpha}}_{nm}^2 (|h_{nm}|^2 + \sigma_r^2) \leq P_n, n = 1, 2, \ldots, N; \tag{6b}$$

$$\sum_{n=1}^{N} \bar{\boldsymbol{\alpha}}_{nm}^2 |\ell_{nm}|^2 \leq \boldsymbol{y}_{nm}, m = 1, 2, \ldots, M; \tag{6c}$$

$$0 \leq \frac{\bar{\boldsymbol{\alpha}}_{nm}}{\sqrt{P_n}} \leq \boldsymbol{x}_{nm}, n = 1, 2, ..., N, m = 1, 2, ..., M, \tag{6d}$$

Note that each φ_m is a convex function in its variables. Indeed, for variable $\boldsymbol{t}_m$ the constraint $\varphi_m(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}) \leq \boldsymbol{t}_m$ is equivalent to the (convex) semi-definite constraint

$$\begin{bmatrix} \boldsymbol{t}_m I_N & \bar{\boldsymbol{\alpha}}_{.m} \odot |\ell_{.m}h_{.m}| \\ (\bar{\boldsymbol{\alpha}}_{.m} \odot |\ell_{.m}h_{.m}|)^T & y_m + \sigma_d^2 \end{bmatrix} \geq 0, \tag{7}$$

where $\bar{\boldsymbol{\alpha}}_{.m} := (\bar{\boldsymbol{\alpha}}_{1m}, \bar{\boldsymbol{\alpha}}_{2m}, ..., \bar{\boldsymbol{\alpha}}_{Nm})^T$, $|\ell_{.m}h_{.m}| = (|\ell_{1m}h_{1m}|, ..., |\ell_{Nm}h_{Nm}|)^T$, $m = 1, 2, ..., M$, and $\odot$ stands for Hadamard product operation.
Then (6) is equivalent to

$$-\min_{\boldsymbol{x},\boldsymbol{y},\bar{\boldsymbol{\alpha}},} [f_1(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}) - f_2(\bar{\boldsymbol{\alpha}}, \boldsymbol{y})] \ : \ (1), (4), (6b), (6c), (6d) \quad (8)$$

with functions

$$f_1(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}) = \max_{m=1,2,...,M} \sum_{i \neq m} \varphi_i(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}_i) \quad (9)$$

$$f_2(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}) = \sum_{m=1}^{M} \varphi_m(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}_m) \quad (10)$$

both convex as maximization of convex functions and as sum of convex functions [9].

3.2. Exact Penalty Function Approach

Note that the integer (discrete) constraint (4) is equivalent to the following continuous constraints

$$0 \leq \boldsymbol{x}_{nm} \leq 1, n = 1, 2, ..., N, m = 1, 2, ..., M, \quad (11a)$$

$$\sum_{n=1}^{N} \boldsymbol{x}_{nm} - \sum_{n=1}^{N} \boldsymbol{x}_{nm}^2 \leq 0, \ m = 1, 2, ..., M \quad (11b)$$

where (11a) is convex and (11b) is reverse convex [9]. For an effective d.c. procedure, we also represent (11b) in another d.c. setting

$$(11) \Leftrightarrow (11a), \upsilon(\boldsymbol{x}) - \psi(\boldsymbol{x}) \leq 0,$$

with convex quadratic functions

$$\upsilon(\boldsymbol{x}) := \sum_{m=1}^{M} [\sum_{n=1}^{N} \boldsymbol{x}_{nm} + (\sum_{n=1}^{N} \boldsymbol{x}_{nm})^2];$$

$$\psi(\boldsymbol{x}) := \sum_{m=1}^{M} [\sum_{n=1}^{N} (\boldsymbol{x}_{nm})^2 + (\sum_{n=1}^{N} \boldsymbol{x}_{nm})^2];$$

Like [10], it can be shown that (8) is equivalent to the following minimization of a d.c. objective function subject to convex only constraints with sufficiently large $\mu > 0$

$$-\min_{\boldsymbol{x},\boldsymbol{y},\bar{\boldsymbol{\alpha}}} [f_1(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}) - f_2(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}) + \mu(\upsilon(\boldsymbol{x}) - \psi(\boldsymbol{x}))] \quad (12)$$
$$\text{s.t. } (1), (6b), (6c), (6d), (11a)$$

