OPTIMAL SEQUENTIAL WAVEFORM DESIGN FOR COGNITIVE RADAR

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ABSTRACT

This paper addresses the problem of adaptive sequential waveform design for system parameter estimation. This problem arises in several applications such as radar, sonar, or tomography. In the proposed technique, the transmit/input signal waveform is optimally determined at each step, based on the measurements in the previous steps. The waveform is determined to minimize the Bayesian Cramér-Rao bound (BCRB) for estimation of the unknown system parameter at each step. The algorithm is tested for spatial transmit waveform design in multiple-input multiple-output radar target angle estimation at very low signal-to-noise ratio. The simulations show that the proposed adaptive waveform design achieves significantly higher rate of performance improvement as a function of the pulse index, compared to identical signal transmission.

***Index Terms*—** Sequential waveform design, Bayesian Cramér-Rao bound (BCRB), Waveform optimization, Cognitive radar (CR).

1. INTRODUCTION

Waveform optimization for system parameter estimation is an emerging topic in signal processing with applications in many areas, such as, radar, sonar, tomopraphy. The basic idea is to optimize a chosen criterion such as, statistical bounds, probability of error, output signal-to-noise ratio (SNR), information theoretic methods, and so on, with respect to the transmit waveform or the transmit auto-correlation matrix. Waveform optimization has been investigated based on the Cramér-Rao bound (CRB), for multiple-input multiple-output (MIMO) radar in case of single target in [1], and generalized for the case of multiple targets in [2]. An information theoretic waveform optimization point of view is investigated for example in [3-5].

In the recent years, the cognitive radar (CR) approach has been proposed [6, 7] and intensively investigated. In this approach, the radar system adaptively interrogates a propagation channel using all available knowledge including previous measurements, task priorities, and external databases. In [8] two different waveform design techniques based on sequential hypothesis testing for active systems operating in a target recognition application were derived. In [9] an optimal waveform design for CR based on maximizing the mutual information and the output SNR is considered. In [10], an adaptive polarized waveform design for target tracking based on sequential Bayesian inference has been presented.

In this paper, we propose a new technique for optimal sequential waveform design. Instead of transmission of a pulse train with predefined waveform, each waveform in the pulse train is adaptively determined based on the previously received data, often referred as memory or history. The considered observation model is general which may be useful in many applications. The Bayesian Cramér Rao bound (BCRB) [11] is used as a criterion for optimality. In other words, we seek to find an optimal waveform design, which minimizes sequentially the BCRB on the system parameter estimation, such as target direction in radars, based on previous received data. The main advantage of the proposed method is that it is capable to automatically focus on the target after a few trials/pulses, at very low signal-to-noise ratios (SNRs).

In the next section, the system model is described and the problem is formulated. The proposed sequential waveform design technique is derived in Section 3. In Section 4, the performance of the proposed technique are evaluated and compared to identical waveform design for radar systems. Finally, our conclusions appear in Section 5.

2. SYSTEM MODEL

Consider the following general data model which is useful in many applications such as radar, sonar, or tomography

$$\mathbf{X}_k = \mathbf{H}\left(\theta\right)\mathbf{S}_k + \mathbf{N}_k \tag{1}$$

where the columns of $\mathbf{X}_k \in \mathbb{C}^{N\times L}$, $\mathbf{S}_k \in \mathbb{C}^{M\times L}$, and $\mathbf{N}_k \in \mathbb{C}^{N\times L}$ represent the L snapshots of the data, transmit/input signal, and noise vectors, respectively, at the kth step. The matrix $\mathbf{H}\left(\theta\right) \in \mathbb{C}^{N\times M}$ denotes the system transfer function, which depends on the unknown random variable θ with *a-priori* probability density function (pdf) $f_\theta(\cdot)$. The columns of $\mathbf{N}_k$ are independent and identically distributed complex circularly symmetric Gaussian random vectors with zero mean and covariance matrix $\mathbf{R}$. This general model represents several signal processing problems. We are interested in optimal design of the transmit signal matrix at the kth step, $\mathbf{S}_k$, given the observations in previous steps (history), $\mathbf{X}^{(k-1)} = [\mathbf{X}_1, \ldots, \mathbf{X}_{k-1}]$. The criterion to be optimized, is the BCRB for estimation of θ from $\mathbf{X}^{(k)}$.

