A NOVEL APPROACH TO DOPPLER COMPENSATION AND ESTIMATION FOR MULTIPLE TARGETS IN MIMO RADAR WITH UNITARY WAVEFORM MATRIX SCHEDULING

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ABSTRACT

In this paper, we present a method of detecting the range and Doppler phase of a point target using multiple antennas. As a key illustrative example, we consider a 4×4 system employing a unitary matrix waveform set, e.g., formed from Golay complementary sequences. When a non-negligible Doppler shift is induced by the target motion, the waveform matrix formed from the complementary sequences is no longer unitary, resulting in significantly degraded target range estimates. To solve this problem, we adopt a subspace based approach exploiting the observation that the receive matrix formed from matched filtering of the reflected waveforms has a (non-trivial) null-space. Through processing of the waveforms with the appropriate vector from the null-space, we can significantly improve the range detection performance. Also, another very important target aspect is the velocity with which the target is moving, and to determine that, the exact Doppler phase shift induced by the target motion needs to be estimated with reasonable accuracy. To accomplish this task, we develop a strategy that uses the MUSIC algorithm to estimate the Doppler phase, and we use simulations to show that the phase estimates obtained are reasonably accurate even at low SNRs.

1. INTRODUCTION

In [1], Howard et al. proposed a new multi-channel radar scheme employing polarization diversity for obtaining multiple independent views of the target. In this scheme, Golay pairs [2] of phase coded waveforms are used to provide synchronization while Alamouti coding is used to coordinate transmission of these waveforms on the horizontal and vertical polarizations and this enables unambiguous radar polarimetry on a pulse-by-pulse basis, thereby reducing signal processing complexity. In [3], the 2×2 case was extended to multiple antennas, and more general waveforms families were developed that allowed for perfect separation in the case of negligible Doppler. In particular, scheduling for Golay pairs was described for a 4×4 system and it was demonstrated that Golay pairs achieve both perfect separation and perfect reconstruction [2]. However, in the presence of Doppler, Golay pairs are known to perform poorly and this is the primary reason that Golay sequences have not found widespread use in radar.

In [4], PTM sequences were used to make the Golay sequence transmissions resilient against Doppler shifts. The method achieves good results for small Doppler shifts, but the number of PRIs needed per transmission of the coded Golay sequence matrix is large, thereby requiring the "radar channel" to stay constant over a relatively long time interval. In this paper, we describe a Doppler compensation and estimation scheme that exploits the subspace structure of the received waveform matrix. We show that the received waveform matrix can be processed in a way that imparts a specific structure on the subspace that it occupies, and the null-space of this matrix can be used to minimize the effects of Doppler. We develop a processing filter using the null-space of this matrix to alleviate the effects of Doppler in target ranging, and demonstrate that the method works over a wide range of target SNRs. In addition to that, we develop a MUSIC algorithm based technique to estimate the Doppler phase. We also show that the scheme works for multiple targets if their separation and velocities follow certain conditions.

2. GOLAY COMPLEMENTARY SEQUENCES AND TARGET DETECTION

A pair of sequences $s_1(n)$ and $s_2(n)$ of length N_c satisfy the Golay property [2] if the sum of their autocorrelation functions satisfy

$$R_{s_1 s_1}(l) + R_{s_2 s_2}(l) = \begin{cases} 2N_c & if \quad l = 0 \\ 0 & if \quad l \neq 0 \end{cases} \tag{1}$$

for $l = -N_c - 1, ..., N_c - 1$. If we take the DFT of the above equation, we get

$$|S_1(k)|^2 + |S_2(k)|^2 = 2N_c \tag{2}$$

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In [3], we showed that if $s_1(n)$ and $s_2(n)$ are Golay complementary, then so are $s_1^*(-n)$ and $s_2^*(-n)$. Using this fact, we can develop a 4-waveform family using Golay complementary sequences by defining