Initialized from a feasible point $(\boldsymbol{x}^{(0)}, \bar{\boldsymbol{\alpha}}^{(0)})$, the following convex quadratic program is iteratively solved to generate a sequence of feasible solutions $(\boldsymbol{x}^{(\kappa)}, \bar{\boldsymbol{\alpha}}^{(\kappa)})$, $\kappa = 1, 2, ...$, which converge to $(\boldsymbol{x}^{(\Omega)}, \bar{\boldsymbol{\alpha}}^{(\Omega)})$ [11]

$$\begin{aligned} - \quad & \min_{\boldsymbol{x},\boldsymbol{y},\bar{\boldsymbol{\alpha}}} \quad [f_1(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}) - (f_2(\bar{\alpha}^{(\kappa)}, \boldsymbol{y}^{(\kappa)}) \\ & -\langle \nabla f_2(\bar{\boldsymbol{\alpha}}^{(\kappa)}, \boldsymbol{y}^{(\kappa)}), (\bar{\boldsymbol{\alpha}} - \bar{\boldsymbol{\alpha}}^{(\kappa)}, \boldsymbol{y} - \boldsymbol{y}^{(\kappa)})\rangle \\ & +\mu(\upsilon(\boldsymbol{x}) - \psi(\boldsymbol{x}^{(\kappa)}) - \langle \nabla\psi(\boldsymbol{x}^{(\kappa)}), \boldsymbol{x} - \boldsymbol{x}^{(\kappa)}\rangle)] \\ \text{s.t.} \quad & (1), (6b), (6c), (6d), (11a). \end{aligned} \quad (13)$$

Our previous works have proved that such an iterative procedure is guaranteed to efficiently and progressively improve the objective function value ([11], [12]).

3.3. Power Allocation

From the solution $(\boldsymbol{x}^{(\Omega)}, \bar{\boldsymbol{\alpha}}^{(\Omega)})$ where $\boldsymbol{x}^{(\Omega)} \in \{0, 1\}^{N \times M}$, the last step of the proposed scheme is to implement the following d.c. program only for power re-allocation,

$$-\min_{\boldsymbol{y},\bar{\boldsymbol{\alpha}},} [f_1(\bar{\boldsymbol{\alpha}}, \boldsymbol{y}) - f_2(\bar{\boldsymbol{\alpha}}, \boldsymbol{y})] : \ (6b), (6c), \boldsymbol{x} = \boldsymbol{x}^{(\Omega)}. \quad (14)$$

3.4. The Choice of Initial Solution

By the variable change $\bar{\boldsymbol{\alpha}}_{nm} = \boldsymbol{x}_{nm}|\boldsymbol{\alpha}_{nm}|^2$, problem (5) is relaxed to

$$\max_{\boldsymbol{x},\bar{\boldsymbol{\alpha}},\boldsymbol{t}} \quad \boldsymbol{t} : (1)$$

$$\sum_{m=1}^{M} \bar{\boldsymbol{\alpha}}_{nm}(|h_{nm}|^2 + \sigma_r^2) \leq P_n, n = 1, 2, \ldots, N; \quad (15a)$$

$$0 \leq \bar{\boldsymbol{\alpha}}_{nm} \leq \boldsymbol{x}_{nm}P_n, n = 1, 2, ..., N, m = 1, 2, ..., M; \quad (15b)$$

$$\sum_{n=1}^{N} \bar{\boldsymbol{\alpha}}_{nm}|\ell_{nm}h_{nm}|^2 \geq \boldsymbol{t} \cdot (\sigma_r^2 \sum_{n=1}^{N} \bar{\boldsymbol{\alpha}}_{nm}|\ell_{nm}|^2 + \sigma_d^2) \quad (15c)$$

which can be solved by parametric linear programming. The feasible solution $(\boldsymbol{x}^{(0)}, \bar{\boldsymbol{\alpha}}^{(0)})$ of (5) for initializing the above iterative procedure is taken from the optimal solution of (15).

4. SIMULATION RESULTS

This section presents several simulation results to illustrate the performance of our proposed method. In all our simulations, we assume the perfect channel state information at both links by relays. AWGN at relays and destination nodes with $\sigma_r^2 = \sigma_d^2 = 0.01$.

In the first scenario, we set $M = 5, N = 10, N_R = 3$, and gradually increase the individual power P_n from 0.1 watt to 9 watt. Under each power constraint, 200 randomly generated network models are tested by [8]'s procedure and the proposed procedures. Then, their performance is averaged. Fig.1 shows the mean worst SNR versus maximum individual power. It is obvious that the performance of the proposed scheme is almost identical with that of the relaxed problem, which is the global optimal upper bound [8]. Therefore, its exceptional ability in locating global optimal solution is revealed . In contrast, although proved better than most other existing approaches, [8]'s procedure shows a consistently inferior performance to the proposed procedure.