3. SEQUENTIAL WAVEFORM OPTIMIZATION

Consider the BCRB on the estimation error of θ. Under some regularity conditions, the BCRB for estimation of θ at step k is given by [11]

$$C_k \geq \left(I_{D_k} + I_P\right)^{-1} \tag{2}$$

where I_{D_k} and I_P denote the Fisher information from data at step k and from the prior statistical knowledge, respectively. In the following, we derive the Fisher information, I_{D_k}, for the model given in (1). Using the definition of I_{D_k} and by applying the Bayes theorem one obtains

$$
\begin{aligned}
I_{D_k} &= -\mathrm{E}\left(\frac{\partial^2 \log f_{\mathbf{X}^{(k)}|\theta}}{\partial\theta^2}\right) \\
&= I_{D_{k-1}} + \Delta I_{D_k}
\end{aligned} \tag{3}
$$

where ΔI_{D_k} is the incremental Bayesian Fisher information (IBFI), defined as

$$
\begin{aligned}
\Delta I_{D_k} &= -\mathrm{E}\left(\frac{\partial^2 \log f_{\mathbf{X}_k|\mathbf{X}^{(k-1)},\theta}}{\partial\theta^2}\right) \\
&= \mathrm{E}\left[\Delta\widetilde{I}_{D_k}\left(\mathbf{X}^{(k-1)}\right)\right]
\end{aligned} \tag{4}
$$

in which

$$
\Delta\widetilde{I}_{D_k}\left(\mathbf{X}^{(k-1)}\right) = -\mathrm{E}\left(\frac{\partial^2 \log f_{\mathbf{X}_k|\mathbf{X}^{(k-1)},\theta}}{\partial\theta^2} \,\middle|\, \mathbf{X}^{(k-1)}\right). \tag{5}
$$

The expectations in (4) and (5) are performed with respect to the joint pdf of $\left(\mathbf{X}^{(k)},\theta\right)$, and the conditional pdf of $\left(\mathbf{X}_k,\theta \mid \mathbf{X}^{(k-1)}\right)$, respectively.

By using the expression for the Fisher information in case of deterministic signal in Gaussian noise [12], and applying the law of total expectation, $\Delta\widetilde{I}_{D_k}\left(\mathbf{X}^{(k-1)}\right)$ can be written as

$$
\begin{aligned}
\Delta\widetilde{I}_{D_k} &= -\mathrm{E}\left(\frac{\partial^2 \log f_{\mathbf{X}_k|\mathbf{X}^{(k-1)},\theta}}{\partial\theta^2} \,\middle|\, \mathbf{X}^{(k-1)}\right) \\
&= \mathrm{E}\left[-\mathrm{E}\left(\frac{\partial^2 \log f_{\mathbf{X}_k|\mathbf{X}^{(k-1)},\theta}}{\partial\theta^2} \,\middle|\, \mathbf{X}^{(k-1)},\theta\right) \,\middle|\, \mathbf{X}^{(k-1)}\right] \\
&= 2L\mathrm{E}\left\{\mathrm{Re}\left[\mathrm{tr}\left(\dot{\mathbf{H}}^H\mathbf{R}^{-1}\dot{\mathbf{H}}\mathbf{R}_{\mathbf{S}_k}\right) \,\middle|\, \mathbf{X}^{(k-1)}\right]\right\}
\end{aligned} \tag{6}
$$

where $\mathrm{Re}(\cdot)$ and $\mathrm{tr}(\cdot)$, denote the real part and trace operators, respectively, and $\dot{\mathbf{H}} \triangleq \frac{\mathrm{d}\mathbf{H}(\theta)}{\mathrm{d}\theta}$, $\mathbf{R}_{\mathbf{S}_k} \triangleq \frac{1}{L}\mathbf{S}_k\mathbf{S}_k^H$. For simplicity of notations, we omit the dependency of $\dot{\mathbf{H}}$ on θ, and of $\Delta\widetilde{I}_{D_k}$ on $\mathbf{X}^{(k-1)}$. Using (4) and (6), the Fisher information in (3) can be expressed as

$$
\begin{aligned}
I_{D_k} &= I_{D_{k-1}} \\
&+ 2L\mathrm{E}\left(\mathrm{Re}\left\{\mathrm{tr}\left[\mathrm{E}\left(\dot{\mathbf{H}}^H\mathbf{R}^{-1}\dot{\mathbf{H}} \,\middle|\, \mathbf{X}^{(k-1)}\right)\mathbf{R}_{\mathbf{S}_k}\right]\right\}\right).
\end{aligned} \tag{7}
$$