$$s_3(n) = s_1^*(-n) \tag{3}$$

and

$$s_4(n) = s_2^*(-n) \tag{4}$$

Now, in the case of negligible Doppler, the received signal over 4 PRIs is given by

$$\mathbf{R}(n) = \mathbf{H}^T\mathbf{S}(n) + \mathbf{N}(n) \tag{5}$$

where $\mathbf{S}(n)$ is the 4×4 transmitted waveform matrix given by [4]. $\mathbf{H}$ is the channel matrix which contains the various round-trip path gains from each transmit antenna to each receive antenna. and $\mathbf{N}(n)$ is the noise matrix. To detect the presence of the target in the delay resolution bin n, We process the received waveform matrix as

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \mathbf{H}^T\mathbf{S}(n) * \mathbf{S}^H(-n) + \mathbf{N}'(n) \tag{6}$$

where $*$ is the pair-wise convolution of two matrices that follows the same order as matrix multiplication. It can be easily shown [4] that

$$\mathbf{S}(n) * \mathbf{S}^H(-n) = \alpha\mathbf{I}\delta(n) \tag{7}$$

From this, it follows that

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \alpha\mathbf{H}\delta(n) + \mathbf{N}'(n) \tag{8}$$

In order to detect the presence of a target in the delay resolution bin n, consider the test stastic

$$z(n) = \left\|\mathbf{R}(n) * \mathbf{S}^H(n)\right\|_F^2 \tag{9}$$

where the subscript F stands for Frobenius norm. A plot of $z(n)$ for target SNRs of 5dB and 10dB, respectively are shown in Figure 1. As we can see from the figure, the unitary waveform matrix signal design greatly facilitates high-resolution time-localization of a target when the Doppler shift is negligible.

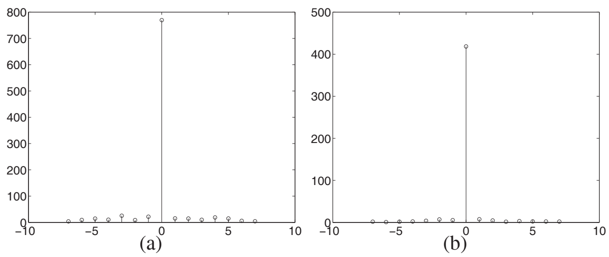

Fig. 1. Plot of $z(n)$ without Doppler for (a) SNR = 5dB (b) SNR = 10dB

3. DOPPLER COMPENSATION AND ESTIMATION

In this section, we develop a signal model that incorporates the effects of Doppler. We assume that the target is moving at a constant speed, which means that between two successive PRIs, the differential Doppler phase shift is constant.

3.1. Effects of Doppler

In the presence of Doppler, the received signal may be expressed as

$$\mathbf{R}(n) = \mathbf{H}^T\mathbf{S}(n)\mathbf{D} + \mathbf{N}(n) \tag{10}$$

The Doppler shift matrix $\mathbf{D}$ is given by

$$\mathbf{D} = diag\{1, e^{j\upsilon'}, e^{j2\upsilon'}, e^{j3\upsilon'}\} \tag{11}$$

where υ' is the Doppler-induced differential phase shift between two successive PRIs. As in the case of negligible Doppler, we process the received waveform matrix as

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \mathbf{H}^T\mathbf{S}(n)\mathbf{D} * \mathbf{S}^H(-n) + \mathbf{N}'(n) \tag{12}$$

In the presence of a non-negligible Doppler phase shift, the condition in (4) is not satisfied in general, i.e.,

$$\mathbf{S}(n)\mathbf{D} * \mathbf{S}^H(-n) \neq \alpha\mathbf{I}\delta(n) \tag{13}$$

and unambiguous range resolution becomes significantly more difficult. To illustrate this graphically, a plot of $z(n)$ for the same target SNRs of 5dB and 10dB are plotted in Figure 2 for the case of $\upsilon' = \pi/3$. For this particular set of round-trip