In the second scenario, with $M = 5, P_n = 2.5$ watt, the network is expanded from $N = 2$ to $N = 16$, and N_R is gradually increased accordingly ($N_R = \min(N, \lceil \frac{N}{M} \rceil)$, where $\lceil \cdot \rceil$ denotes ceiling function). 200 randomly generated channel models are tested by both procedures at each point, Fig.2 demonstrates how mean minimum SNR performance responds to the changing number of relays with both schemes. The diversity of the network increases as the network expands, and so does the performance of the cooperative network. Again, the performance of the proposed procedure closely follows that of the relaxed upper bound. This justifies its solutions can be regarded as global optimal. However, [8]'s procedure is evidently outperformed. Fig.2 also indicates that the superior performance of the proposed is robust to the increasing size of the network. Moreover, the computational complexity in terms of mean CPU time is also analyzed, as illustrated in Fig.3. Unlike other iterative algorithms normally of much higher complexity, the proposed scheme is shown to locate the global optimal solution within few seconds. It should also be noted that its complexity is not significantly increased as network expands, which indicates its advanced practical applicability and efficiency.

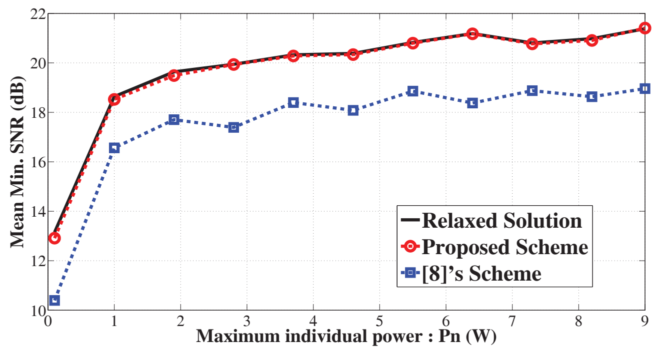

Fig. 1. Mean minimum SNR versus Individual relay power

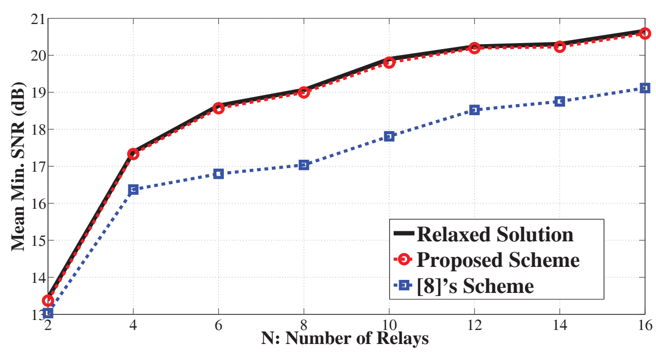

Fig. 2. Mean minimum SNR versus Number of relays

5. CONCLUSION

The optimized schemes for multi-relay selection in amplify-and-forward wireless relay network have been studied in this paper. The existing attempts have either been suboptimal or computationally unaffordable. In contrast, we first recast the original NP-hard combinatorial relay selection problem into a d.c. program over convex constraints by employing an effective representation of the integer constraints. The global optimal solution is then obtained by iteratively solving a convex quadratic d.c. program. In the end, the simulation performance of the proposed scheme has well validated its capacity of efficiently locating the global optimal solution.

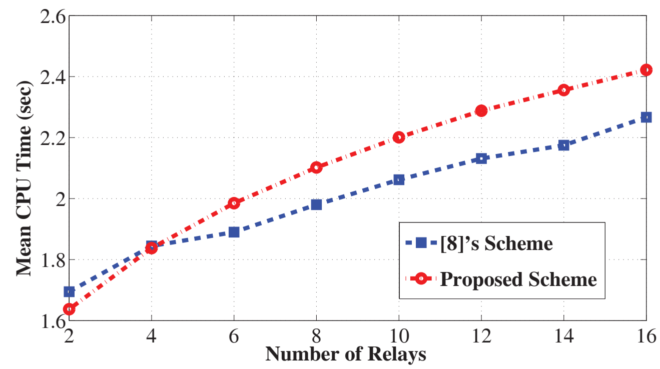

Fig. 3. CPU time versus Number of relays

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