As mentioned above, we aim to find the transmit signal matrix, $\mathbf{S}_k$, which minimizes the BCRB at the kth step, C_k. Based on (7) the BCRB depends on the transmit waveform only through $\mathbf{R}_{\mathbf{S}_k}$. Accordingly, we aim to find the optimal transmit signal auto-correlation matrix $\mathbf{R}_{\mathbf{S}_k}$. In addition, we will assume that the transmit signal energy is limited. Under the total energy constraint, i.e. $\mathrm{tr}\left(\mathbf{R}_{\mathbf{S}_k}\right) \le P$, using (2), and the fact that I_P is independent of $\mathbf{R}_{\mathbf{S}_k}$, the optimization problem is given by

$$
\begin{aligned}
\overline{\mathbf{R}}_{\mathbf{S}_k} &= \arg\max_{\mathbf{R}_{\mathbf{S}_k}} I_{D_k} \\
&\text{s.t. } \mathrm{tr}\left(\mathbf{R}_{\mathbf{S}_k}\right) \le P \\
&\qquad \mathbf{R}_{\mathbf{S}_k} \succeq 0.
\end{aligned} \tag{8}
$$

The first component in the r.h.s of (7), $I_{D_{k-1}}$, denotes the Fisher information at step $k-1$, and it depends only on the observations $\mathbf{X}^{(k-1)}$, which are independent of $\mathbf{R}_{\mathbf{S}_k}$. Therefore, for optimization of (7), only the second term should be considered. The optimization of (7) can be performed by maximizing the inner term in the outer expectation of (7) independently for each $\mathbf{X}^{(k-1)}$. Hence, the optimization problem can be rewritten as

$$
\begin{aligned}
\overline{\mathbf{R}}_{\mathbf{S}_k} &= \arg\max_{\mathbf{R}_{\mathbf{S}_k}} \mathrm{Re}\left\{\mathrm{tr}\left[\mathrm{E}\left(\dot{\mathbf{H}}^H\mathbf{R}^{-1}\dot{\mathbf{H}} \,\middle|\, \mathbf{X}^{(k-1)}\right)\mathbf{R}_{\mathbf{S}_k}\right]\right\} \\
&\text{s.t. } \mathrm{tr}\left(\mathbf{R}_{\mathbf{S}_k}\right) \le P \\
&\qquad \mathbf{R}_{\mathbf{S}_k} \succeq 0.
\end{aligned} \tag{9}
$$

Let

$$
\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right) \triangleq \mathrm{E}\left(\dot{\mathbf{H}}^H(\theta)\,\mathbf{R}^{-1}\dot{\mathbf{H}}(\theta) \,\middle|\, \mathbf{X}^{(k-1)}\right). \tag{10}
$$

Then by using the singular value decomposition (SVD) of $\mathbf{R}_{\mathbf{S}_k}$: $\mathbf{R}_{\mathbf{S}_k} = \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{U}_k^H$, the optimization problem in (9) becomes

$$
\begin{aligned}
\left(\overline{\mathbf{U}}_k,\overline{\mathbf{\Lambda}}_k\right) &= \arg\max_{\mathbf{U}_k,\mathbf{\Lambda}_k} \mathrm{Re}\left\{\mathrm{tr}\left[\mathbf{U}_k^H\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right)\mathbf{U}_k\mathbf{\Lambda}_k\right]\right\} \\
&\text{s.t. } \mathrm{tr}\left(\mathbf{\Lambda}_k\right) \le P \\
&\qquad \mathbf{\Lambda}_k \succeq 0, \text{ and } \mathbf{U}_k \text{ is unitary.}
\end{aligned} \tag{11}
$$