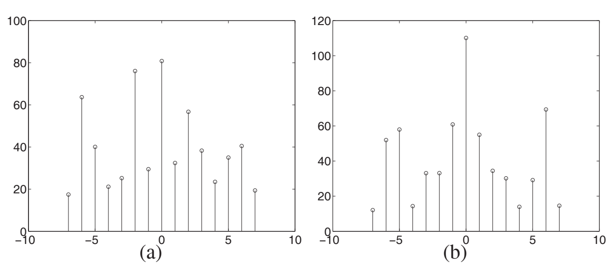

Fig. 2. Plot of $z(n)$ with Doppler ($\upsilon' = \pi/3$) for (a) SNR = 5dB (b) SNR = 10dB

channel gains, the presence of Doppler makes it impossible to detect the target.

3.2. Doppler Processing and Phase Estimation

Towards combatting this problem, consider the matrix

$$\hat{\mathbf{R}}(n) = \mathbf{R}(n) * \mathbf{S}^H(-n) \tag{14}$$

Each term of this matrix is a sum of four individual convolution sequences. Next, consider the 4×4 matrix $\mathbf{Y}_i$, given by

$$\mathbf{Y}_i(n) = \left[\begin{array}{cccccc} r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) \\ r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) \\ r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) \\ r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) \end{array} \right] \quad (15)$$

Note that column j above contains the individual convolution sequences that are summed up to yield the ij^{th} term in $\hat{\mathbf{R}}(n)$.

Consider the vector

$$\mathbf{w} = \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array} \right]^T \quad (16)$$

In the case of no Doppler, and ignoring the noise, it is easy to verify that

$$\mathbf{w}^H \mathbf{Y}_i(n) = \gamma \mathbf{h}_i \delta(n) \quad (17)$$

where γ is just a scaling constant, and

$$\mathbf{h}_i = \left[\begin{array}{cccc} h_{1i} & h_{2i} & h_{3i} & h_{4i} \end{array} \right] \quad (18)$$

The index i is associated with one of the receive antennas.

When Doppler is present, it is likewise easy to verify that

$$\mathbf{w}_D^H \mathbf{Y}_i(n) = \gamma \mathbf{h}_1 \delta(n) \quad (19)$$

where

$$\mathbf{w}_D = \left[\begin{array}{cccc} 1 & e^{j\upsilon'} & e^{2j\upsilon'} & e^{3j\upsilon'} \end{array} \right]^T \quad (20)$$

and this holds for all i. This means that for $n \neq 0$, the matrices $\mathbf{Y}_i$ are singular, and the vector producing the desired output lies in the null-space of these matrices. Also, because of the same waveform inputs, the matrices $\mathbf{Y}_i$ share the same null-space. Thus, we can form a concatenated matrix as

$$\mathbf{Y}_C(n) = \left[\begin{array}{cccc} \mathbf{Y}_1(n) & \mathbf{Y}_2(n) & \mathbf{Y}_3(n) & \mathbf{Y}_4(n) \end{array} \right] \quad (21)$$

It is easy to verify that

$$\mathbf{w}_D^H \mathbf{Y}_C(n) = \gamma \mathbf{h} \delta(n) \quad (22)$$

where $\mathbf{h} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \mathbf{h}_3 \ \mathbf{h}_4]$. Now, since we don't know the true-target delay for which $\mathbf{Y}_C(n)$ is non-singular, we cannot simply find the null-space vector at every n. That is, an approach is need to circumvent the fact $\mathbf{w}_D$ is not in the null-space of $\mathbf{Y}_C(n)$ at the true target delay. Again, WLOG the true target delay is assumed to be $n = 0$. The concatenated matrix $\mathbf{Y}_C(n)$ is formed to exploit the fact that the different matrices share the same null-space vector.