Denoting $\mathbf{\Lambda}_k = \mathrm{diag}\left(\lambda_{1,k},\ldots,\lambda_{M,k}\right)$, and $\mathbf{U}_k = \left[\mathbf{u}_{1,k},\ldots,\mathbf{u}_{M,k}\right]$, where $\mathrm{diag}(\cdot)$ denotes the diagonal operator, the optimization problem in (11) can be rewritten as

$$
\begin{aligned}
\left(\overline{\mathbf{U}}_k,\overline{\mathbf{\Lambda}}_k\right) &= \arg\max_{\mathbf{U}_k,\mathbf{\Lambda}_k} \sum_{i=1}^{M} \gamma_k\left(\mathbf{u}_{i,k}\right)\lambda_{i,k} \\
&\text{s.t. } \sum_{i=1}^{M} \lambda_{i,k} \le P \\
&\qquad \lambda_{i,k} \ge 0, \quad i = 1,\ldots,M
\end{aligned} \tag{12}
$$

where $\gamma_k\left(\mathbf{u}_{i,k}\right) = \mathbf{u}_{i,k}^H\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right)\mathbf{u}_{i,k}$. Since $\{\lambda_{i,k}\}_{i=1}^M$ are non-negative, then the objective function in (12) is maximized by assigning all the available power towards the subspace with maximum $\gamma_k\left(\mathbf{u}_{i,k}\right)$, and zero power towards the complement subspace. The vector $\mathbf{u}_{i,k}$ which maximizes $\gamma_k\left(\mathbf{u}_{i,k}\right)$ is given by the eigenvector corresponding to the maximum eigenvalue of $\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right)$. Denoting this eigenvector by $\overline{\mathbf{u}}_k$, the solution of the optimization problem in (12) is given by

$$
\begin{aligned}
\overline{\mathbf{\Lambda}}_k &= \mathrm{diag}\left(P,0,\ldots,0\right) \\
\overline{\mathbf{U}}_k &= \left[\overline{\mathbf{u}}_k,\mathbf{V}_k\right]
\end{aligned} \tag{13}
$$

where $\mathbf{V}_k$ denotes a matrix of size $M \times (M-1)$, whose columns are orthonormal and perpendicular to $\overline{\mathbf{u}}_k$. Based on (13), the optimal transmit signal auto-correlation matrix is given by

$$
\overline{\mathbf{R}}_{\mathbf{S}_k} = P \cdot \overline{\mathbf{u}}_k\overline{\mathbf{u}}_k^H. \tag{14}
$$

In order to compute $\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right)$ in (10), one needs to calculate the posterior pdf $f_{\theta|\mathbf{X}^{(k-1)}}$, which can be sequentially updated as follows

$$
f_{\theta|\mathbf{X}^{(k-1)}} = \frac{F_{k-1}(\theta)}{f_{\mathbf{X}^{(k-1)}}}, \quad k = 2,3,\ldots \tag{15}
$$

where

$$
F_{k-1}(\theta) = F_{k-2}(\theta) \cdot f_{\mathbf{X}_{k-1}|\mathbf{X}^{(k-2)},\theta} \tag{16}
$$

and

$$F_0(\theta) = f_\theta, \quad f_{\mathbf{X}_1|\mathbf{X}^{(0)},\theta} = f_{\mathbf{X}_1|\theta} \tag{17}$$

where $\left(\mathbf{X}_k \mid \mathbf{X}^{(k-1)}, \theta\right) \sim \mathcal{N}^c\left(\mathbf{H}(\theta)\mathbf{S}_k\left(\mathbf{X}^{(k-1)}\right), \mathbf{R}\right)$. The denominator in (15) is a normalization factor, which only scales the matrix $\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right)$. Accordingly, for calculation of $\overline{\mathbf{u}}_k$, one can ignore this normalization factor, which scales only the eigenvalues of $\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right)$. Correspondingly, in the optimization problem, one can compute

$$\begin{aligned}\widetilde{\mathbf{\Gamma}}\left(\mathbf{X}^{(k-1)}\right) &= \mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right) f_{\mathbf{X}^{(k-1)}} \\ &= \int_\Theta \dot{\mathbf{H}}^H(\theta)\mathbf{R}^{-1}\dot{\mathbf{H}}(\theta)F_{k-1}(\theta)\,\mathrm{d}\theta \end{aligned} \tag{18}$$

instead of $\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right)$, where Θ is the parameter space.