To counter the effect of $\mathbf{Y}_C(0)$ on the null-space, instead of working with a single chip interval, we take a length $2q+1$ lag window and form the matrix

$$\mathbf{X}_C(n) = \left[\begin{array}{ccccc} \mathbf{Y}_C(n-q) & \ldots & \mathbf{Y}_C(n) & \ldots & \mathbf{Y}_C(n+q) \end{array} \right] \quad (23)$$

The SVD of $\mathbf{X}_C$ can be written as

$$\mathbf{X}_C(n) = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^H \quad (24)$$

Since $\mathbf{X}_C(n)$ and $\mathbf{X}_C(n)\mathbf{X}_C^H(n)$ share the same singular vectors, we will work with the latter. The idea is to subtract out $\mathbf{Y}_C(0)$ in order to obtain the correct the null space of $\mathbf{R}_{X_C X_C}(n)$. To do this, we compute the SVD of

$$\mathbf{R}_{X_C X_C}(n) - \mathbf{Y}_C(k)\mathbf{Y}_C^H(k) \quad (25)$$

for $n - q \leq k \leq n + q$ and store the singular vector associated with the smallest eigenvalue. Note that out of the $2q + 1$ singular vectors that we store for each n, there is at most one singular vector that corresponds to $\mathbf{Y}_C(0)$ and this happens whenever $0 \in \{n - q, ..., n + q\}$. Again, WLOG the true target delay here is $n = 0$.

The inclusion of $\mathbf{Y}_C(0)$ alters the null-space structure. In order to find which matrix to subtract, since we don't know the true target delay, for each of the $2q+1$ singular vectors, we compute its inner product with the other $2q$ singular vectors, and choose that singular vector that yields the smallest inner product (magnitude) with the rest of the vectors.

This process is mathematically described as follows. Let $\mathbf{U}_{min}(n)$ be the matrix with the $2q + 1$ singular vectors as columns. The inner product (Grammian) matrix is formed as

$$\mathbf{M}(n) = \mathbf{U}_{\min}^H(n)\mathbf{U}_{\min}(n) \quad (26)$$

We can write $\mathbf{M}(n)$ as

$$\mathbf{M}(n) = \left[\begin{array}{c} \mathbf{m}_{n-q} \\ \vdots \\ \mathbf{m}_{n+q} \end{array} \right] \quad (27)$$

The index of the singular vector of interest is obtained as

$$k_{opt} = \arg\min_k \|\mathbf{m}_k\| \quad (28)$$

To check the presence of target in the delay bin n, we process the vector $\mathbf{Y}_C(n)$ as

$$z(n) = \left\| \mathbf{u}_{k_{opt}}^H \mathbf{Y}_C(n) \right\|_F^2 \quad (29)$$

Ideally, the magnitude of $z(n)$ should exhibit a sharp peak at the true target delay and be near zero for all other values of n.

Now, to estimate the Doppler phase, we make use of the fact that the Doppler processing vector we are looking for lies in the null space of the matrix $\mathbf{Y}_C(n)$, and we use the MUSIC algorithm to estimate this vector. Consider the eigendecomposition

$$\mathbf{U}(n)\Sigma(n)\mathbf{U}^H(n) = \mathbf{R}_{X_C X_C}(n) - \mathbf{Y}_C(k_{opt})\mathbf{Y}_C^H(k_{opt}) \quad (30)$$

where $\mathbf{U}$ is a 4×4 matrix. Since the matrix for which we compute the SVD is singular, we can write $\mathbf{U}$ can be written as

$$\mathbf{U}(n) = \left[\begin{array}{cc} \mathbf{S}'(n) & \mathbf{G}'(n) \end{array}\right] = \left[\begin{array}{cccc} \mathbf{s}_1(n) & \mathbf{s}_2(n) & \mathbf{g}_1(n) & \mathbf{g}_2(n) \end{array}\right] \tag{31}$$

where $\mathbf{S}'$ are the 'signal' eigenvectors and $\mathbf{G}'$ are the null-space eigenvectors. Now, from our earlier analysis, we know that the Doppler processing vector that we seek lies in the space spanned by the vectors of $\mathbf{G}'$, and since that vector has the structure of a complex sinusoid, we can estimate the phase of that vector by using the MUSIC algorithm. Specifically, we estimate the Doppler frequency from the psuedospectrum as

$$\arg\max_{\upsilon} \left\{ \frac{1}{\mathbf{a}^H(\upsilon)\mathbf{G}'(n)\mathbf{G}'^H(n)\mathbf{a}(\upsilon)} \right\} \quad , \quad -\pi \leqslant \upsilon < \pi \tag{32}$$

where

$$\mathbf{a}(\upsilon) = \left[\begin{array}{cccc} 1 & e^{j\upsilon} & e^{j2\upsilon} & e^{j3\upsilon} \end{array}\right] \tag{33}$$

4. SIMULATION RESULTS

We simulate a 4×4 baseband system and use Golay complementary sequences of length 10. The channel entries are i.i.d complex Gaussian with unit variance. The relative motion between the target and the radar introduces a Doppler phase shift of $\pi/3$ and $2\pi/3$ between adjacent PRIs for the two targets which are separated N_c+1 samples apart in delay. In Figure 3, we plot the estimated delay-Doppler image of the target scene. The Doppler axis goes from 0-256 where 256 is equivalent to 2π. As we can see, the peaks occur at the correct target locations in both delay and Doppler. In Figure 4, we plot the phase estimation error, defined as

$$E_{\upsilon} = \frac{1}{N} \sum_{n=1}^{N} \|\hat{\upsilon}(n) - \upsilon'\|_F^2 \tag{34}$$

as a function of SNR. As we can see from this figure, even at low SNRs, the Doppler phase estimation is not poor and it still gives a relatively accurate estimate of the target velocity. The phase estimation error improves with the SNR.

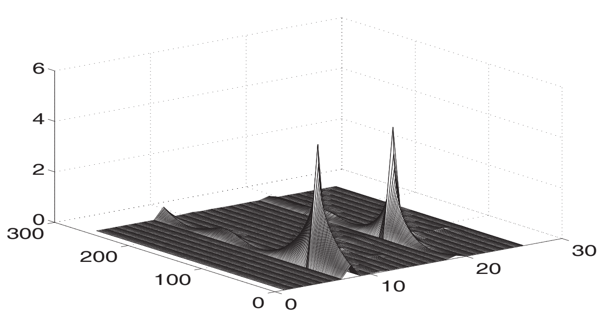

Fig. 3. Delay-Doppler image of the two targets

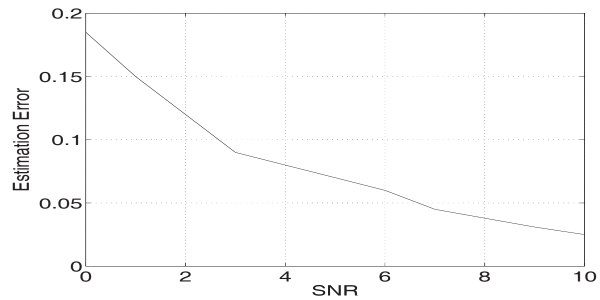

Fig. 4. Phase estimation error as a function of SNR

5. CONCLUSIONS

We have developed a technique for accurate target ranging and velocity estimation in the presence of Doppler using Golay complementary sequences when multiple targets are present. The technique is based on finding the null-space of the waveform matrix after matched filtering at the receiver, and then using an appropriate vector from the null-space to process the matched filtered received waveforms over multiple PRIs. Simulation results were presented that show how the proposed technique diminishes the effects of Doppler while still facilitating high-resolution, accurate target ranging over a wide range of target SNRs. The target velocity estimation is dependent on the Doppler phase, which is estimated by using the MUSIC algorithm. The phase estimates obtained using this algorithm are reasonably accurate for SNRs as low as 0 dB, and improve with SNR.

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