In summary, the steps of the proposed sequential algorithm are described in Algorithm 1.

Algorithm 1

<u>**Initialization**</u>: $F_0 = f_\theta$
for k = 1,2,... **do**
1. Compute $F_{k-1}(\theta)$ for a predefined grid of θ using (16).
2. Compute $\widetilde{\mathbf{\Gamma}}\left(\mathbf{X}^{(k-1)}\right)$ using (18).
3. Pick the eigenvector corresponding to the maximum eigenvalue of $\widetilde{\mathbf{\Gamma}}\left(\mathbf{X}^{(k-1)}\right)$, $\overline{\mathbf{u}}_k$.
4. Construct $\mathbf{R}_{\mathbf{S}_k}$ according to (14).

end for

---

4. NUMERICAL RESULTS

An example of the model used in (1) is the mono-static MIMO radar with N_R receiving and N_T transmitting antennas. In this case, the sequential observation model is given by [13]

$$\mathbf{X}_k = \alpha \cdot \mathbf{a}_R(\theta)\mathbf{a}_T^T(\theta)\mathbf{S}_k + \mathbf{N}_k \tag{19}$$

where θ is the target direction, $\mathbf{a}_R(\theta) \in \mathbb{C}^{N_R \times 1}, \mathbf{a}_T(\theta) \in \mathbb{C}^{N_T \times 1}$ are the steering vectors for the receiving and transmitting arrays, respectively, and α is the complex amplitude. We assume that the receive and transmit arrays are uniform, linear and that we choose the reference point for the arrays such that $\mathbf{a}_R^H\dot{\mathbf{a}}_R = 0, \mathbf{a}_T^H\dot{\mathbf{a}}_T = 0$, where $\dot{\mathbf{a}}_R$ and $\dot{\mathbf{a}}_T$ denote the derivatives of $\mathbf{a}_R$ and $\mathbf{a}_T$, respectively, which means that the transmit and receive arrays share the same reference point. Also, we assume that $\mathbf{R} = \sigma^2\mathbf{I}_{N_R}$, where $\mathbf{I}_{N_R}$ denotes an identity matrix of size N_R. In practice, α is usually unknown. However, for simplicity of the demonstration of the main idea, in this paper we assume that it is known. According to (14), in order to find the optimal sequential transmit auto-correlation matrix, we need to calculate the eigenvector corresponding to the maximum eigenvalue of $\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right)$. For the model given in (19), and under the assumptions we made above, $\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right)$ is given by:

$$\begin{aligned}&\mathbf{\Gamma}\left(\mathbf{X}^{(k-1)}\right) = \\ &= |\alpha|^2\,\mathrm{E}\left[\|\dot{\mathbf{a}}_R\|^2\left(\mathbf{a}_T\mathbf{a}_T^H\right)^* + \|\mathbf{a}_R\|^2\left(\dot{\mathbf{a}}_T\dot{\mathbf{a}}_T^H\right)^* \,\middle|\, \mathbf{X}^{(k-1)}\right].\end{aligned} \tag{20}$$

This result coincides with [1, 2] in the case of deterministic known θ, where $f_{\theta|\mathbf{X}^{(k-1)}}$ is given by a delta function at the true angle.

In the simulations, both the transmit and receive arrays are assumed to be uniform and linear with reference point chosen to be at the center of the arrays. We define the array signal-to-noise ratio (ASNR) as ASNR $\triangleq |\alpha|^2 PN_R/\sigma^2$, where P denotes the total transmitted power. In the simulations we use a uniform *a-priori* distribution i.e. $\theta \sim U\left(-\frac{\pi}{2}+\epsilon, \frac{\pi}{2}-\epsilon\right)$, where $\epsilon = 0.1$ rad . Notice that the BCRB does not exist for uniform prior distribution since the regularity conditions are not satisfied. Accordingly, we assume that I_P is constant over $\left(-\frac{\pi}{2}+\epsilon, \frac{\pi}{2}-\epsilon\right)$, which is an artificial, but reasonable assumption.

Fig. 1 illustrates the process of waveform adaptation by the proposed algorithm for various pulse indices, in which the target direction is fixed to $\theta = 30°$. The first and third rows in the figure show the posterior pdf, while the second and fourth rows show the beampattern defined as $p(\theta) = \mathbf{a}_T^H(\theta)\mathbf{R}_{\mathbf{S}_k}^*\mathbf{a}_T(\theta)$. In the simulations, we use $N_T = 7, N_R = 9$ antennas, with half wavelength inter-element spacing for both transmit and receive arrays, and the target has a unit complex amplitude $|\alpha| = 1$. We use ASNR of -6 dB. It can be seen that as the iteration step increases, the beampattern peak location is closer to the target direction similar to regular sum beam, as expected. Respectively, the spread in the posterior pdf's decreases, which means better estimation performance of θ.

Fig. 2 shows the root mean-square-error (RMSE) for estimation of θ as a function of the pulse index using the optimized auto-correlation matrix and uncorrelated waveforms, i.e. $\mathbf{R}_{\mathbf{S}_k} = (P/N_T)\mathbf{I}_{N_T}$ for each k. The same array configuration and signal wavelength as in the previous example were considered. We use 500 Monte Carlo trials with independent noise and target direction randomization. We use ASNR of -12 dB. In order to estimate the unknown parameter we used the minimum mean-square-error (MMSE) estimator. It can be seen that by using the sequential technique, the estimation performance are significantly better compared to using identical and orthonormal waveforms without optimization.

5. CONCLUSION

In this paper, we proposed a new technique for optimal adaptive sequential waveform optimization. Instead of transmission of identical waveforms, in the proposed technique, the waveform is adjusted at each step, in order to minimize the BCRB for system parameter estimation with respect to the transmit/input waveform. The proposed technique was tested via simulations for adaptive spatial transmit waveform design in the presence of a single target with a very weak ASNR. The simulations show that the proposed technique enables a significantly higher rate of reduction in the RMSE, compared to identical orthonormal waveform transmission.

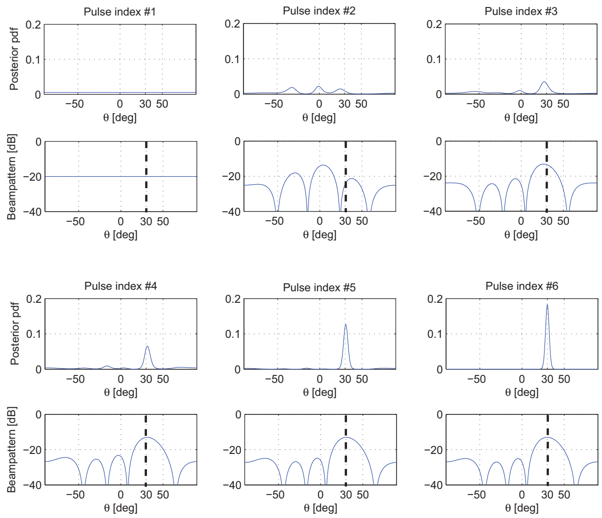

Fig. 1. Posterior pdf's (first and third rows) and optimal transmit beampatterns (second and fourth rows) against θ for various pulse steps, with $\theta = 30^{\circ}$ and ASNR = -6 dB.

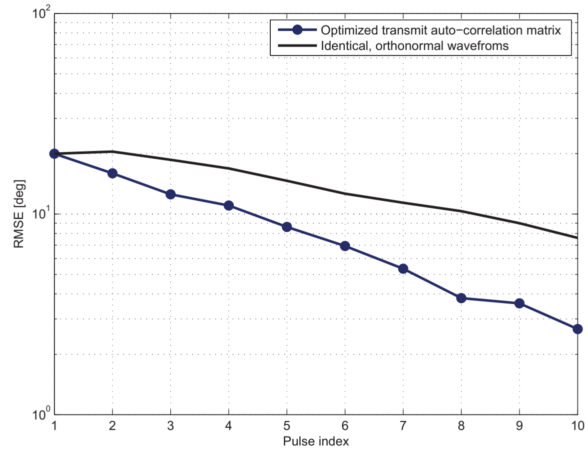

Fig. 2. Target direction estimation RMSE versus pulse index, with ASNR = -12 dB.

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